Chapter 3

DIFFERENTIAL CODING

- Instead of encoding a signal directly, the ***differential coding*** technique encodes the difference between the signal itself and its prediction. Therefore it is also known as ***predictive coding***.

- Assume: 8 bits/sample in quantization.

  Difference signal: between pixels

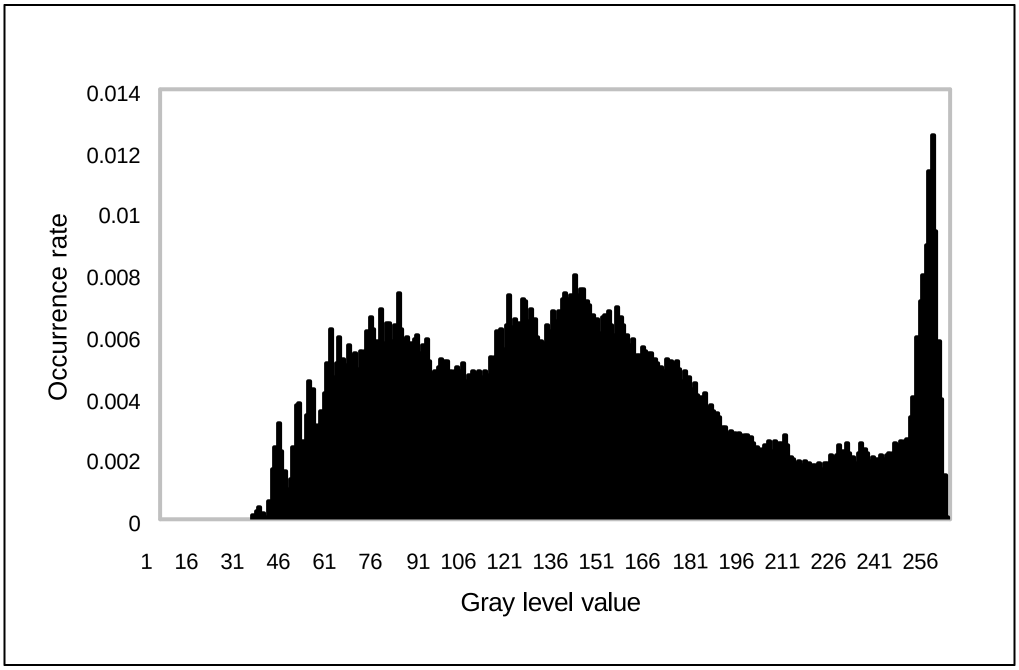
  Then: dynamic range: from 256 to 512

  variance: actually much smaller.

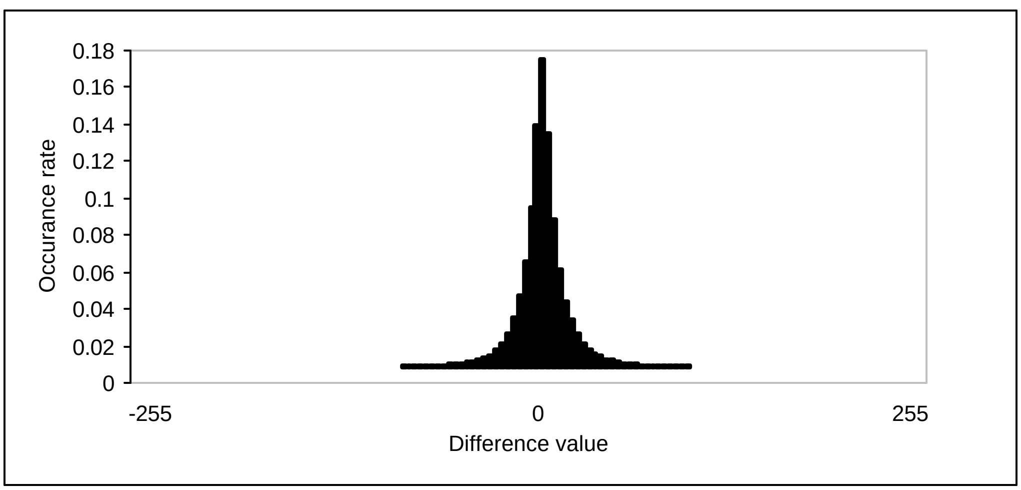
- $\Rightarrow$ **efficient**

  and yet **computationally simple**

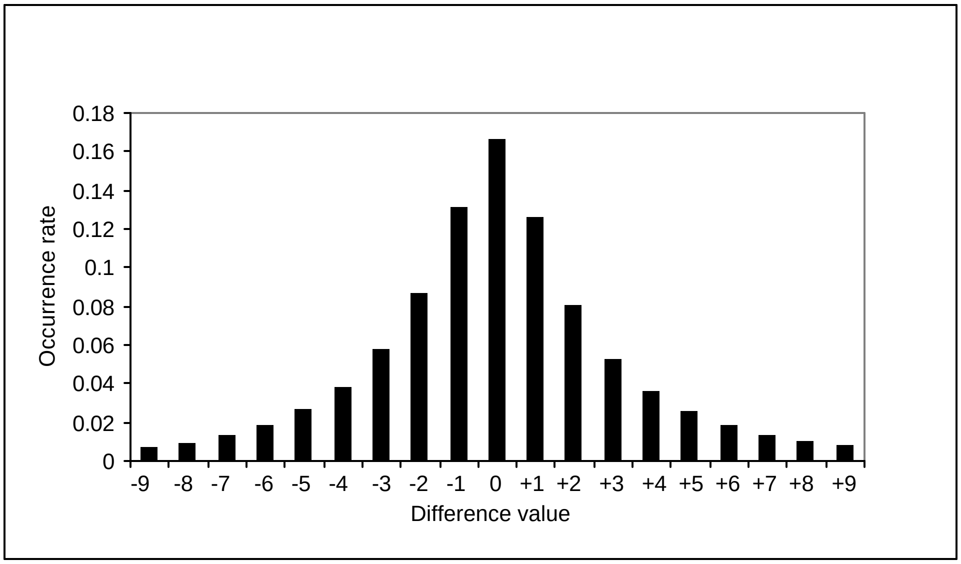
"Boy and Girl" image



(a)



(b)



(c)

Figure 3.1 (a) Histogram of the original “Boy and Girl” image. (b) Histogram of the difference image obtained by using horizontal pixel-to-pixel differencing. (c) A close-up of the central portion of the histogram of the difference image.

