Chapter 8

Wavelet Transform (WT) & JPEG-2000

8.1 A Review of WT

8.1.1 Wave vs. Wavelet [castleman]

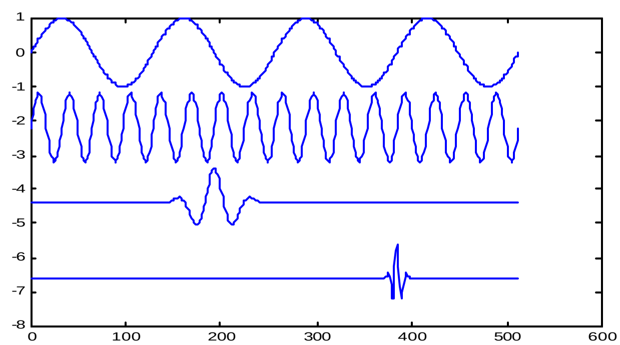

Figure 8.1 Sinusoidal waves (top two) and wavelets (bottom two) (horizontal axis: time; vertical axis: magnitude)

- Wave: Does not have compact support (extends to infinity)
- Transient signal: Have compact support (non-zero only in a short interval)
- Many image features (e.g., edges) highly localized in spatial position.

8.1.2 Trends vs. Anomaly [shapiro 1993]

Anomaly, burst, transient signal

8.1.3 Non-Statioanry Signal Analysis

- Stationary signal:

  properties not evolve in time

  FT is suitable

- Non-Stationary signal:

  Time-Frequency Analysis

- Time-Frequency Analysis
  - 2-D time-frequency space
    Can be derived from Figure 8.1
  - Preceding WT
  - Started with Gabor's **windowed FT**
  - $\Rightarrow$ short-time Fourier transform (STFT)
  - Another approach: WT

8.1.4 STFT vs. WT [rioul 1991]

- STFT:
  - Resolution in time and frequency
    cannot be arbitrarily small
  - Due to Heisenberg inequality:
    $\Delta t \cdot \Delta f \geq 1/(4p)$
  - Once window is chosen:
    Δt and Δf fixed – Figure 8.2 (left)

- Meaning anomaly (burst) and trend cannot be analyzed with good resolution (Δt or Δf) simultaneously

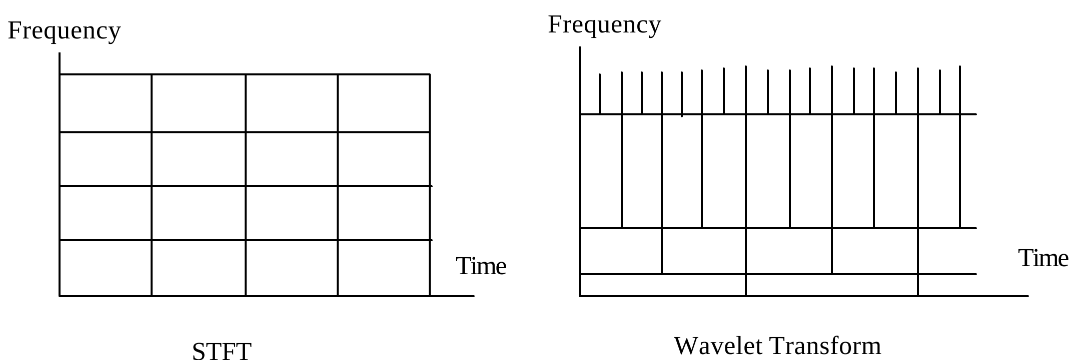

Figure 8.2 Comparison of STFT and WT in the time-frequency plane

- WT:
  - Constant relative bandwidth (const. Q): $\Delta f / f = constant$

- Meaning:
  - $\Delta t \downarrow$ as $f \uparrow (\Delta f \uparrow)$, and $\Delta f \downarrow$ as $f \downarrow$
  - as $f \uparrow$, obtain high time resolution
  - as $f \downarrow$, obtain high freq. resolution
  - Figure 8.2 (right)

