ECE 643 Digital Image Processing I

Chapter 4 (2nd Part)

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Acknowledgement

Mr. Patchara Sutthiwan made some slides in this set.

Useful Fourier Transform Properties

- Discrete Convolution

$$f(x,y) * h(x,y) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n)$$

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v) \cdot H(u,v)$$

$$f(x,y) \cdot h(x,y) \Leftrightarrow F(u,v) * H(u,v)$$

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Useful Fourier Transform Properties

- Impulse function of strength A

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)\, A\, \delta(x-x_0, y-y_0) = A\, s(x_0, y_0) \quad \text{sifting property}$$

$$\delta(x,y) * h(x,y) \Longleftrightarrow H(u,v)$$

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Basics of Filtering in Frequency Domain

- Filtering techniques in the frequency domain are based on modifying the Fourier transform to achieve a specific objective and then computing the inverse DFT to get back to the image domain.

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Frequency Domain Filtering Fundamentals

$$g(x,y) = F^{-1}[H(u,v)F(u,v)]$$

F(u,v) = DFT of the input image

H(u,v) = Filter Function

g(x,y) = the filtered (output) image

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Frequency Domain Filtering Fundamentals

- Restriction and Suggestion

F, H, g are arrays of size M by N

The design of H(u,v) is simplified considerably by using functions that are symmetric about their center which also requires that F(u,v) be centered

The nature of an image in frequency domain

- Low-frequency components of any image are related to slowly varying intensity in an image such as the walls of a room or a cloudless sky in an outdoor scene. In other words, they define the global identity of an image.
- High-frequency components of any image are caused by sharp transitions in intensity, such as edges and noise. So we say they represent details of an image.

Filters to be discussed

- Notch filter which rejects (or passes) frequencies in a predefined neighborhood about the center of the frequency rectangle
- Low-pass filters which attenuate high frequencies while pass low frequencies. Based on what we have previously discussed, it would blur an image.
- High-pass filters which attenuate low frequencies while pass high frequencies. This type of filter would enhance sharp detail, but cause a reduction in contrast in the image.

Correspondence Between Filtering in the Spatial and Frequency Domains

- The link between filtering in the spatial and frequency domains is the convolution theorem
- The inverse transform of the frequency domain filter is the corresponding filter in the spatial domain
- In frequency domain, filtering is defined as the multiplication of a filter function and the Fourier transform of the input image; while in spatial domain, filtering is the spatial convolution between an image and a filter mask.

Correspondence Between Filtering in the Spatial and Frequency Domains

- In practice, filtering in spatial domain is preferable due to speed and ease of implementation in hardware and/or firmware.
- However, filtering concepts are more intuitive in the frequency domain.
- We can exploit these two domains by
  1) computing its IDFT of a filter function
  2) using the resulting, full-size spatial filter as a guide for constructing smaller spatial filter masks which can most capture the properties of the filter function

Correspondence Between Filtering in the Spatial and Frequency Domains

- Here 1-D Gaussian filters are used to illustrate how frequency domain filters can be used as guide for specifying the coefficients of some of the small masks discussed in Chapter 3. This concept can later be extended into 2-D framework.
- Filter based on Gaussian functions are of interest because both the forward and inverse Fourier transforms of a Gaussian function are real Gaussian functions.

Correspondence Between Filtering in the Spatial and Frequency Domains

- Gaussian filter, GLPF

$$\begin{cases} H(u) = A e^{-\frac{u^2}{2\sigma^2}} \\ h(x) = \sqrt{2\pi}\,\sigma A e^{-2\pi^2\sigma^2 x^2} \end{cases} \quad \text{(Problem 4.4)}$$

· Both are Gaussian and real.

· Reciprocal in standard deviation

if $\sigma \to \infty$, $H(u) \to$ constant, $h(x) \to$ impulse

Fig. 4.9 2nd ed.