3.1 Introduction to DPCM

3.1.1 Simple Pixel-to-Pixel DPCM

- Notations:
  - $z_i, i=1,\cdots,M$: gray level values of pixels along a row of an image
  - $\hat{z}_i$: a prediction of gray level value of the present pixel,
  - $d_i = z_i - \hat{z}_i$: the difference signal
  - $\hat{d}_i$: quantized version of the difference
  - $\hat{d}_i = Q(d_i) = d_i + e_q$

    e_q: quantization error
  - $\bar{z}_i = \hat{z}_i + \hat{d}_i$: reconstructed gray value

- Now, let: $\hat{z}_i = \bar{z}_{i-1}$

  (previous reconstructed = prediction)

- So, $d_i = z_i - \bar{z}_{i-1}$

  $\bar{z}_i = \bar{z}_{i-1} + \hat{d}_i$

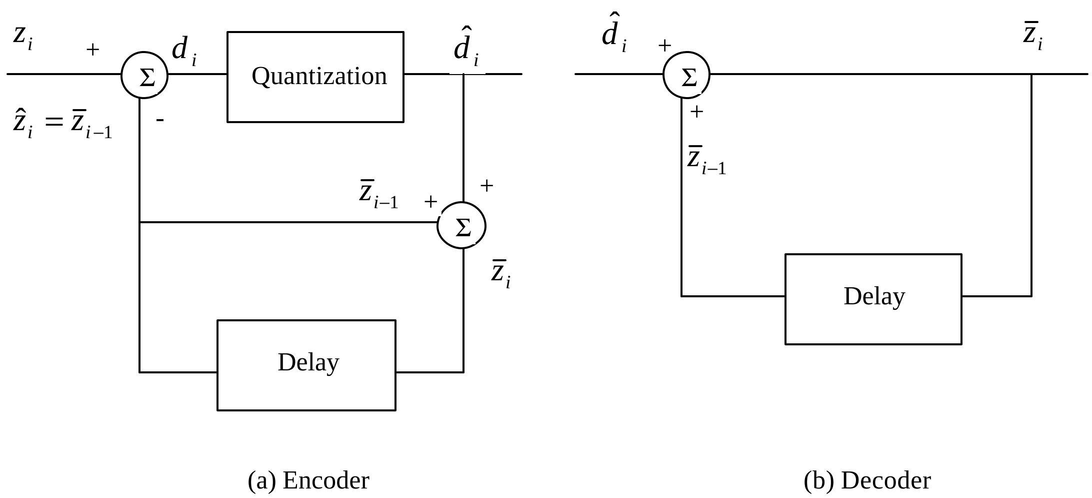


Figure 3.2 Block diagram of a practical pixel-to-pixel differential coding system

In this way, we have

$$\begin{aligned} as \quad i=1, \quad & d_1 = z_1 - z_0 \\ & \hat{d}_1 = d_1 + e_{q,1} \\ & \bar{z}_1 = z_0 + \hat{d}_1 = z_0 + d_1 + e_{q,1} = z_1 + e_{q,1}. \end{aligned} \qquad (3.\ 1)$$

Similarly, we have

$$\begin{aligned} as \quad i=2, \quad & d_2 = z_2 - \bar{z}_1 \\ & \hat{d}_2 = d_2 + e_{q,2} \\ & \bar{z}_2 = \bar{z}_1 + \hat{d}_2 = z_2 + e_{q,2}. \end{aligned} \qquad (3.\ 2)$$

- In general,

$$\bar{z}_i = z_i + e_{q,i} \qquad (3.\ 3)$$

$\Rightarrow$ Error will not be accumulated.

3.1.2 <u>General DPCM Systems</u>

$$\hat{z}_i = f(\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n}). \qquad (3.\ 4)$$

- **Linear prediction**, i.e., the function f in Equation 3.14 is linear, is of particular interest and is widely used in differential coding.

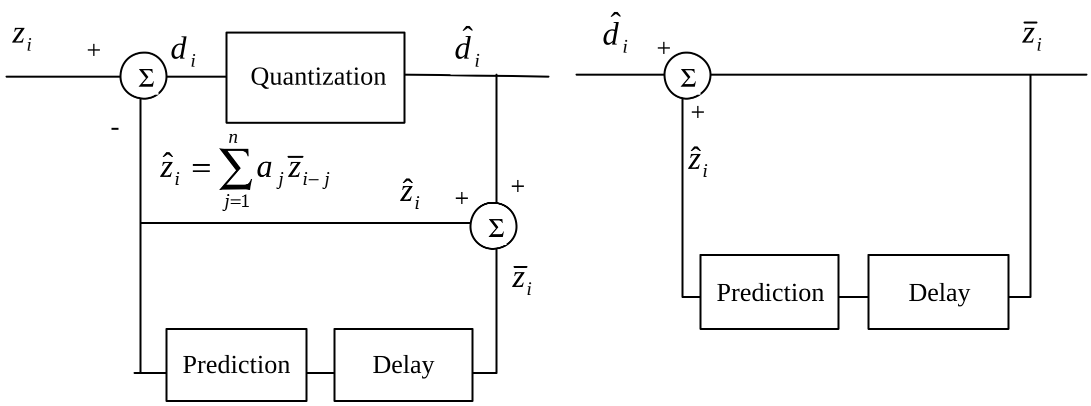


Figure 3. 3 Block diagram of a general DPCM system

Prediction error, e_p :

The difference between the original input and the predicted input is called prediction error. That is,

$$e_p = z_i - \hat{z}_i, \qquad (3.\ 5)$$

Reconstruction error (or coding error) e_r :

Difference between the original signal, z_i, and the reconstructed signal, $\bar{z}_i$.

$$e_r = z_i - \bar{z}_i$$

Quantization error is equal to the reconstruction error or coding error when the transmission is error free [gish 1967]:

$$\begin{aligned} e_q &= d_i - \hat{d}_i \\ &= (z_i - \hat{z}_i) - (\bar{z}_i - \hat{z}_i) \\ &= z_i - \bar{z}_i = e_r . \end{aligned} \qquad (3.6)$$

Meaning: quantization error is the only source of information loss with an error free transmission channel.

Another name:

The DPCM system depicted Figure 3.4 is also called closed-loop DPCM with feedback around the quantizer [jayant 1984]. This term reflects the feature in DPCM structure.

History:

The first theoretical and experimental approaches to image coding involving linear prediction began in 1952 at the Bell Telephone Laboratories [oliver 1952, kretzmer 1952, harrison 1952].

The concepts of DPCM and DM were also developed in 1952 [cutler 1952 and dejager 1952].

Predictive coding capable of preserving information for a PCM signal was established at the Massachusetts Institution of Technology [elias 1955].

An excellent survey article on differential image coding [musmann 1979].

