ECE 643 Digital Image Processing I

Chapter 5

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Chapter 5
Image Restoration and Reconstruction

Introduction

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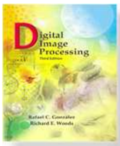

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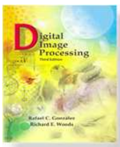
- *Image restoration* and *image enhancement* share a common goal: to improve image for human perception
- Image enhancement is mainly a subjective process in which individuals' opinions are involved in process design.
  - For instance: Image sharpening

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).



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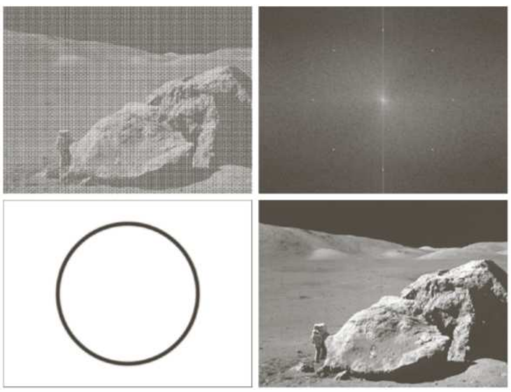
- Image restoration is mostly an objective process which
  - utilizes a prior knowledge of degradation phenomenon to recover image.
  - models the degradation and then to recover the original image.
    - For instance: Image denoising

a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)



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5.1 A Model of the Image Degradation/Restoration Process

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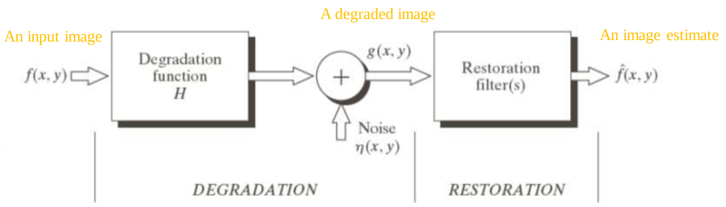
Image Restoration and Reconstruction

FIGURE 5.1 A model of the image degradation/restoration process.

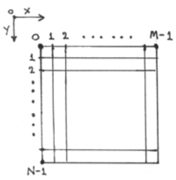


- If H is a linear, position-invariant process (filter), the degraded image is given in the spatial domain by

$$g(x,y)=h(x,y)\otimes f(x,y)+\eta(x,y)$$

- whose equivalent frequency domain representation is

$$G(u,v)=H(u,v)\bullet F(u,v)+N(u,v)$$



The frame of reference

- The objective of restoration is to obtain an image estimate which is as close as possible to the original input image.
- A typical difference measurement is the *mean square error (MSE)*:

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

- Generally, the more H and noise are known, the lower MSE will become.

Remark: In Sections 5.2, 5.3 and 5.4, H is assumed to be the identity operator.

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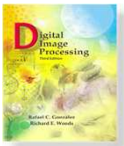
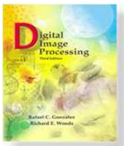
2. Noise Models



- The principal sources of noise in digital images arise during:
  - Image <u>acquisition</u>
    - For instance, with a CCD camera, light levels and sensor temperature introduce noise to the resulting image.
  - Image <u>transmission</u>
    - For instance, an image transmitted over a wireless network might be corrupted as a result of lighting or other atmospheric disturbance.
- Noise:
  - Negative side: noise degrades image quality.
  - <u>Positive side</u>: camera noise pattern can be useful in digital image forensics such as camera identification.



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5.2.1 Noise Models
—Spatial and Frequency Properties of Noise

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- ❖ Spatial properties:
  - Spatial periodicity of noise
  - Spatial dependency between noise and image
    - o Noise is assumed herein to be <u>independent</u> of spatial coordinates

      Or, there is no correlation between pixel values and the values of noise components.
    - o This assumption is invalid in some applications: X-ray, nuclear-medicine imaging and so on.
- ❖ Frequency properties:
  - Frequency content of noise in the Fourier sense
    - o For example, if the Fourier spectrum of noise is constant, the noise is usually called *white noise*.

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5.2.2 Noise Models

- Some Important Noise Probability Density Functions

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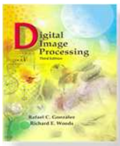

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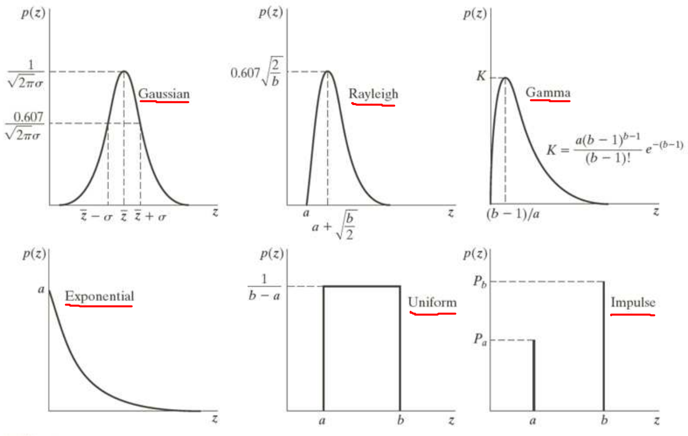
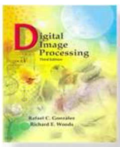


FIGURE 5.2 Some important probability density functions.