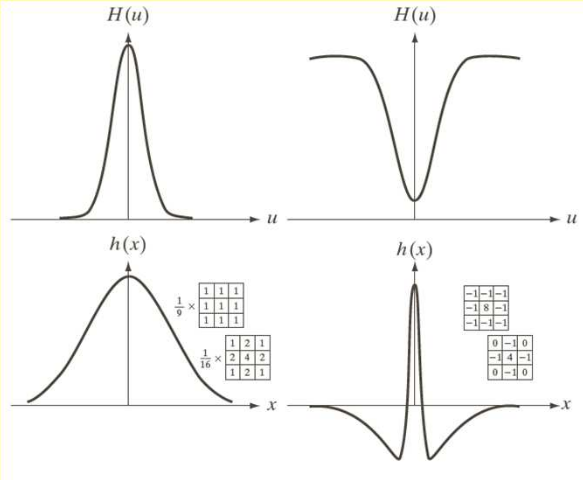
Correspondence Between Filtering in the Spatial and Frequency Domains

- GHPF

$$\begin{cases} H(u) = A e^{-\frac{u^2}{2\sigma_1^2}} - B e^{-\frac{u^2}{2\sigma_2^2}} \\ A \geq B \quad \& \quad \sigma_1 > \sigma_2 \\ h(x) = \sqrt{2\pi}\,\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\,\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2} \end{cases}$$

- Easy to design a filter in frequency domain
  Easy to implement a filter in spatial domain with a small mask.

Correspondence Between Filtering in the Spatial and Frequency Domains



a c
b d

FIGURE 4.37 (a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

Correspondence Between Filtering in the Spatial and Frequency Domains

- For LPF, based on Figure 4.37 (a) and (b), we conclude that we can implement low-pass filtering in the spatial domain by using a mask with all positive coefficients
- For HPF, based on Figure 4.37 (c) and (d), we notice that h(x) has a positive center term with negative terms on either side. Therefore, we can implement high-pass filtering in the spatial domain by using a mask with a positive coefficient at center surrounded by negative coefficients.

Notch Filter

- Notch filter is the most useful of the selective filter
- It rejects (or passes) frequencies in a predefined neighborhood about the center of the frequency rectangle.

○ <u>Notch filter</u>

$$H(u,v) = \begin{cases} 0 & \text{if } (u,v) = (\frac{M}{2}, \frac{N}{2}) \\ 1 & \text{otherwise} \end{cases}$$

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Image Smoothing

- 3 types of lowpass filters in the frequency domain to be discussed
  - Ideal Lowpass Filters (ILPF)
  - Butterworth Lowpass Filters (BLPF)
  - Gaussian Lowpass Filters (GLPF)

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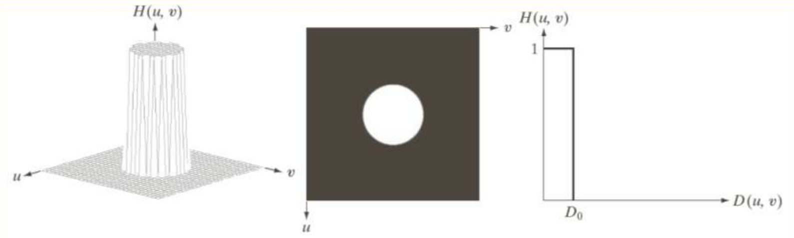
Ideal Lowpass Filter

o Ideal Lowpass Filter (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

Here and throughout the slides, D_0 is a positive constant and D(u,v) is the Distance between a point (u,v) in the frequency domain and the center of the Frequency rectangle