Appilcations: The differential coding technique has played an important role in image and video coding.

In the international coding standard for still images, JPEG, the differential coding is used in lossless mode,

in DCT-based mode for coding DC coefficients.

In all the international video coding standards, such as H.261 and H.263, MPEG ½:

Motion compensated (MC) coding is essentially predictive coding.

3.2 Optimum Linear Prediction

3.2.1 Formulation

Consider a discrete-time random process z. At a typical moment i, it is a random variable z_i. We have n previous observations $\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n}$ available and would like to form a prediction of z_i, denoted by $\hat{z}_i$. The output of the predictor, $\hat{z}_i$, is a linear function of the n previous observations. That is,

$$\hat{z}_i = \sum_{j=1}^{n} a_j \bar{z}_{i-j}, \tag{3.7}$$

with $a_j, j = 1,2,\cdots,n$ being a set of real coefficients.

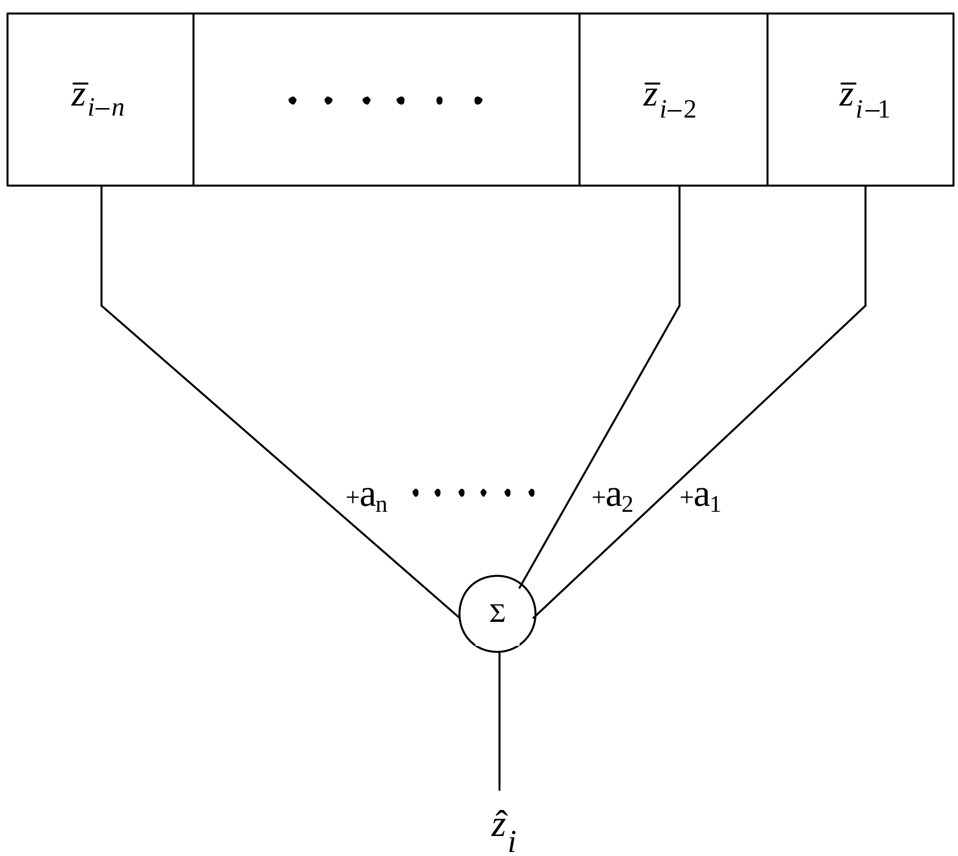


Figure 3. 4 An illustration of a linear predictor.

The mean square prediction error, MSE_p, is

$$MSE_p = E[(e_p)^2] = E[(z_i - \hat{z}_i)^2] \qquad (3.8)$$

The optimum prediction then refers to the determination of a set of coefficients $a_j, j = 1,2,\cdots,n$ such that the mean square prediction error, MSE_p, is minimized.

This optimization problem turns out to be computationally intractable for most practical cases due to the feedback around the quantizer shown in Figure 3.4, and the nonlinear nature of the quantizer.

Therefore, the optimization problem is solved in two separate stages.

The best linear predictor is first designed ignoring the quantizer.

Then, the quantizer is optimized for the distribution of the difference signal [habibi 1971].

Although the predictor thus designed is suboptimal, ignoring the quantizer in the optimum predictor design allows us to substitute the reconstructed $\bar{z}_{i-j}$ by z_{i-j} for $j = 1, 2, \cdots, n$, according to Equation 3.19.

Consequently, we can apply the theory of optimum linear prediction to handle the design of the optimum predictor as shown below.

3.2.2 Orthogonality Condition and Minimum Mean Square Error

By taking the differentiation of MSE_p with respect to coefficient a_j s, one can derive the following necessary conditions, which are usually referred to as the ***orthogonality condition***.

$$E[e_p \cdot z_{i-j}] = 0 \quad for \quad j = 1, 2, \cdots, n. \qquad (3.\ 9)$$

The interpretation of Equation 3.23 is that the prediction error, e_p, must be orthogonal to all the observations, which are now the preceding samples: z_{i-j}, $j = 1, 2, \cdots, n$.

These are equivalent to

$$R_z(m) = \sum_{j=1}^{n} a_j R_z(m-j) \quad for \quad m = 1,2,\cdots,n, \qquad (3.10)$$

where R_z represents the autocorrelation function of z.

In a vector-matrix format, the above orthogonal conditions can be written as

$$\begin{bmatrix} R_z(1) \\ R_z(2) \\ \vdots \\ \vdots \\ R_z(n) \end{bmatrix} = \begin{bmatrix} R_z(0) & R_z(1) & \cdots & \cdots & R_z(n-1) \\ R_z(1) & R_z(2) & \cdots & \cdots & R_z(n-2) \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ R_z(n-1) & R_z(n) & \cdots & \cdots & R_z(0) \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix} \qquad (3.11)$$

Equations 3.24 and 3.25 are called **Yule-Walker equations**.

The minimum mean square prediction error is then found to be

$$MSE_p = R_z(0) - \sum_{j=1}^{n} a_j R_z(j). \qquad (3.12)$$

3.2.3 Solution to Yule-Walker Equations

Once autocorrelation data are available, the Yule-Walker equation can be solved by matrix inversion.

A recursive procedure was developed by Levinson to solve the Yule-Walker equations [leon-garcia 1994]. When the number of previous samples used in the linear predictor is large, i.e., the dimension of the matrix is high, the Levinson recursive algorithm becomes more attractive.