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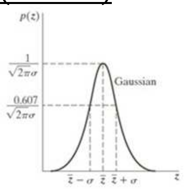
Gaussian (normal) noise



$$p(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\bar{z})^2/2\sigma^2}$$

z represents intensity

$\bar{z}$ is the mean of z

σ is the standard deviation of z

σ^2 is the variance of z

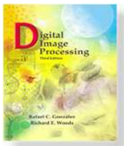
−frequently used in practice since it is mathematically tractable in both the spatial and frequency domains

−70% of z's values fall into the range $[(\bar{z}-\sigma),(\bar{z}+\sigma)]$

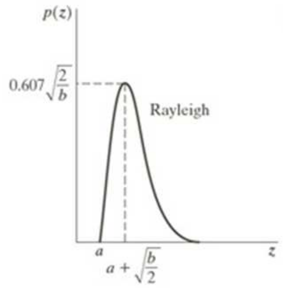
−95% of z's values fall into the range $[(\bar{z}-2\sigma),(\bar{z}+2\sigma)]$

−arising in an image due to factors such as electronic circuit noise and sensor noise due to poor illumination and/or high temperature

−Central limit theorem

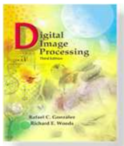


Raleigh noise



$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \qquad \bar{z} = a + \sqrt{\pi b/4} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

−Displacement from origin; and skewed to the right; useful for approximating skewed histograms
−characterizing noise phenomena in range imaging



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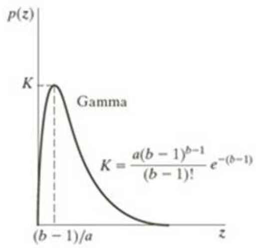
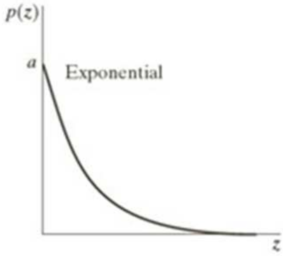
Erlang (Gamma) noise



$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!}e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad \bar{z} = \frac{b}{a}, \quad \sigma^2 = \frac{b}{a^2} \qquad a \in R^+, b \in Z^+$$

−developed by Erlang to model telephone traffics
−called Gamma noise if the denominator is the gamma function, $\Gamma(b)$
−useful in laser imaging

Exponential noise



$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad \bar{z} = \frac{1}{a} \qquad \sigma^2 = \frac{1}{a^2}$$

− a special case of the Erlang density, with b=1

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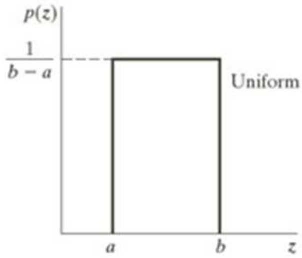
Uniform noise

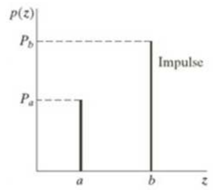


$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \qquad \bar{z} = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

−each noise intensity being equally probable
−the least descriptive of practical situations;
−useful as the basis for numerous random number generators used in simulations

Impulse (salt-and-pepper) noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



−bipolar if neither P_a or P_b is zero; in practice, for an 8-bit image, b=255 (white) and a = 0 (black)
−bipolar one, also known as salt-and-pepper, data-drop-out and spike noise
−called unipolar if either P_a or P_b is zero
−caused by either sensors' failure to respond (**pepper**, *black*) or sensors' saturation in color (**salt**, *white*)

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5.2.2 Noise Models

−Example 5.1: Noisy images and their histograms

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Figure 5.3 shows a test pattern well suited for illustrating the noise models just discussed,

– composed of simple constant areas that span the gray scale from black to near white in only three increments
– facilitating visual analysis of the characteristics of the various noise components added to the image

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

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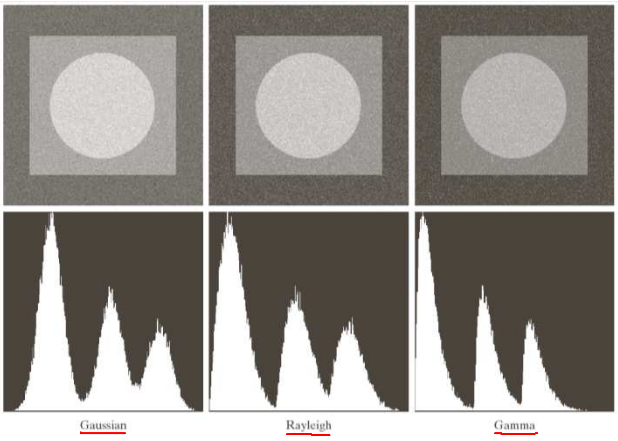

a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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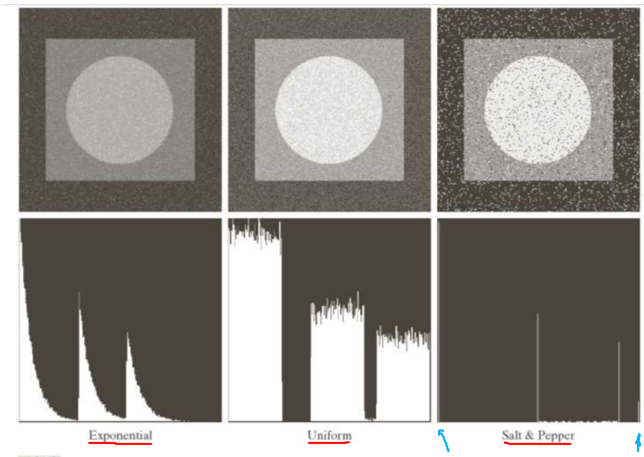

g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

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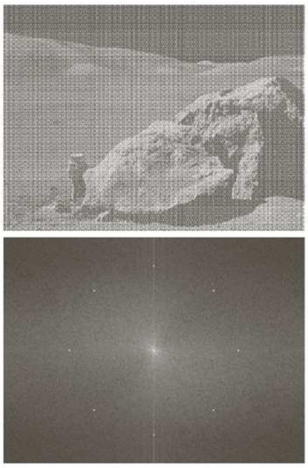
5.2.3 Noise Models **-- Periodic Noise**