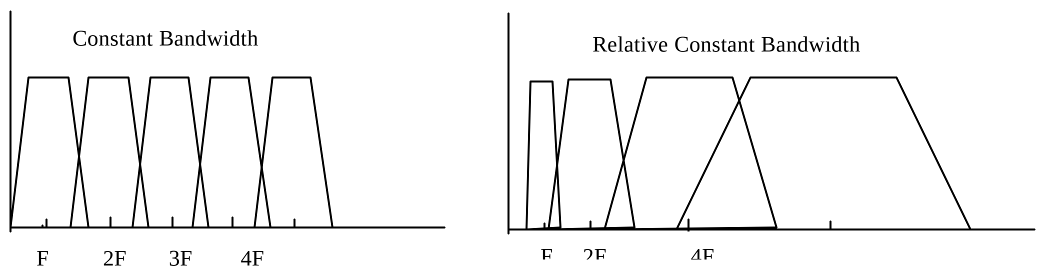

Figure 8.3 Constant bandwidth analysis (for FT) and relative constant bandwidth analysis (for WT)

<u>8.1.5 Example</u> [castleman]

- A two tone burst corrupted by random noise (Figure 8.4(a))

castlman figure 14.10 (a)

- FT: not easy to be interpreted (Figure 8.4(b))

  In particular, phase spectrum

castlman figure 14.10 (b)

- Filter Bank Theory (WT)

castleman Figure 4-12

Figure 8.5 Implementation of a bandpass filter bank

$$\sum_{\forall i} H_i(S) = 1 \quad \Rightarrow \quad \sum_{\forall i} g_i x) = f(x)$$

fig8.6

castleman Figure 14-14

Figure 8.6 Smooth bandpass filters: (a) impulse responses; (b) transfer functions

castleman Figure 14-15

Fig.8.7 Smooth bandpass filter bank output

8.1.6 Unification of Several Techniques

- ✓Filter Bank Analysis
- ✓Pyramid Coding
- ✓Subband Coding

8.1.7 Three Types of WT

- ✓CWT (Continuous WT)
- ✓Wavelet series expansion
- ✓DWT (Discrete WT)

8.1.8 DWT

- Most closely resembles unitary transforms
  Most useful in image compression

- Given a set of orthonormal basis fct.'s
  $\Rightarrow$ DWT just like unitary transform

- Orthonormal wavelets with compact support
  (by Daubechies):

$$\{{}_r y(x)\}=\{2^{j/2} r y(2^j x-k)\}$$

j,*k*: integers
compact support: [*0*,*2r*-*1*]
shift: *k*
dilation (scaling): 2^j

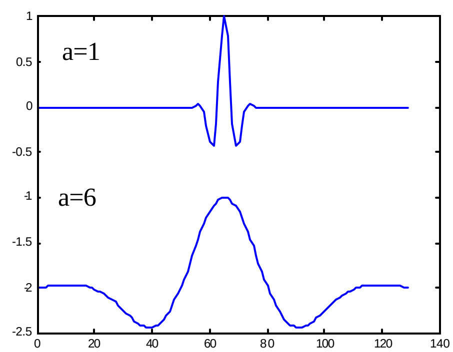

Figure 8.8: Scaling (A Morlet wavelet with a=1 (top) and a=6 (bottom))

- N-point signal $\Rightarrow$ N coefficients
  $N \times N$ image $\Rightarrow$ N^2 coefficients

8.2 DWT for Image Compression

8.2.1 Block Diagram

Similar to the other TC

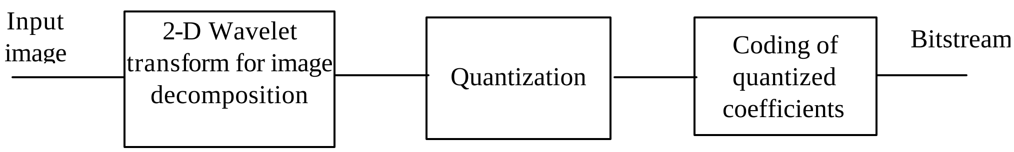

Figure 8.9: Block diagram of image coding with DWT

8.2.2 Image Decomposition

- Scale 1 (Figure 8.10 (a)):

  4 subbands: LL_1, HL_1, LH_1, HH_1

  Each coeff. $\leftrightarrow$ a 2×2 area in the original image

  Low frequencies: $0 < |w| < p/2$

  High frequencies: $p/2 < w < p$

- Scale 2 (Figure 8.8 (b)):