Note that in the field of image coding the autocorrelation function of various types of video frames is derived from measurements [o'neal 1966, habibi 1971].

3.3 Some Issues in the Implementation of DPCM

3.3.1 1-D, 2-D and 3-D DPCM

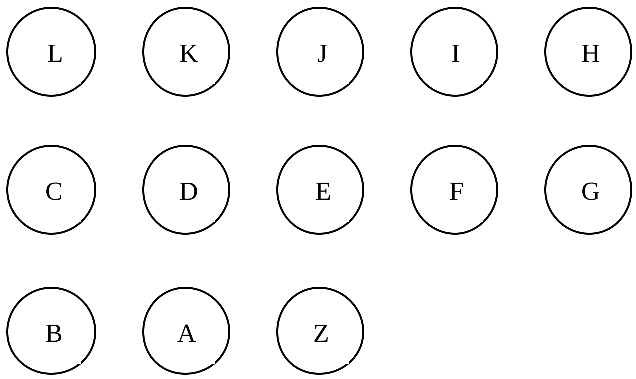


Figure 3.5 Pixel arrangement in 1-D and 2-D prediction

1-D DPCM may use the reconstructed gray level values of more than one preceding pixels **within the same scan line** to predict that of a pixel being coded.

By far, however, the immediately preceding pixel in the same scan line is most frequently used in 1-D DPCM. That is, pixel A in Figure 3.6 is often used as a prediction of pixel Z, which is being DPCM coded.

2-D DPCM: Sometimes in DPCM image coding, both the decoded intensity values of adjacent pixels within the same scan line and the decoded intensity values of neighboring pixels in the **different scan lines** are involved in the prediction. This is called 2-D DPCM.

A typical pixel arrangement in 2-D predictive coding is shown in Figure 3.6.

Only those pixels, which have been coded, available in both the transmitter and the receiver, are used in the prediction.

In 2-D system theory, this support is referred to as **recursively_computable** [bose 1982]. An often used 2-D prediction involves pixels A, D, and E.

2-D predictive coding outperforms 1-D predictive coding by decreasing the prediction error by a factor of two, or equivalently 3dB in *SNR*. The improvement in subjective assessment is even larger [musmann 1979].

the transmission error in 2-D predictive image coding is much less severe than in 1-D predictive image coding. This is discussed in Section 3.6.

3-D DPCM: In the context of image sequences, neighboring pixels along the time dimension are also involved. If the prediction of a DPCM system involves three types of neighboring pixels: those along the same scan line, those in the same image frame, and those **in the different frames**, the DPCM is then called 3-D differential coding.

3.3.2 Order of Predictor

The number of coefficients in the linear prediction, n, is referred to as the order of the predictor.

The relation between the MSE_p and the order of the predictor, n:

Figure 3.7

The MSE_p decreases as n increases quite effectively

The performance improvement becomes negligible as $n>3$ [habibi 1971].

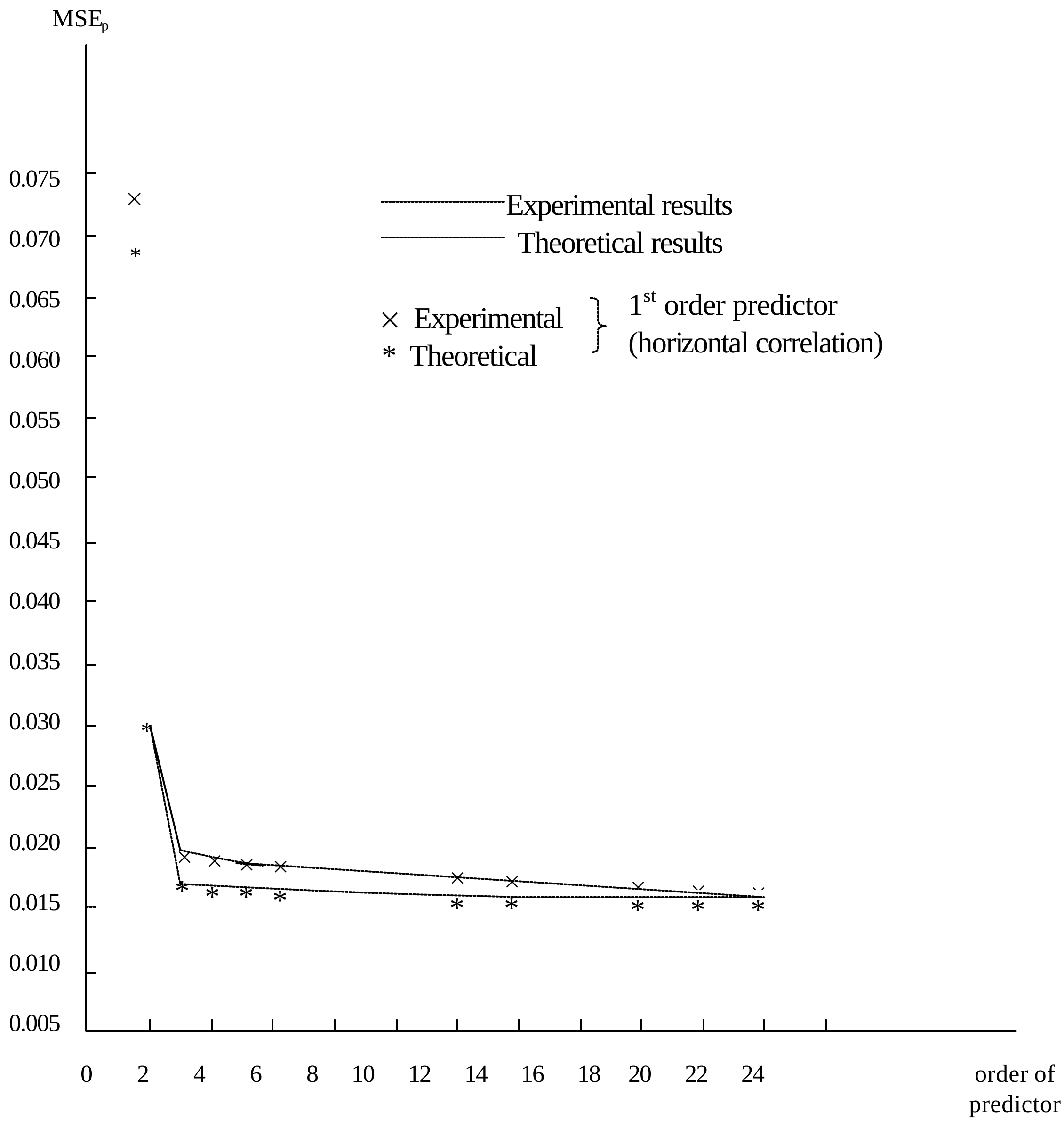


Figure 3.6 Mean square prediction error versus order of predictor after [habibi 1971].