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.
  - only spatially dependent noise considered herein
  - maybe reduced significantly via frequency domain filtering (Figure 5.5)



a
b
FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

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5.2.4 Noise Models
—Estimation of Noise Parameters

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- The parameters of periodic noise
  typically estimated by inspecting the image's Fourier spectrum
- The parameters of noise PDFs
  maybe known partially from sensor specifications
  often necessary to be estimated for a particular imaging arrangement
  - capturing a set of images of "flat" environments
- Possible to be estimated from small patches of *reasonably constant background intensity*, when only images already generated by a sensor are available
  - e.g., the vertical strips of 150x20 pixels

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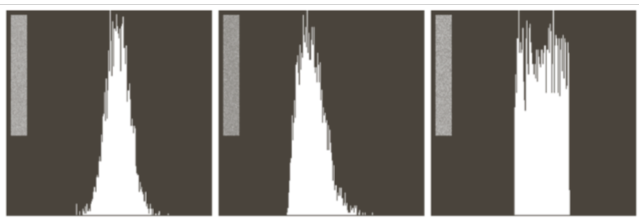

a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Let S denote a strip and the probability estimates are denoted by

$$p_S(z_i), i = 0,1,2,\ldots,L-1$$

where L is the number of possible intensity in the entire image

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_S(z_i) \qquad \sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

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5.3 Restoration in the Presence of Noise Only —**Spatial Filtering**

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A backward glance on the generic degraded image equations:

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y) \Longleftrightarrow G(u,v) = H(u,v) \bullet F(u,v) + N(u,v)$$

DFT

When the only degradation present is noise , $h(x,y) = \delta(x,y)$

$$g(x,y) = f(x,y) + \eta(x,y) \Longleftrightarrow G(u,v) = F(u,v) + N(u,v)$$

DFT

Is it possible to obtain a perfect estimate by

$$\hat{f}(x,y) = f(x,y) = g(x,y) - \eta(x,y)$$
$$\hat{F}(u,v) = F(u,v) = G(u,v) - N(u,v)$$

?

Answer:

✓Generally **NO**, since noise is unknown.
✓The answer will be the other way around, provided noise is **PERIODIC**, being able to be estimated from Fourier spectrum and hence **KNOWN** to a certain extent.

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Spatial filtering is suitable when only additive random noise is present.

In the next several slides, the following spatial filters will be discussed:

- ❑Mean Filters
  - –Arithmetic mean filter
  - –Geometric mean filter
  - –Harmonic mean filter
  - –Contraharmonic mean filter
- ❑Order-Statistic Filters
  - –Median filter
  - –Max and min filters
  - –Midpoint filter
  - –Alpha-trimmed mean filter
- ❑Adaptive Filters
  - –Adaptive, local noise reduction filter
  - –Adaptive median filter

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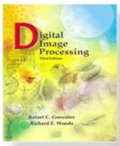
5.3.1 Restoration in the Presence of Noise Only
—**Mean Filters**

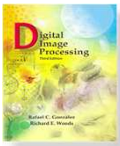


Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

S_{xy}, so-called a filter window, represents a rectangular sub-image of size $m \times n$, centered at (x,y)

- the simplest mean filters
- representing the restored pixel value at (x,y) by the arithmetic mean computed within the filter window
- smoothing local variations in an image → blurring
- noise-reducing as a by-product of blurring



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Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

- each restored pixel value given by the product of all the pixel values in the filter window, raised to the power 1/mn
- achieving smoothing comparable to the arithmetic mean filter, but tending to <u>lose less image detail</u> in the process

Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- working well for <u>salt</u> and Gaussian noises
- but failing for pepper noise

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Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- Q called the order of the filter and $Q \in R$
- well handling or virtually eliminating the effects of salt-and-pepper noise.
- however, unable to eliminate both salt and pepper noises simultaneously
    - eliminating pepper noise when $Q \in R^{+}$
    - eliminating salt noise when $Q \in R^{-}$

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5.3.1 Restoration in the Presence of Noise Only
—Example 5.2: Illustration of mean filters

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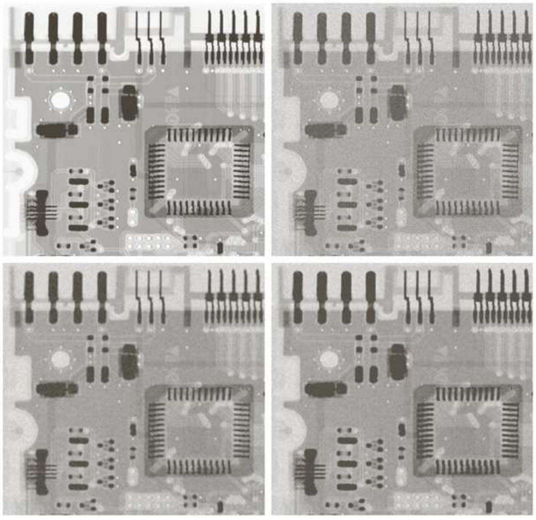