  4 subbands: LL_2, HL_2, LH_2, HH_2

Each coeff. ↔ a 2×2 area in scale 1 image

Low Frequency: $0<|w|<p/4$

High frequencies: $p/4<w<p/2$

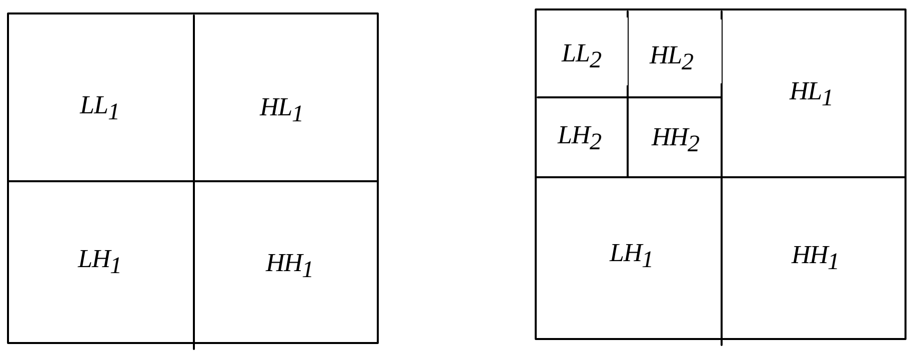

Figure 8.10 2-D DWT: (left) scale 1, and (right) scale 2 decomposition.

- At a coarser scale, coefficients represent a larger spatial area of the image but a narrow band of frequencies.

- Parent
- Children

- Descendants: corresponding coeff. at finer scales

- Ancestors: corresponding coeff. at coarser scales

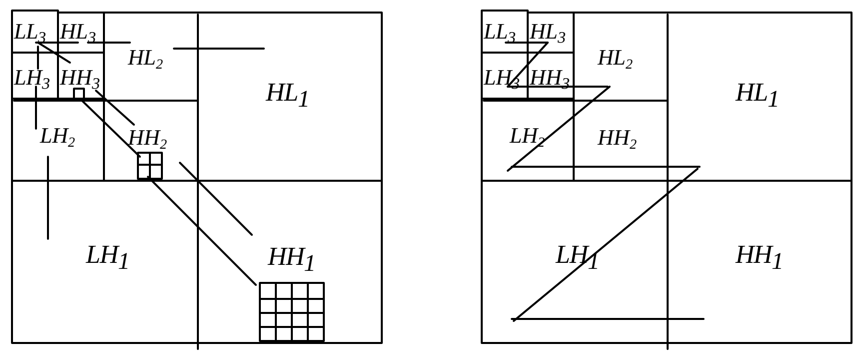

Figure 8.11 (a) Parent-children dependencies of subbands, arrow points from the subband of parents to the subband of children. (b) The scanning order of the subbands for encoding the significance map.

- Feature 1:

  Energy distribution similar to other TC:

  Concentrated in low frequencies

- Feature 2:
  Spatial self-similarity across subbands

8.2.3 DWT Image Compression

- Thresholding (deterministic)
  $\Rightarrow$ many zeros, data compression
- Differences from DCT Technique
  - In conventional TC
    - ✓Anomaly (edge) $\leftrightarrow$ many nonzero coeff.
      Insignificant energy
    - ✓TC allocates
      too many bits to "trend"
      few bits left to "anomalies"
    - ✓Problem at Very Low Bit-rate Coding:
      block artifacts

  - DWT
    - ✓Trends & anomalies infor. available

✓Major difficulty:

fine detail coefficients associated with anomalies ⇔the largest no. of coeff.

✓⇒ Problem:

how to efficiently represent *position* information?

✓Successful techniques

- Embedded zerotree wavelet (EZW) [shapiro 1993]
- Set partitioning in hierarchical trees (SPIHT) [said 1996]

8.3 EZW Image Coding [shapiro 1993]

8.3.1 Embedded Coding

Having all lower bit rate codes of the same image embedded at the beginning of the bit stream.

$\Leftrightarrow$ Bits are generated in order of importance.

$\Rightarrow$ Encoder can terminate encoding at any point, allowing a target rate to be met exactly

Suitable for scalability applications

8.3.2 Zerotree of DWT Coefficients

- Significance map:
  Binary decision as to a pixel = 0 or not
- Total encoding cost = cost of encoding singnificance map + cost of encoding nonzero values

- Target rate $\downarrow \Rightarrow$ more zero coeff. i.e.,

Cost of encoding position of nonzero coeff. $\uparrow$

♦ **An element of zerotree**:

A coeff.: itself and <u>all</u> of its descendants are insignificant w.r.t. thereshold T

- **Zerotree root**:

  An element of zerotree, & not a descendant of a zero element at a coarser scale

- **Isolated zero**:

  Insignificant, but has some significant descendant

- Significance map can be efficiently represented as a string of four symbols:
  - Zerotree root
  - Isolated zero
  - Positive significant coeff.
  - Negative significant coeff.