3.3.3 Adaptive Prediction

Adaptive prediction can be done in two different ways:

Forward adaptive prediction

Based on the input of a DPCM system

More sensitive to variation of local statistics

Side information: prediction coefficients

Backward adaptive prediction

Based on the output of the DPCM

Less sensitive to variation of local statistics

No side inforamtion

In either case, the data (either input or output) has to be buffered. Autocorrelation coefficients are analyzed, based on which the prediction parameters are determined.

3.3.4 Effect of Transmission Errors

Transmission error caused by channel noise:

reverse the binary bit information from 0 to 1 or 1 to 0 with what is known as *bit error probability*, or *bit error rate*.

The effect of transmission error on reconstructed images varies depending on different coding techniques.

In the case of the PCM-coding technique,

each pixel is coded independently of the others. Therefore bit reversal in the transmission only affects the gray level value of the corresponding pixel in the reconstructed image. It does not affect other pixels in the reconstructed image.

In DPCM, however, the effect caused by transmission errors becomes more severe.

The transmission error propagates.

It is reported that the error propagation is more severe in 1-D differential image coding than in 2-D differential coding.

In 1-D differential coding, an error will be propagated along the scan line until the beginning of the next line, where the pixel gray level value is reinitialized.

In 2-D differential coding, the prediction of a pixel gray level value depends not only on the reconstructed gray level values of pixels along the same scan line but also on the reconstructed gray level values of the vertical neighbors. Hence, the effect caused by a bit reversal transmission error is less severe than in the 1-D differential coding.

For this reason, the bit error rate required by DPCM coding is lower than that required by PCM coding. For instance, while a bit error rate less $5 \cdot 10^{-6}$ is normally required for PCM to provide broadcast TV quality, for the same application a bit error rate less than 10^{-7} and 10^{-9} are required for DPCM coding with 2-D prediction and 1-D prediction, respectively [musmann 1979].

- Channel encoding with an error correction capability was applied to lower the bit error rate. For instance, to lower the bit error rate from the order of 10^{-6} to the order of 10^{-9} for DPCM coding with 1-D prediction, an error correction code by adding 3% redundancy in channel coding has been used [bruders 1978].

3.4 Delta Modulation (DM)

DM: an important, simple, special case of DPCM

It has been widely applied

DM is essentially a special type of DPCM, with the following two features.

The linear predictor is of the first order, with the coefficient a_1 equal to 1.

The quantizer is a one-bit quantizer. That is, depending on whether the difference signal is positive or negative, the output is either $+\Delta/2$ or $-\Delta/2$.

$$\hat{z}_i = \bar{z}_{i-1}. \qquad (3.13)$$

$$\hat{d}_i = \begin{cases} +\Delta/2 & if \quad z_i > \bar{z}_{i-1} \\ -\Delta/2 & if \quad z_i < \bar{z}_{i-1} \end{cases}. \qquad (3.14)$$

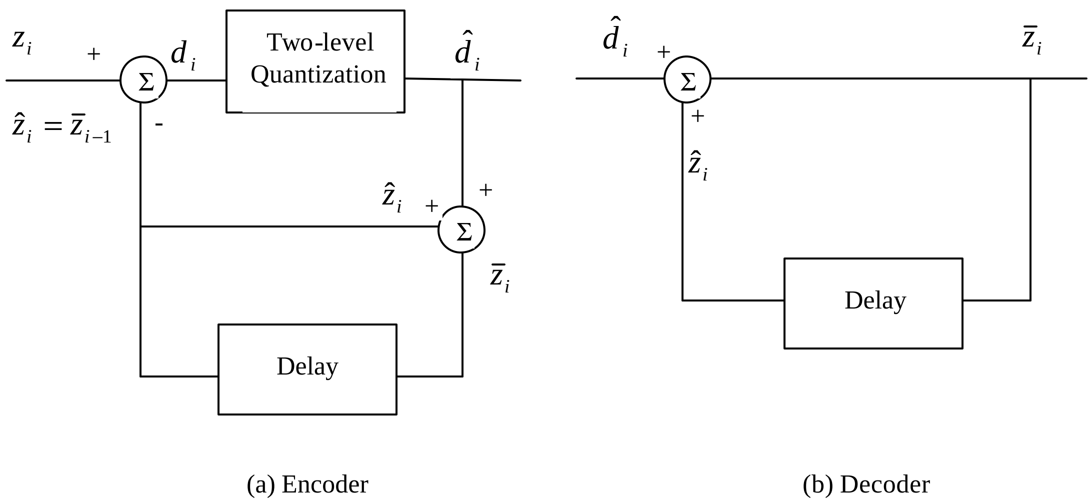


Figure 3. 7 Block diagram of DM systems.

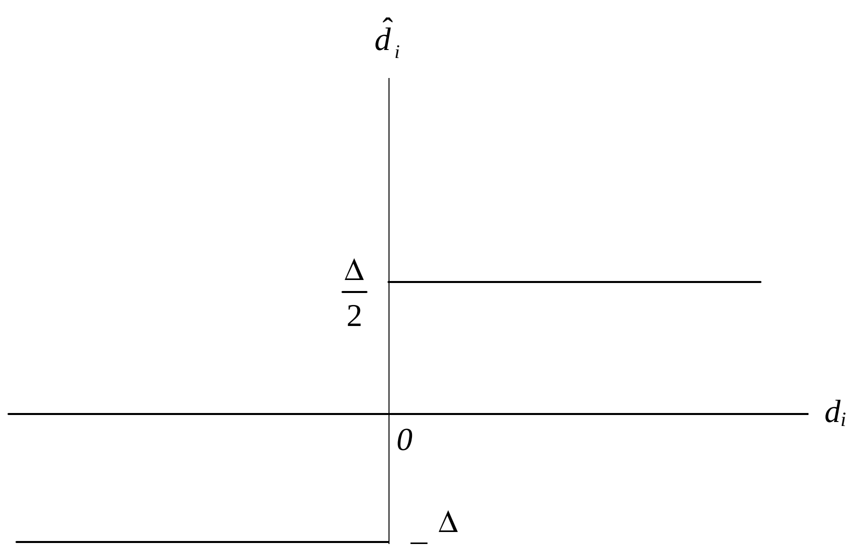


Figure 3. 8 Input-output characteristic of two-level quantization in DM.