a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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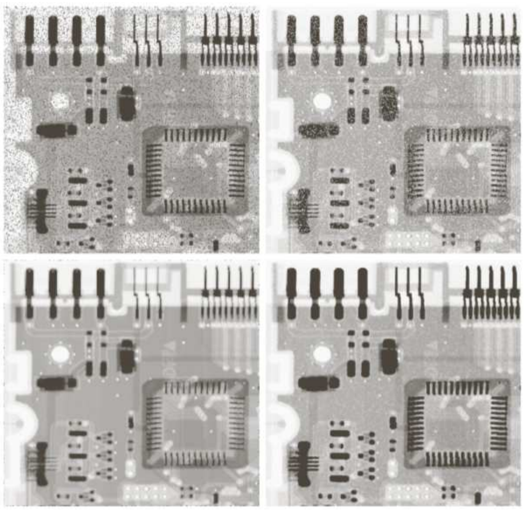
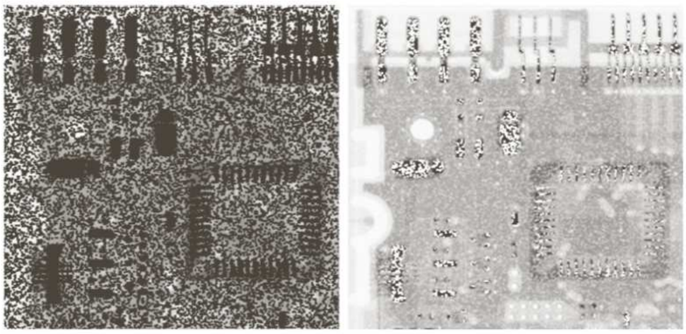


a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

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5.3.2 Restoration in the Presence of Noise Only — **Order-Statistic Filters**

Order-statistic filters (OSF)

- ❑ whose response is based on ordering (ranking) the values of the pixels contained in the filter window
- ❑ previously introduced in Section 3.5.2
- ❑ more extensively discussed herein with some additional OSFs

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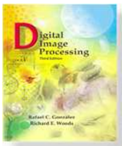
Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

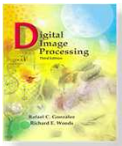
- the best-known of order-statistic filters
- representing the restored pixel value at (x,y) by the median (ranked in the 50^{th} percentile) of intensity levels in the filter window
- for certain types of noise, providing excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters on the same basis (of similar size)
- particularly effective in the presence of both bipolar and unipolar impulse noise



Max and min filters

$$\hat{f}(x,y)=\begin{cases}\max\limits_{(s,t)\in S_{xy}}\{g(s,t)\} & \text{for the max filter}\\ \min\limits_{(s,t)\in S_{xy}}\{g(s,t)\} & \text{for the min filter}\end{cases}$$

- representing the restored pixel value at (x,y) by the maximum/minimum of intensity levels in the filter window
- the max filter greatly reducing pepper noise (black dots)
- the min filter greatly reducing salt noise (white dots)



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Midpoint filter

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

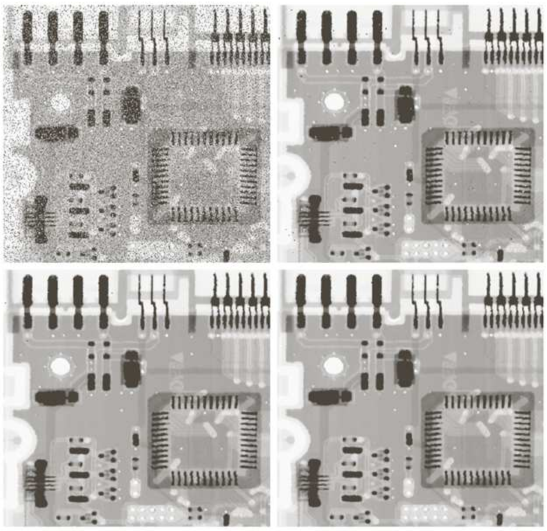
- representing the restored pixel value at (x,y) by the midpoint between the darkest and brightest points in the filter window
- working best for randomly distributed noise, e.g., Gaussian or uniform noise

Alpha-trimmed mean filter

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

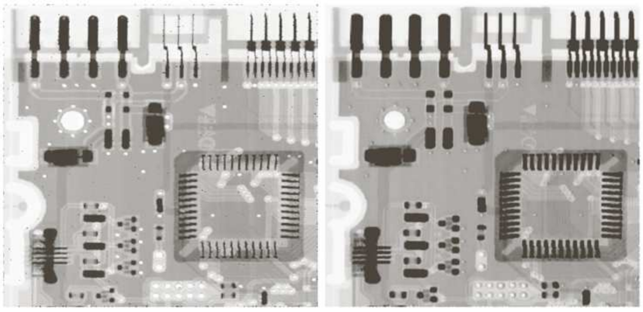
- $g_r(x,y)$ representing the trimmed filter window of size $mn-d$ after deleting the d/2 lowest and the d/2 highest values out of the original filter window
- becoming a median filter when $d = mn - 1$
- efficiently handling mixture noise, e.g., a combination of salt-and-pepper and Gaussian noise

5.3.2 Restoration in the Presence of Noise Only
—Example 5.3: Illustration of order-statistic filters



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

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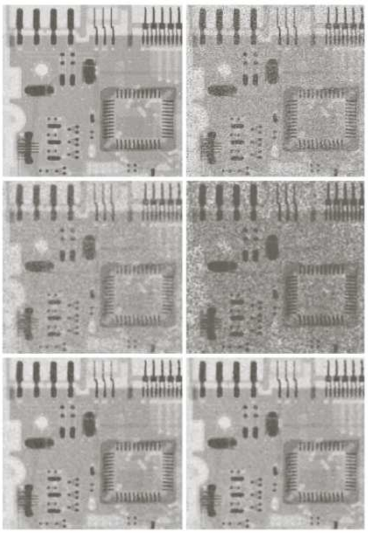

a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5 × 5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

5.3.3 Restoration in the Presence of Noise Only —**Adaptive Filters**

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The filters discussed thus far are non-adaptive filters.