Ideal Lowpass Filter



a b c

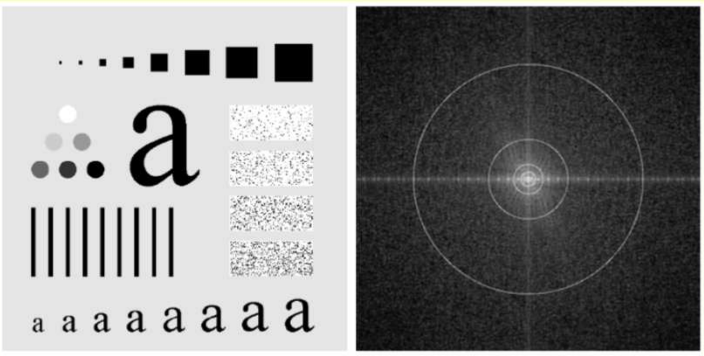
FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Ideal Lowpass Filter

- The filters discussed here are compared by studying their behavior as a function of the same cutoff frequencies. One way to establish a set of standard cutoff frequency loci is to compute circles that enclose specified amounts of total image power P_T

$$P_T = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} P(u,v) \qquad P(u,v) = |F(u,v)|^2$$

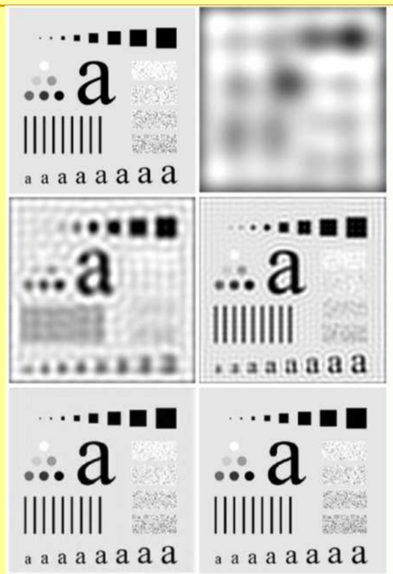
Ideal Lowpass Filter



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Ideal Lowpass Filter



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

The more power removed by an LPF, the more blurry the resulting image becomes

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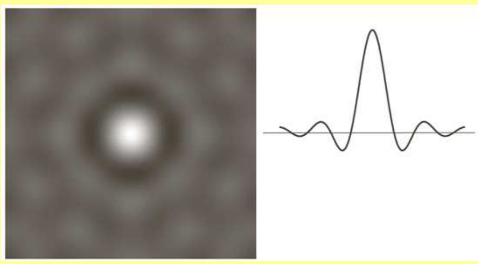
Ideal Lowpass Filter



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

- Since a cross section of the ILPF in the frequency domain looks like a box filter, it is not unexpected that a cross section of the corresponding spatial filter has the shape of a sinc function.
- Convolving a sinc with an impulse copies the sinc at the location of the impulse.
- The center lobe of the sinc is the principal cause of blurring, while the outer, smaller lobes are mainly responsible for ringing.

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Butterworth Lowpass Filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

$$\text{where } D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$$

The transfer function of a Butterworth lowpass filter (BLPF) of order n, and with cutoff frequency at a distance D_0 from the origin

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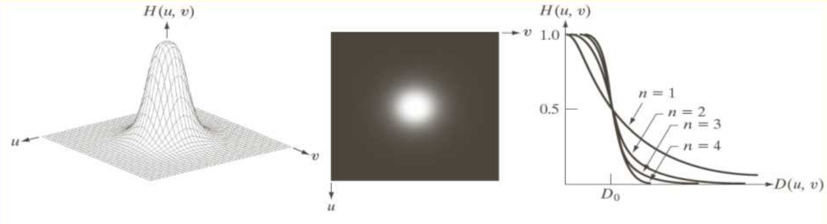
Butterworth Lowpass Filter



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that gives a clear cutoff between passed and filtered frequencies, but as n Approaches Infinity, BLPF turns to be ILPF.