- Figure 8.12 Flow chart for encoding a coefficient of the significance map

Shapiro Figure 6

8.3.3 Comparison

- DCT coding

  Run-length (RLC) [within the same scale]
  End of block (EOB) [within the same scale]

- EZW coding

  - More efficient due to:
    Using self-similarity across different scales [e.g., zerotree root]

  - Higher quality of reconstructed image:

- ✓ Due to more efficient in position encoding
- ✓ No possibility that a significant coeff. be obscured by a statistical energy measure

- Experimental results reported

  "Barbara" image:

  2.4 dB better for same bit rate
  0.12 bpp savings for the same PSNR

8.4 Wavelet Transform for JPEG-2000

- DCT: core technique of image decomposition

  for most standards

- Things change recently:

- MPEG-4: adopt WT for still image coding [mpeg4].

- JPEG-2000: considering using WT as its core technique [jpeg2000].

- Reason: WT provides:
  - excellent coding efficiency, especially for VLBC
  - wonderful spatial and quality scalable functionality

- JPEG-2000 also aims at:

  accessing, manipulating, editing in compressed domain.

- Requirements of JPEG-2000 include:

  - <u>Low bit-rate compression performance</u>:

- ✓Lower than 0.25 bit/pixel for highly detailed gray level images
- ✓JPEG fails for the case
- ✓Primary feature of JPEG-2000.

- Lossless and lossy compression

- Large images: JPEG does not allow for images greater than 64K by 64K without tiling.

- Single decomposition architecture: JPEG has 44 modes. Desired to have a single decomposition architecture that can encompass interchange among applications.
- Transmission in noisy environments:
  - ✓error robustness,
  - ✓important for wireless communication.
- Computer generated imagery

- Compound documents
  - ✓both continuous-tone and bi-level images
  - ✓⇒ compress images from 1-bit to 16-bit for each color component
  - ✓JPEG does not work well for binary image
- Progressive transmission
  - ✓Allowing images be transmitted with increasing pixel resolution
  - ✓e.g., World Wide Web, image archiving.

- Real-time encoding and decoding
- Fixed-rate, fixed-size and limited workspace memory
- Other requirements:

  e.g., backwards compatibility with JPEG, interface with MPEG-4.

- All these requirements are seriously considered during the development of JPEG-2000.
- Too early, however, to comment whether all targets can be reached.
- No doubt:

  The core requirement on coding efficiency at VLBC will be achieved by using WT

- Seoul meeting [March 1999]:
  - Lots of changes
  - EBCOT (embedded block coding with optimized truncation)

References

[cohen 1989] L. Cohen, "Time-Frequency Distributions - A Review," Proceeding IEEE, Vol. 77, No. 7, July 1989, pp.941-981.

[daubechies 1992] I. Daubechies, *Ten Lectures on Wavelets*, CBMS series, Philadelphia, SIAM, 1992.

[grossman 1984] A. Grossman and J. Morlet, "Decompositions of hardy functions into square integrable wavelets of constant shape," SIAM Journal of Mathematical Analysis, July 1984, 15(4), pp.723-736.

[jpeg2000 vm] ISO/IEC JTC 1/SC 29/WG 1, JPEG 2000 Verification Model Version 1.2, August 14, 1998.

[mpeg4] ISO/IEC 14496-2, Coding of audio-visual objects, Nov. 1998.

[rioul 1991] O. Rioul and M. Vetterli, “Wavelets and signal processing,” IEEE Signal Processing Magazine, pp. 14-38, October 1991.

[said 1996] A. Said and W. A. Pearlman, "A new fast and efficient image codec based on set partitioning in hierarchical trees," IEEE Trans. on Circuits and Systems for Video Technology, June 1996, pp.243-250.

[shapiro 1993] J. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients," IEEE Trans. on Signal Processing, Dec. 1993, pp.3445-3462.

[vetterli 1995] M. Vetterli and J. Kovacevic, "Wavelets and subband coding," Englewood Cliffs, NJ: Prentice-Hall, 1995.