- whose coefficients are static, collectively forming the transfer function
- applied to an image regardless of how image characteristics vary from one point to another

In this section, two adaptive filters are discussed.

- whose behavior changes according to statistical characteristics of the image inside the filter window
- whose performance is superior to that of non-adaptive filters having discussed

Adaptive, local noise reduction filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

- based on local mean (average intensity) m_L and local variance (contrast) σ_L^2
- if $\sigma_\eta^2 = 0$, no change
- if $\sigma_L^2 > \sigma_\eta^2$, edge, keep unchanged or less changed
- if $\sigma_L^2 \approx \sigma_\eta^2$, the m_L returns

- only the variance of corrupting noise $\hat{f}(x,y) \geq 0$ needed to be known or estimated
- assume $\sigma_\eta^2 \leq \sigma_L^2$, otherwise, set the ratio =1

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5.3.3 Restoration in the Presence of Noise Only — **Example 5.4: Illustration of adaptive, local noise-reduction filtering**

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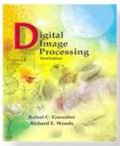

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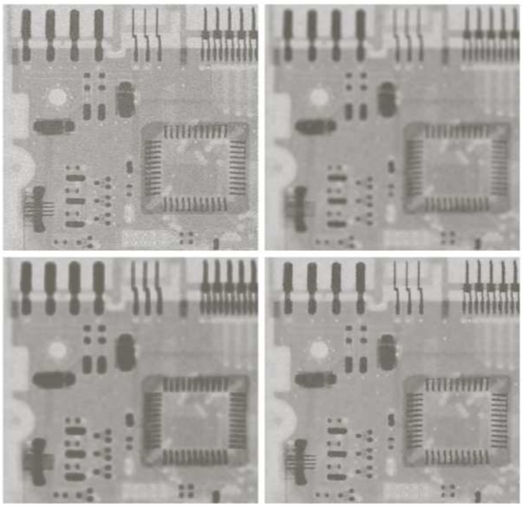
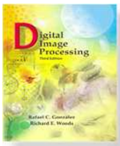


a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



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Adaptive median filter

Stage A: $A1 = z_{med} - z_{min}$
$A2 = z_{med} - z_{max}$
If $A1 > 0$ AND $A2 < 0$, go to stage B
Else increase the window size
If window size $\leq S_{max}$ repeat stage A
Else output z_{med}

Stage B: $B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
If $B1 > 0$ AND $B2 < 0$, output z_{xy}
Else output z_{med}

z_{min} = minimum intensity value in S_{xy}
z_{max} = maximum intensity value in S_{xy}
z_{med} = median of intensity values in S_{xy}
z_{xy} = intensity value at coordinates (x, y)
S_{max} = maximum allowed size of S_{xy}

- representing the restored pixel value at (x,y) by executing pseudocode
- the size of filter window is adaptive
- three purposes: to remove salt-and-pepper noise (capable of handling large P_a and P_b), to smooth non-impulsive noise, and to reduce distortion, e.g., excessive thinning or thickening of object boundaries
- performance is better than un-adaptive median filter
- for more detail, read text

5.3.3 Restoration in the Presence of Noise Only — Example 5.5: Illustration of adaptive median filtering

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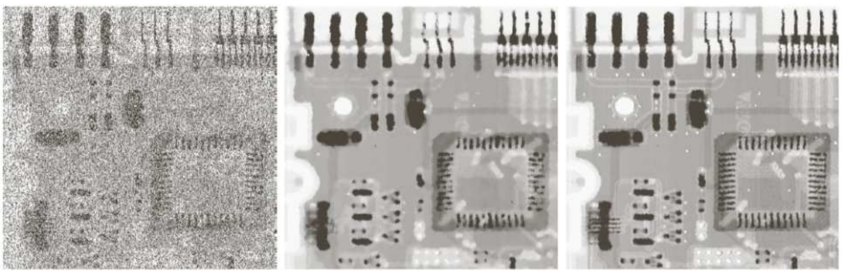
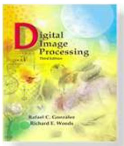
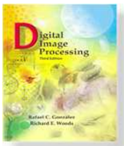


a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.



5.4 Periodic Noise Reduction by -**Frequency Domain Filtering**



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Periodic noise can be analyzed and filtered quite effectively

- ❑ using frequency domain techniques
- ❑ in other words, using a selective filter to isolate noise

Three types of selective filters will be discussed herein

- ❑ Band-reject
- ❑ Band-pass
- ❑ Notch

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5.4.1 Periodic Noise Reduction by Frequency Domain Filtering
—Band-reject Filters

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Bandreject Filters

a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Suitable for noise removal when the noise component(s) in the frequency domain is approximately known

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5.4.1 Periodic Noise Reduction by Frequency Domain Filtering
—Example 5.6: Use of bandreject filtering for periodic noise removal

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a b
c d

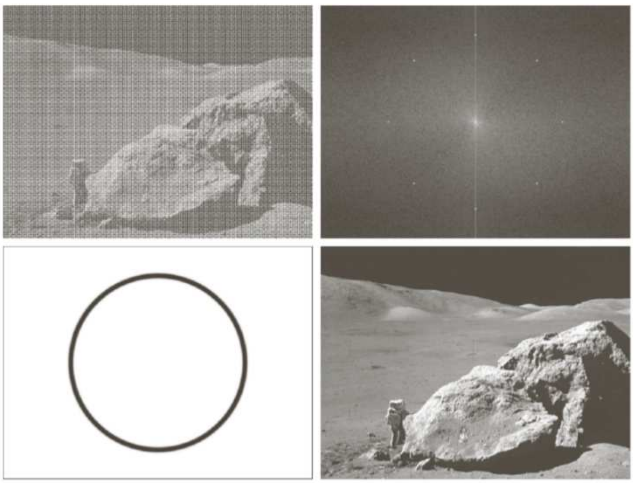

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

5.4.2 Periodic Noise Reduction by Frequency Domain Filtering
—Bandpass Filters

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Bandpass (BP) Filters

- ❑ performing the opposite operation of bandreject (BR) filters.
- ❑ which can be obtained from those of corresponding bandreject filters

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

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5.4.2 Periodic Noise Reduction by Frequency Domain Filtering

—Example 5.7: Bandpass filtering for extracting noise patterns.