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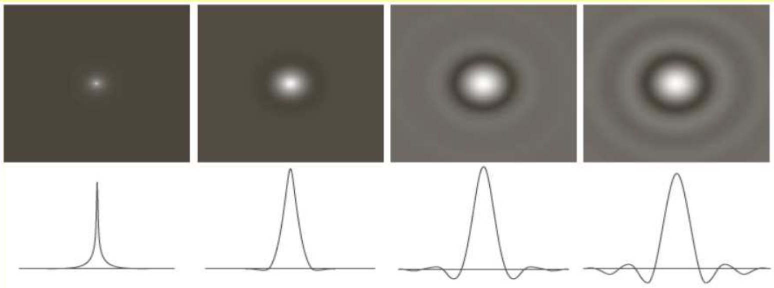
Butterworth Lowpass Filter

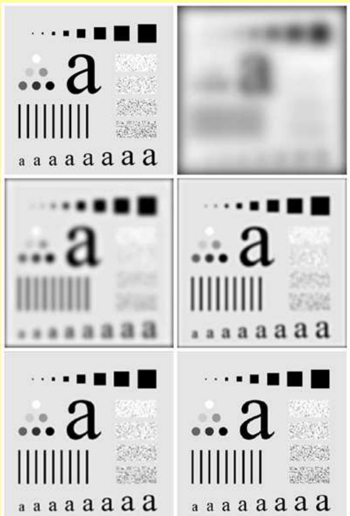


a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

A BLPF of order 1 has no ringing in the spatial domain. Ringing generally is imperceptible in the filters of order 2, but can become significant in filters of higher orders

Butterworth Lowpass Filter



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Ringing effect is Hardly perceivable

Gaussian Lowpass Filter

Gaussian LPF

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}} = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

σ , standard deviation, is a measure of spread about the center which is essentially equivalent to D_0 or cutoff frequency

Gaussian Lowpass Filter

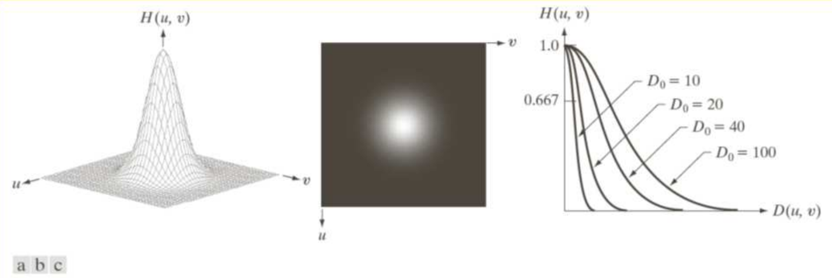


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

GLPF establishes no ringing effect at all

Gaussian Lowpass Filter

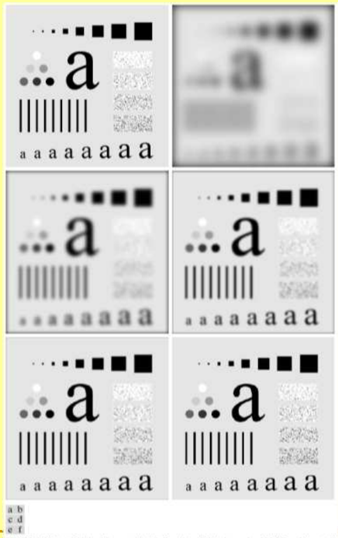


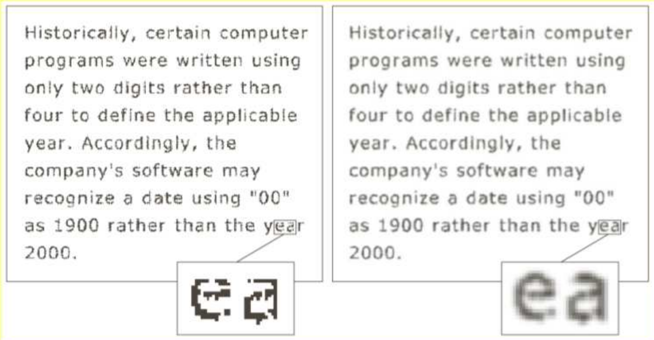
FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Summary Table for Lowpass Filters

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

Applications of LPF (Smoothing)



a b

FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Applications of LPF (Smoothing)