$$\bar{z}_i = \hat{z}_i + \hat{d}_i. \qquad (3.15)$$

Combining Equations 3.28, 3.29, and 3.30, we have

$$\bar{z}_i = \begin{cases} \bar{z}_{i-1} + \Delta/2 & if \quad z_i > \bar{z}_{i-1} \\ \bar{z}_{i-1} - \Delta/2 & if \quad z_i < \bar{z}_{i-1} \end{cases}. \qquad (3.16)$$

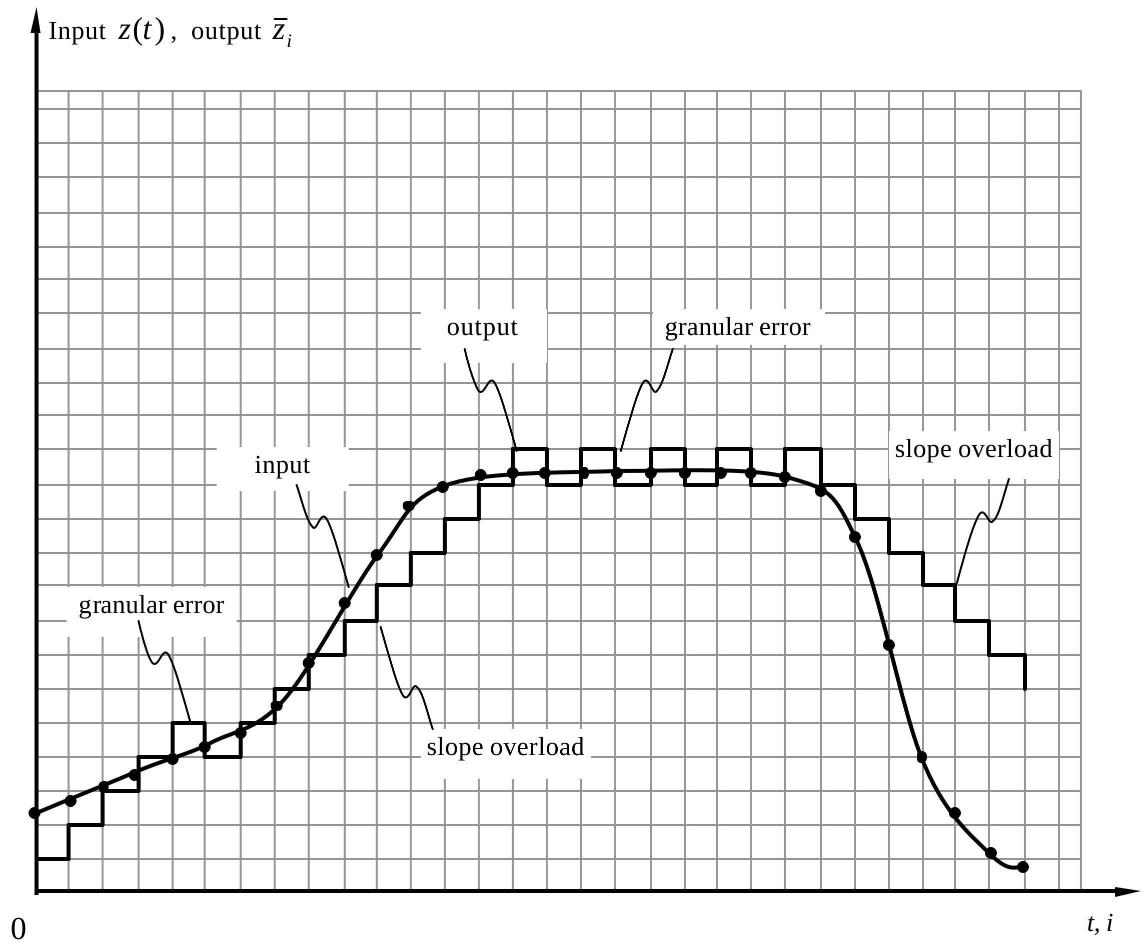


Figure 3.9 DM with fixed step size.

To improve the performance of DM, **an oversampling technique** is often applied. That is, the input is oversampled prior to the application of DM. By oversampling, we mean that the sampling frequency is higher than the sampling frequency used in obtaining the original input signal. The increased sample density caused by oversampling decreases the magnitude of the difference signal. Consequently, a relatively small step size can be used so as to decrease the granular noise without increasing the slope overload error. At the last, the resolution of the DM coded image is kept the same as that of the original input [jayant 1984, lim 1989].

Adaptive technique:

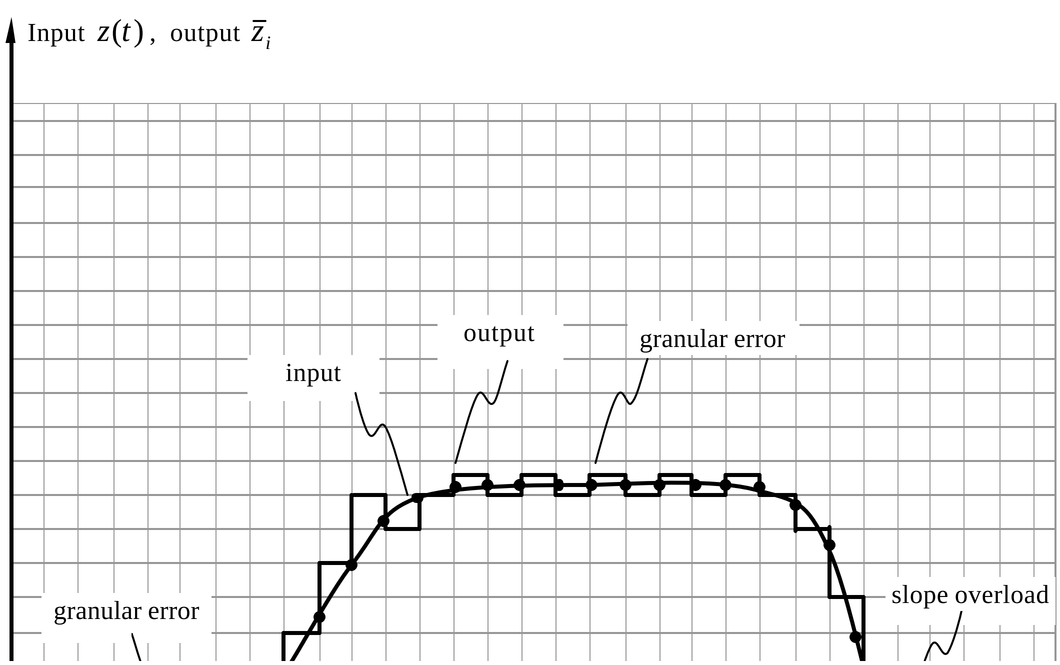


Figure 3.10 Adaptive DM.