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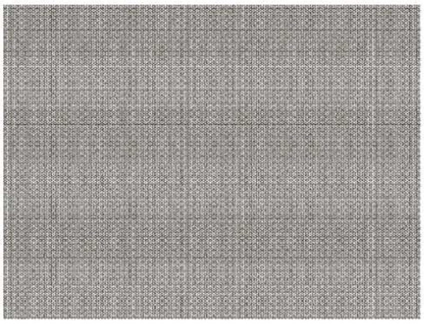

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

Figure 5.17 generated by

1) Obtaining the bandpass filter corresponding to the bandreject filter used in Fig. 5.16
2) Taking the inverse transform of the bandpass-filtered transform

5.4.3 Periodic Noise Reduction by Frequency Domain Filtering
—Notch Filters

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Notch Filters

- Either rejecting (NR) or passing (NP) frequencies in predefined neighborhoods about a center frequency.

The transfer functions of NP and NR have the following relationship

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

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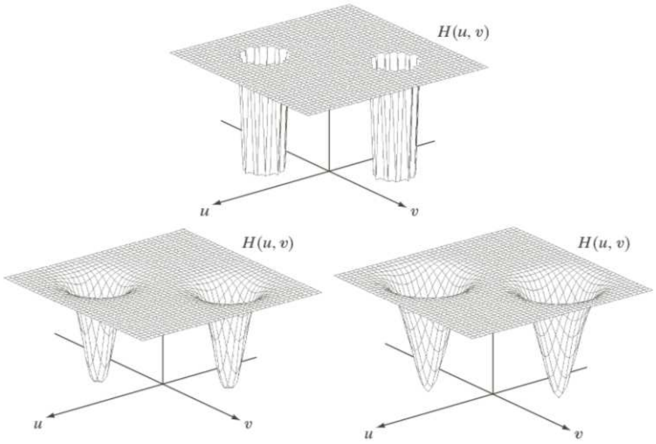
a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



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5.4.3 Periodic Noise Reduction by Frequency Domain Filtering
—Example 5.8: Removal of periodic noise by notch filtering

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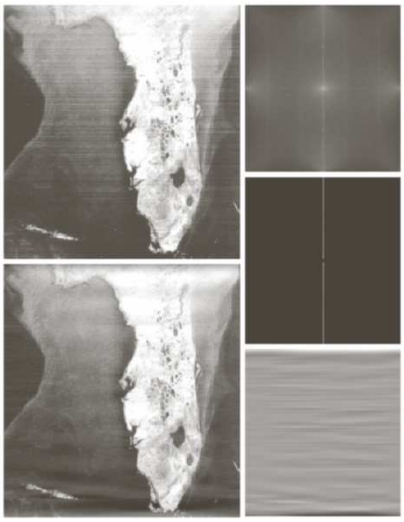

a b
c
e d

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

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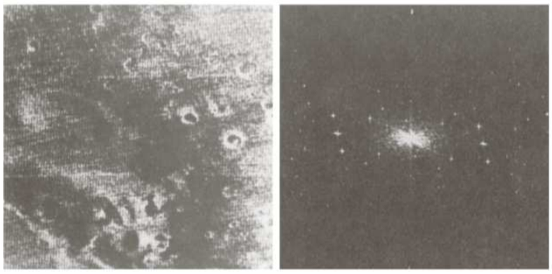
5.4.4 Periodic Noise Reduction by Frequency Domain Filtering
—Optimum Notch Filtering



a b
FIGURE 5.20 (a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

Generally, interference components consisting of multiple interference components

- traditional notch filters may remove too much information (unacceptable)

The way-out of this problem

- to use an optimum method by minimizing local variances of the restored estimate

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Optimum Notch Filtering Procedure (read text for detail)

1) Isolating the principal contributions of the interference pattern

- with traditional NP (notch pass) placed at the location of each spike,

$$N(u,v) = H_{NP}(u,v)G(u,v) \longrightarrow \eta(x,y) = \Im^{-1}\{N(u,v)\}$$

2) The effect of components not present in the estimate of $\eta(x,y)$ can be minimized by subtracting a weighted portion of the $\eta(x,y)$ to obtain an estimate of *f(x,y)*.