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Applications of LPF (Smoothing)

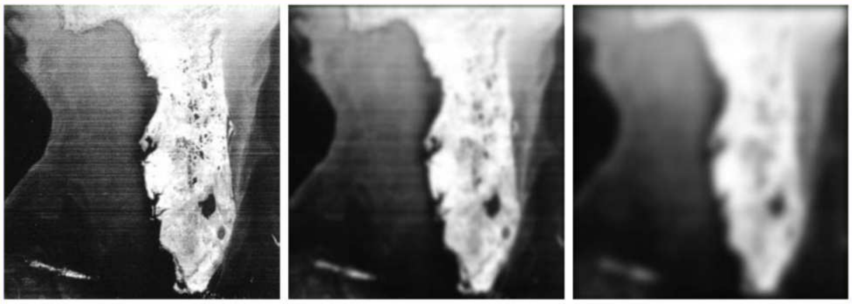


FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Image Sharpening

- It's an application of highpass filtering
- A highpass filter function can be derived from any given lowpass filter function as follows

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Image Sharpening

- 3 types of highpass filters in the frequency domain to be discussed
  - Ideal Highpass Filters (IHPF)
  - Butterworth Highpass Filters (BHPF)
  - Gaussian Highpass Filters (GHPF)

Ideal Highpass Filter

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Because of the way in which we relate HPF to LPF, we can expect IHPF
To have the same ringing effect properties as ILPF.

This can be obviously seen in Figure 4.54 in the next slide

Ideal Highpass Filter

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

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Butterworth Highpass Filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}$$

As with lowpass filters, we can expect BHPFs to behave smoother than IHPFs as shown in Figure 4.55

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Butterworth Highpass Filter

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

The boundaries are much less distorted than in Fig. 4.54, even for the smallest value of cutoff frequency

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Gaussian Highpass Filter

$$H(u,v) = 1 - e^{-\frac{D(u,v)}{2D_0^2}}$$

The same story as that of GLPF goes to GHPF.
We can expect no ringning effect from GHPF