3.5 Interframe Differential Coding

3-D differential coding involves an image sequence.

Consider applications such as videophony and videoconferencing,

The sensor is fixed in position for a while and it takes pictures. As time goes by, the images form a temporal image sequence.

The coding of such an image sequence is referred to as **interframe coding**.

The subject of image sequence and video coding is addressed in Part IV. In this section, we briefly discuss how differential coding is applied to interframe coding.

3.5.1 Conditional Replenishment

Frame replenishment (FR) [mounts 1969]:

the first real demonstrations of interframe coding exploiting interframe redundancy [netravali 1979].

In this scheme, the previous frame is used as a reference for the present frame.

Consider a pair of pixels: one in the previous frame, the other in the present frame -both occupying the same spatial position in the frames.

If the gray level difference between the pair of pixels exceeds a certain criterion, then the pixel is considered a ***changing*** pixel.

The present pixel gray level value and its position information are transmitted to receiving side, where the pixel is replenished.

Otherwise, the pixel is considered ***unchanged***. At receiver its previous gray level is repeated.

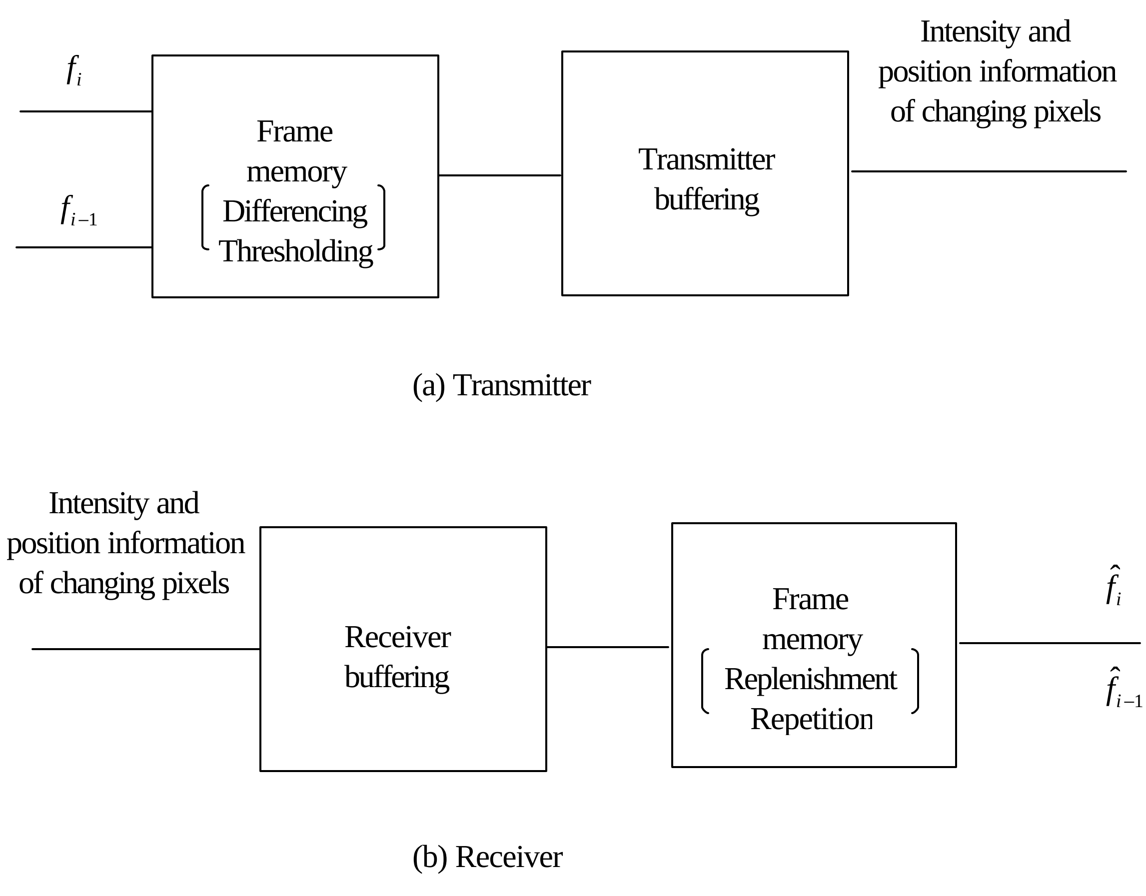


Figure 3. 11 Block diagram of conditional replenishment

A buffer in the transmitter is used to smooth the transmission data rate. This is necessary because the data rate varies from region to region within an image frame and from frame to frame within an image sequence.

Experiments in real time using the head-and-shoulder view of a person in animated conversation as the video source demonstrated **an average bit rate of 1 bit/pixel** with a quality of reconstructed video comparable with standard 8 bit/pixel PCM transmission [mount 1969].

Compared with pixel-to-pixel 1-D DPCM, the most popularly used coding technique at the time, conditional replenishment technique is more efficient due to the exploitation of high interframe redundancy.

As pointed in [mount 1969] there is more correlation between television pixels along the frame-to-frame temporal dimension than there is between adjacent pixels within a signal frame. That is, the temporal redundancy is normally higher than spatial redundancy for TV signals.

Tremendous efforts have been made to improve the efficiency of this rudimentary technique.

For an excellent review, readers are referred to [haskell 1972,1979].

3-D DPCM coding is among the improvements and is discussed next.

3.5.2 3-D DPCM

It is soon realized that it is more efficient to transmit the gray level difference than to transmit the gray level itself, resulting in **interframe differential coding**.

Furthermore, instead of treating each pixel independently of its spatial neighboring pixels, it is more efficient to utilize spatial redundancy as well as temporal redundancy, resulting in 3-D DPCM.