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$ (5.4-5)

a weighting or modulation function

3) For derivation: read p. 341 of the text)

$\sigma^2(x,y)$ denotes the local variance of the restored estimate

- in a neighborhood of size $(2a+1)\times(2b+1)$ *(a = b = 15)*
- with some approximations, $\sigma^2(x,y)$ becomes

$$\sigma^2(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\{[g(x+s,y+t)-w(x,y)\eta(x+s,y+t)]-[\overline{g}(x,y)-w(x,y)\overline{\eta}(x,y)]\}^2$$

- to obtain $w(x,y)$, solving $\frac{\partial\sigma^2(x,y)}{\partial w(x,y)}=0$ (i.e., minimizing $\sigma^2(x,y)$)

$$w(x,y)=\frac{\overline{g(x,y)\eta(x,y)}-\overline{g}(x,y)\,\overline{\eta}(x,y)}{\overline{\eta^2}(x,y)-\overline{\eta}^2(x,y)} \quad (5.4\text{-}13)$$

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5.4.4 Periodic Noise Reduction by Frequency Domain Filtering
—Example 5.9: Illustration of optimum notch filtering

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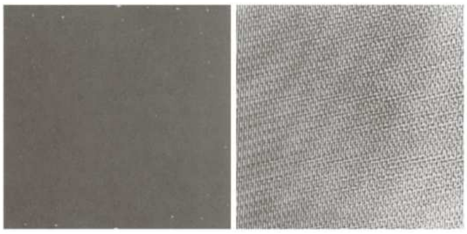
FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

FIGURE 5.23 Processed image. (Courtesy of NASA.)

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5.5 Linear, Position-Invariant Degradations

❑ Many formulae are in this section. They describe linear position-invariant systems. (ECE601)

❑ If needed, you may go over this section.

In the absence of noise, the degraded image is expressed as

$$g(x, y) = H[f(x, y)]$$

H is **linear** if the following two properties are fulfilled

- additivity (superposition)
- homogeneity (scaling)

$$H[af_1(x,y)+bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

H is **position (or space) invariant** if $H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$

If noise is present and H is **linear, position-invariant (LPI)**, then

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y) \Longleftrightarrow G(u,v) = H(u,v) \bullet F(u,v) + N(u,v)$$

DFT

5.6 Estimating the Degradation Function

In image restoration, there are three principal ways to estimate the degradation function

1) Observation: laborious, used in very specific circumstances, e.g., restoring an old photograph of historical value
2) Experimentation
3) Mathematical modeling

The image restoration process using an estimated degradation function is sometimes called **blind deconvolution**

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5.6.1 Estimating the Degradation Function **—Estimation by Image Observation**

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Estimation by Image Observation

- based on the assumption that the image was degraded by LPI

1) gathering image information from a sub-image $g_s(x,y)$ whose signal content is strong (e.g., an area of high contrast)

2) process the subimage, to obtain $\hat{f}_s(x,y)$, against the degradation phenomenon (e.g., if an image is blurred, de-blur it)

Then, the degradation function can be deduced by the transfer function of the observed window (under LPI assumption)

$$H(u,v) = H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)} = \frac{\Im\{g_s(x,y)\}}{\Im\{\hat{f}_s(x,y)\}}$$

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5.6.2 Estimating the Degradation Function **—Estimation by Experimentation**

Estimation by Experimentation

➢ only if the equipment similar to that used to acquire the degraded image is available

1) Obtain image similar to the degraded one by varying system settings until achieving approximately the same degradation level as that of degraded image

2) With such a settings, obtain the impulse response of the degradation by imaging an impulse (small dot of light) with an arbitrary strength A (A is the FT of the impulse)

The degradation function can be deduced by the impulse response of the observed image.

$$H(u,v) = \frac{G(u,v)}{A}$$

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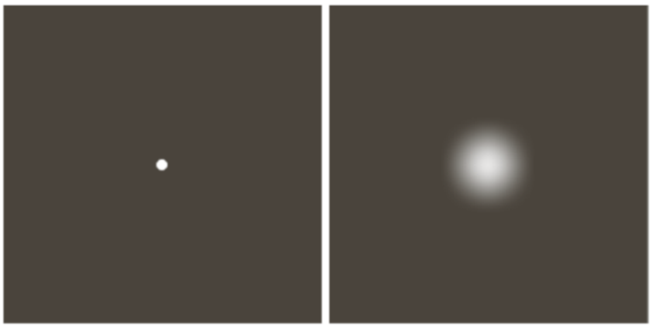

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

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5.6.3 Estimating the Degradation Function
—Estimation by Modeling
Example 1: Atmospheric turbulence model

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Degradation modeling has been widely used because of the insight it affords into the image restoration problem.

In some case, the model can even take into account environmental conditions that cause degradation.

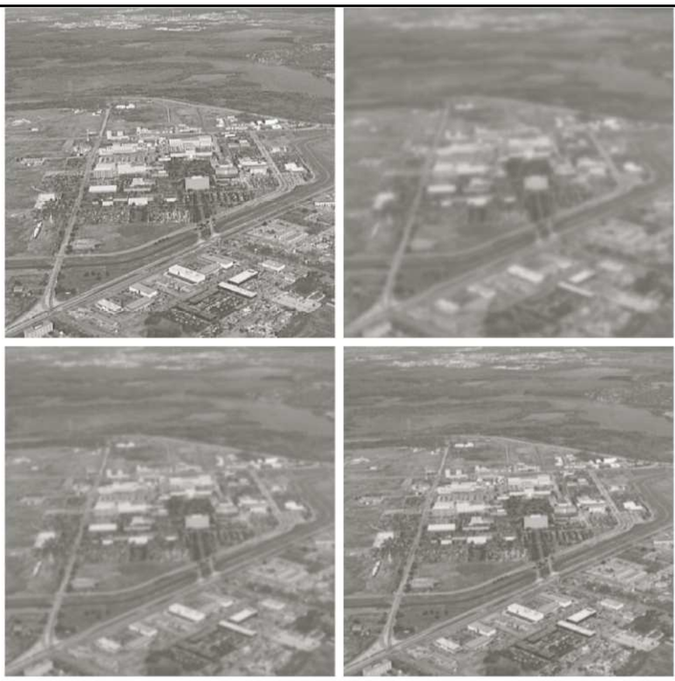
For example, the atmospheric turbulence model of Hufnagel and Stanley