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Gaussian Highpass Filter

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Image Sharpening

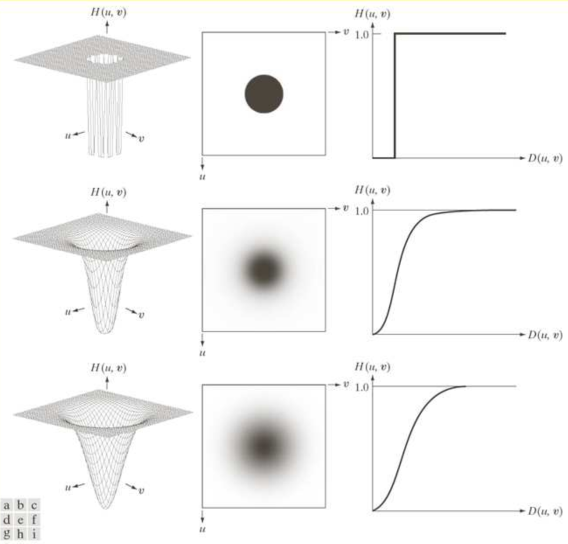

FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Image Sharpening

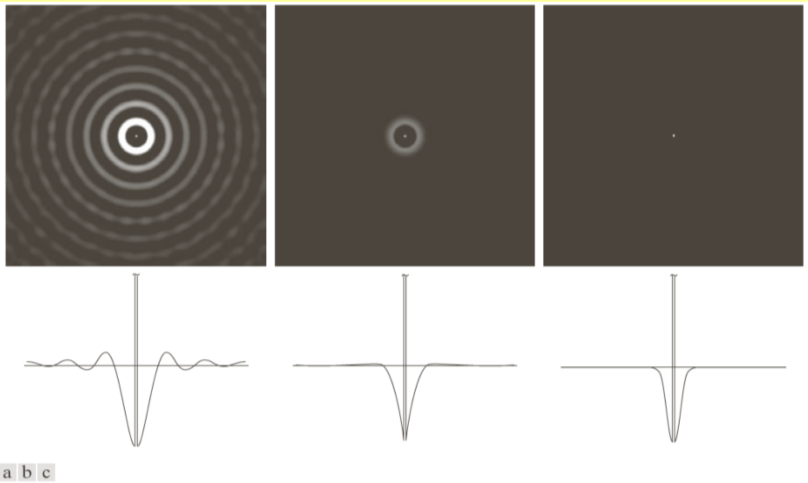

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

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Summary Table for Highpass Filters

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

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Application of HPF

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Application of HPF

- It can be noticed from Fig. 4.57 (b) that the highpass-filtered image lost its gray tones because the dc term was reduced to 0.
- An additional processing (thresholding) is required to enhance details of interest.
- The net result in Fig. 4.57 (c) provides much clearer thumb print.

Other high-frequency-emphasis filters

- For all high-passed images we have discussed, they have one thing in common: Their average background intensity has been reduced to near black due to the fact that the highpass filters applied to those images eliminate the DC component.
- The solution to this problem consists of adding a portion of the image back to the filtered result.
- Here we shall discuss two types of such a filter in the frequency domain:
  - Laplacian
  - High-boost

The Laplacian in the Frequency Domain

- In Chapter 3, we used the Laplacian for image enhancement in the spatial domain. Here, we are to show that it yields equivalent results using frequency domain techniques.

$$\mathcal{F}\left\{\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right\} = (ju)^2 F(u,v) + (jv)^2 F(u,v)$$

$$= -(u^2+v^2)\, F(u,v)$$

⇒ Laplacian is a filter

has $H(u,v) = -(u^2+v^2)$

⇒ centering. $H(u,v) = -\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]$

i.e. $\nabla^2 f(x,y) \overset{FT}{\Longleftrightarrow} -\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right] F(u,v)$

Laplacian enhance image $g(x,y) = f(x,y) - \nabla^2 f(x,y)$

The Laplacian in the Frequency Domain

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Edges are enhanced without much darkening the filtered image

Highboost Filter

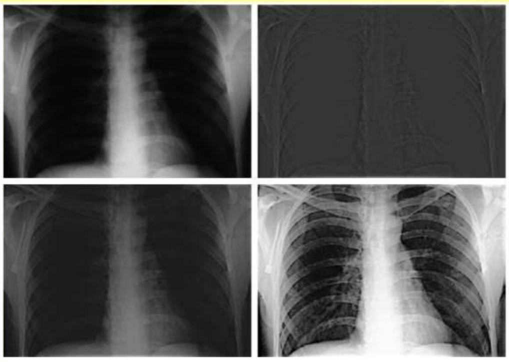
- Highboost filtering is able to increase the contribution made by the original image to the overall filtered result. This concept was once introduced in the spatial domain in Chapter 3 and here we repeat it in the frequency domain.

$$f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y) \qquad A \geq 1.$$

i.e. $$H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$$

- When A = 1, high-boost filter reduces to regular highpass filter. As A increases beyond 1, the contribution made by the image itself becomes more significant.

Highboost Filter



a b
c d

FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

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Homomorphic Filtering

- Any image f(x,y) can be expressed as the product of its illumination i(x,y) and reflectance r(x,y) components

$$f(x,y) = i(x,y)r(x,y)$$

The above representation cannot be used directly to operate on the frequency components of illumination and reflectance because the Fourier transform of a product is not the product of the transforms. This problem is solved by taking natural logarithm to f(x,y)

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Homomorphic Filtering

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\mathcal{F}\{z(x,y)\} = \mathcal{F}\{\ln i(x,y)\} + \mathcal{F}\{\ln r(x,y)\}$$