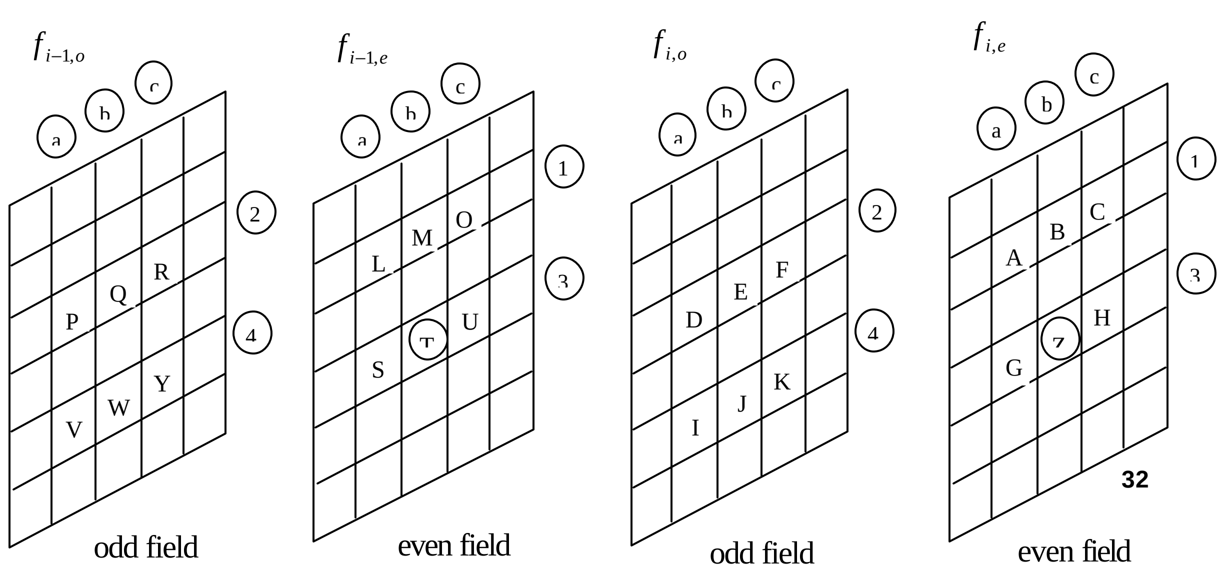


Figure 3. 12 Pixel arrangement in two TV frames after [haskell 1979].

The prediction can only be based on the previously encoded pixels.

Table 3. 1 Some linear prediction schemes after [haskell 1979]

	Original signal (Z)	Prediction signal ($\hat{Z}$)	Differential signal (d_z)
Element difference	Z	G	$Z-G$
Field difference	Z	$\frac{E+J}{2}$	$Z-\frac{E+J}{2}$
Frame difference	Z	T	$Z-T$
Element difference of frame difference	Z	$T+G-S$	$(Z-G)-(T-S)$

Line difference of frame difference	Z	$T+B-M$	$(Z-B)-(T-M)$
Element difference of field difference	Z	$T+\frac{E+J}{2}-\frac{Q+W}{2}$	$(Z-\frac{E+J}{2})-(T-\frac{Q+W}{2})$

It was found [haskell 1979] that the element difference of field difference generally corresponds to the lowest entropy, meaning that this prediction is the most efficient.

The frame difference and element difference correspond to higher entropy.

It is recognized that, in the circumstances, transmission error will be propagated if the pixels in the previous line are used in prediction [connor 1973]. Hence, the linear predictor should use only pixels from the same line or the same line in the previous frame when bit reversal error in transmission needs to be considered.

Combining these two factors, the element difference of frame difference prediction is preferred.

3.5.3 Motion Compensated Predictive Coding

When frames are taken densely enough, changes in successive frames can be attributed to the motion of objects during the interval between frames.

Under this assumption, if we can analyze object motion from successive frames, then we should be able to predict objects in the next frame based on their positions in the previous frame and the estimated motion.

The difference between the original frame and the predicted frame thus generated, and the motion vectors are then quantized and coded.

If the motion estimation is accurate enough, the motion compensated prediction error can be smaller than 3-D DPCM.

In other words, the variance of the prediction error will be smaller, resulting in more efficient coding.

Take motion into consideration - this differential technique is called motion compensated predictive coding.

This has been a major development in image sequence coding since the 1980s.

It has been adopted by all international video coding standards. A more detailed discussion is provided in Part IV.

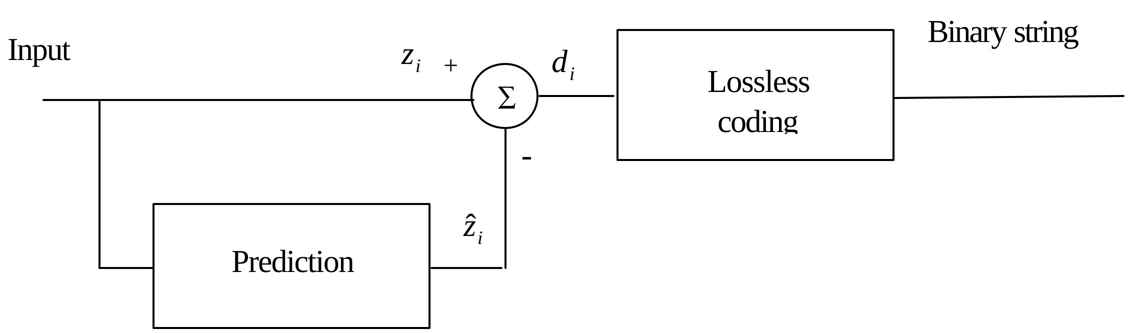
3.6 Information Preserving Differential Coding

DPCM involves quantization and, hence, is lossy coding.

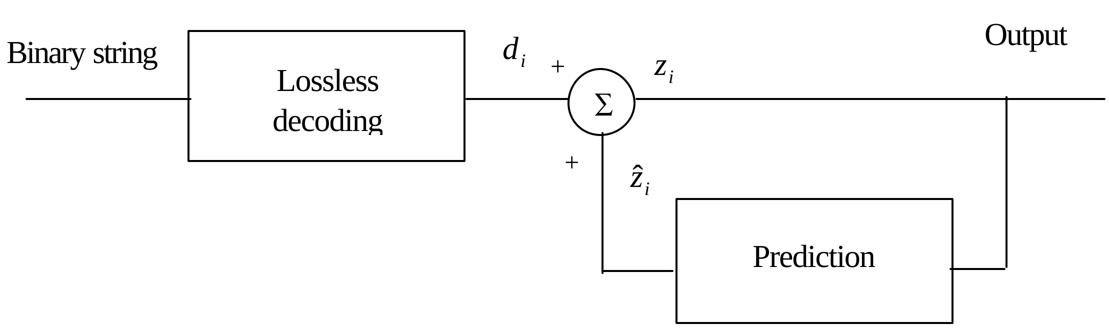
Information reserving – Lossless coding.

In applications such as those involving scientific measurements, information preserving is required.

Lossless differential coding:



(a) Encoder



(b) Decoder

Figure 3. 13 Block diagram of information preserving differential coding

First, there is no quantizer.

Second,, the differential (predictive) technique still applies.

Third, an efficient lossless coder is utilized.

Huffman coding

Arithmetic coding

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