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

where a constant k depends on the nature of the turbulence

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



5.6.3 Estimating the Degradation Function
—Estimation by Modeling
Example 2: Image blurring due to motion

Another major approach in modeling is to derive a mathematical model starting from basic principles

For example, an image has been blurred by uniform linear motion between the image and the sensor during image acquisition with exposure time (shutter speed) T.

$$g(x,y)=\int_0^T f[x-x_0(t),y-y_0(t)]dt \overset{\Im}{\Longleftrightarrow} G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt=F(u,v)H(u,v)$$

Eq.5.6-8

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$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

Eq. 5.6-11: $x_0(t)=at/T$, $y_0(t)=bt/T$

In T, x direction move a, y: move b

Detail: text or PDF file

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

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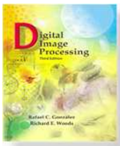
5.7 Inverse Filtering



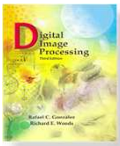
Inverse Filtering

If *H* is given or estimated, the simplest approach to restoration is direct inverse filtering

$$G(u,v) = H(u,v) \bullet F(u,v) + N(u,v) \xrightarrow[\bullet 1/H(u,v)]{} \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \cdots F(u,v) + \frac{N(u,v)}{H(u,v)}$$

(Eq. 5.7-1) (Eq. 5.7-2)

- Even if *H* completely known, *F* cannot be exactly recovered because *N* is not known.
- If *H* has zero or very small values, the ratio *N/H* may predominate
  - One way out is to limit the filter frequencies to values near the origin



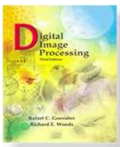
5.7 Inverse Filtering
—Example 5.11: Inverse filtering

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$H(u,v) = e^{-k[(u-M/2)^2+(v-N/2)^2]^{5/6}}$$

$$k = 0.0025, M = N = 480$$



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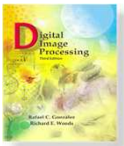
5.8 Minimum Mean Square Error (Wiener) Filtering

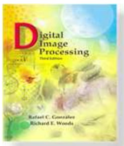


Traditional inverse filtering makes no explicit provision for handling noise.

In this section, an approach incorporating both the degradation and statistical characteristic of noise is discussed.

The objective of this approach is to find an image estimate such that the mean square error (MSE) between the uncorrupted image f and an estimate $\hat{f}$ is minimized.

$$e^2 = E\left\{ (f - \hat{f})^2 \right\}$$



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Based on aforementioned conditions, the image estimate in the frequency domain is given by

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \qquad \text{(Eq. 5.8-2)}$$

$H^*(u,v) =$ complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2 =$ power spectrum density of the noise

$S_f(u,v) = |F(u,v)|^2 =$ power spectrum density of the undegraded image

Measures based on the power spectra of noise and of the undegraded image characterize the performance of restoration algorithms.

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2} \text{ or } \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y) - \hat{f}(x,y)\right]^2}$$

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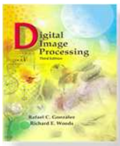
When noise spectrum is constant (white noise), things can be considerably simplified; however, the spectrum of undegraded image seldom is known.

Under this circumstance, the image estimate, Eq. 5.8-2, may be rewritten as

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right]G(u,v) \qquad \text{(Eq. 5.8-6)}$$

where K is a specified constant.

This equation is often actually utilized for Wiener filtering.



5.8 Minimum Mean Square Error (Wiener) Filtering
—Example 5.12: Comparison of inverse and Wiener filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

K manually adjusted to yield the best visual results

Wiener filtering yielded a result very closer to the original image.

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5.8 Minimum Mean Square Error (Wiener) Filtering
—Example 5.13: Further Comparisons of Wiener filtering

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a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

5.9 Constrained Least Squares Filtering

Drawback of Wiener filtering:

- Need to know power spectrum of the original image (difficult)
- Or, need to use a constant estimate of the ratio of power spectra (before and after degradation) (not very reasonable)
- Minimization of $E[(f - \hat{f})^2] \rightarrow$ optimal in an average sense

This method:

- Only requires mean and variance of noise (more reasonable)
- Optimization for each image

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- $g(x,y) = h(x,y)*f(x,y) + \eta(x,y)$ (Eq. 5.5.-16) can be written in vector-matrix format as follows.
- $\underline{g} = \underline{H} \cdot \underline{f} + \underline{\eta}$
  image size MxN, $\underline{g}$, $\underline{\eta}$: MNx1, $\underline{H}$: MNxMN

- Large dimensionality – very difficulty to numerically solve
- Using FFT to solve – detail is in old version of text and notes

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Constrained optimization

- Minimization of image smoothness:

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left|\nabla^2 f(x,y)\right|^2$$

- Subject to the constraint

$$\|g - Hf\|^2 = \|\eta\|^2$$

- where $\|w\|^2 = w^T w$

- See reference (old notes) for detail.

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a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

In a and b, this method is obviously better than Wiener filtering.

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a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

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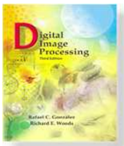
5.10 Geometric Mean Filter

(generalized Wiener filter)

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

with α and β being positive, real constants.



When α=1,the filter reduces to the inverse filter.

With α=0, it is parametric Wiener filter, which reduces to the standard Wiener filter when β=1.

If α=1/2, it becomes product of two filters (with same power), like geometric mean, hence geometric mean filter.

When β=1, as α<1/2, the filter more like Wiener filter
as α>1/2, the filter more like inverse filter.

When α=1/2 and β=1, the filter is called spectrum equalization filter.

Equation (5.10-1) is useful when implementing restoration filters because it represents a family of filters combined into a single expression.