FT ↓

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

$$S(u,v) = H(u,v)\,Z(u,v)$$

$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

IFT ↓

$$s(x,y) = i'(x,y) + r'(x,y)$$

enhanced image $g(x,y) = e^{s(x,y)} = e^{i'(x,y)} \cdot e^{r'(x,y)}$

$$= i_0(x,y)\,r_0(x,y)$$

Homomorphic Filtering

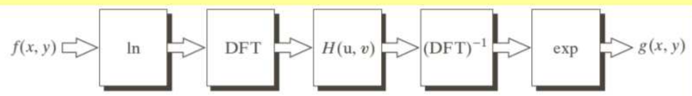


FIGURE 4.60 Summary of steps in homomorphic filtering.

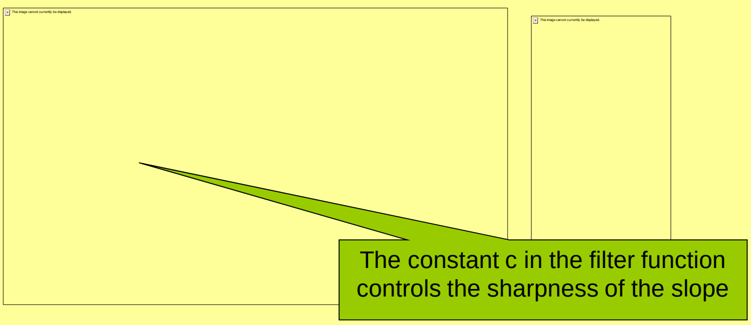
Homomorphic Filtering

- The homomorphic filter function H(u,v) can operate on illumination and reflectance components separately.
- The illumination component of an image generally is characterized by slow spatial variations (low frequency compoents) , while the reflectance component tends to vary abruptly (high frequency components)
- With homomorphic filter, we are able to control over the illumination and reflectance components of any image by specifying the following two parameters: γ_L and γ_H
- The generic form of homomorphic filter function is

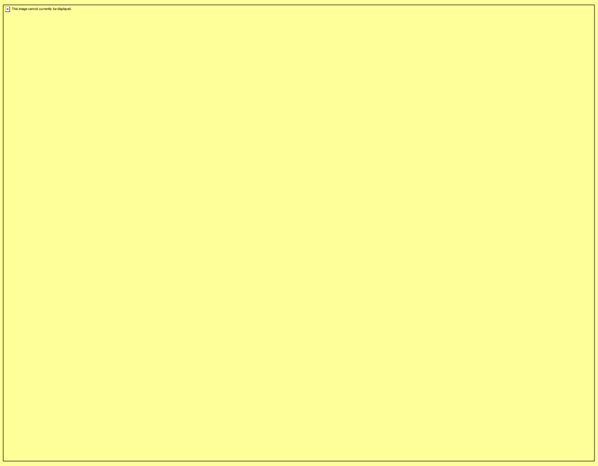
$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\frac{D^2(u,v)}{D_0^2}}\right] + \gamma_L$$

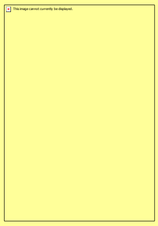
Homomorphic Filtering

- When $\gamma_L < 1$ and $\gamma_H > 1$, the filter function tends to attenuate the contribution made by the low frequencies (illumination) and amplifying the contribution made by high frequencies (reflectance). The net result is simultaneous dynamic range compression and contrast enhancement.



Homomorphic Filtering





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Homomorphic Filtering

- Fig. 4.62 (b) was obtained with the following parameters

$$\gamma_L = 0.25, \gamma_H = 2, c = 1, \text{ and } D_0 = 80$$

- By reducing the effects of the dominant illumination components (the hot spots), it became possible for the dynamic range of the display to allow lower intensities to become much more visible.
- Similarly, because the high frequencies are enhanced by homomorphic filtering, the reflectance components of the image (edge information) were sharpened considerably.
- The enhanced image in Fig. 4.62 (b) is a significant improvement over the original.

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END

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