Wireless Networking with Selfish Agents

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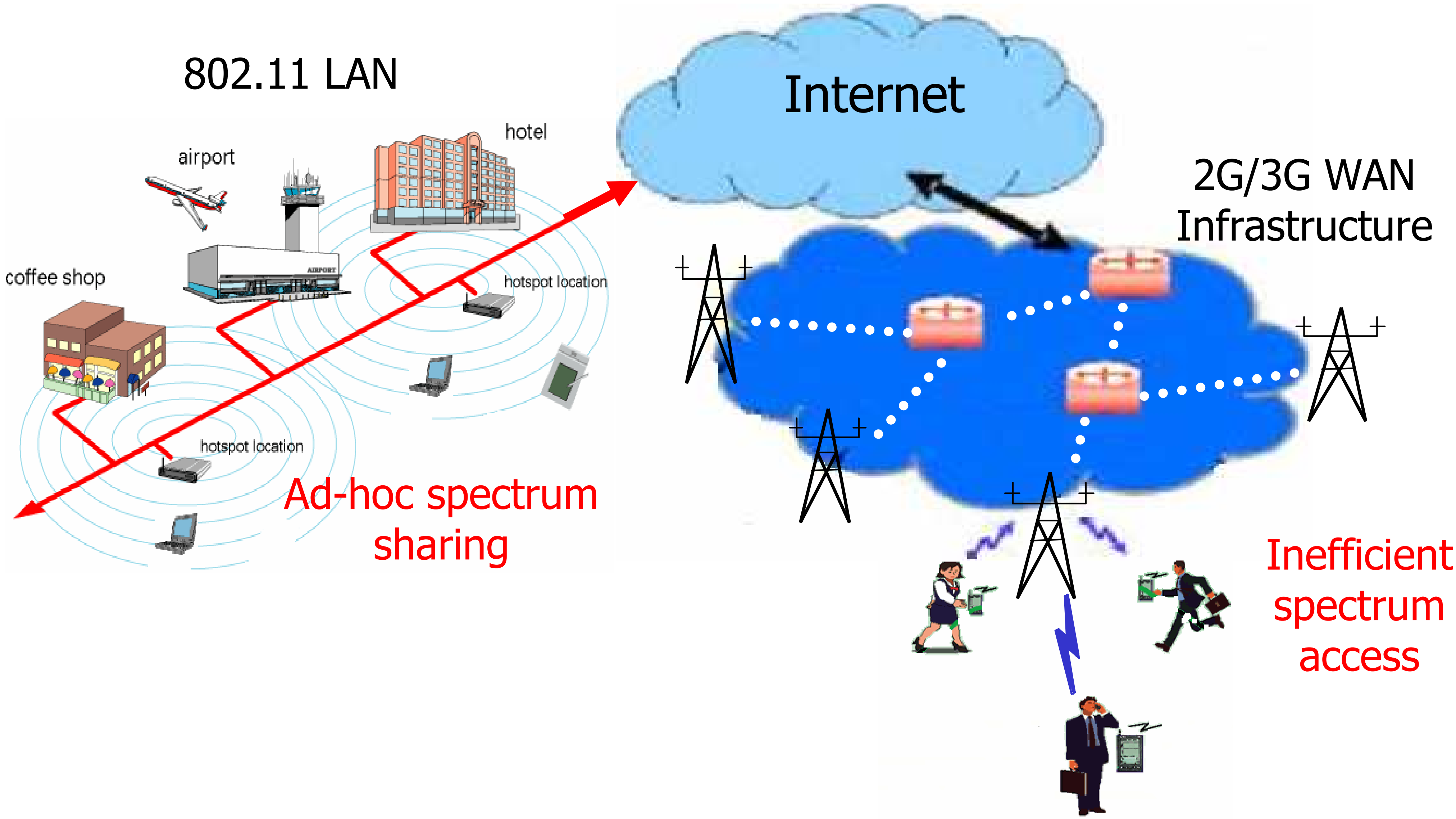
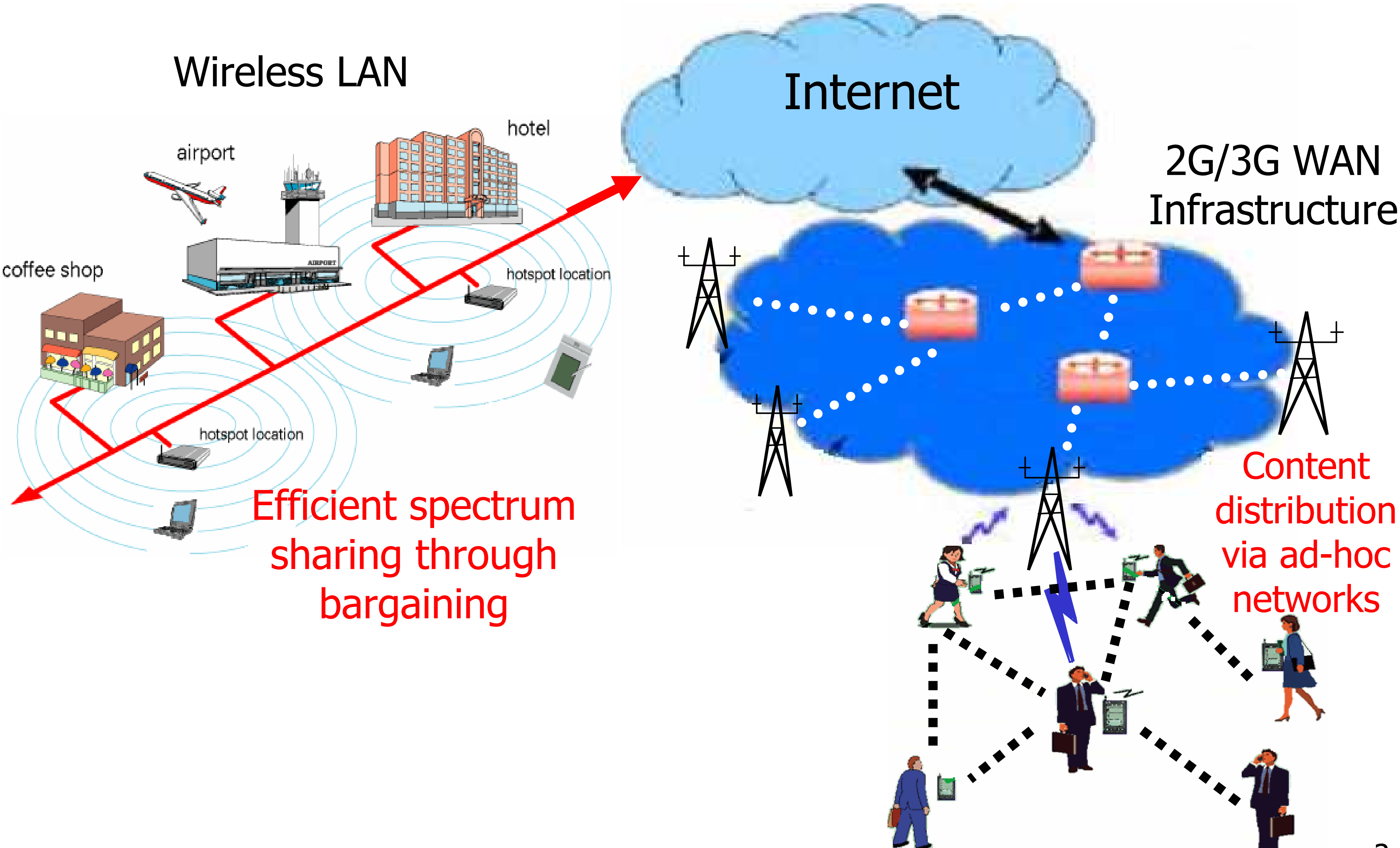
Today's Wireless Internet



Wireless Internet with Selfish Agents



Efficient spectrum sharing through bargaining

Content distribution via ad-hoc networks

Outline

- Improving 3G spectrum efficiency through content distribution in ad-hoc network
  - M. Goemans, L. Li, V. Mirrokni and M. Thottan, “Market sharing game applied to content distribution in ad-hoc netoworks”, MobiHoc’04

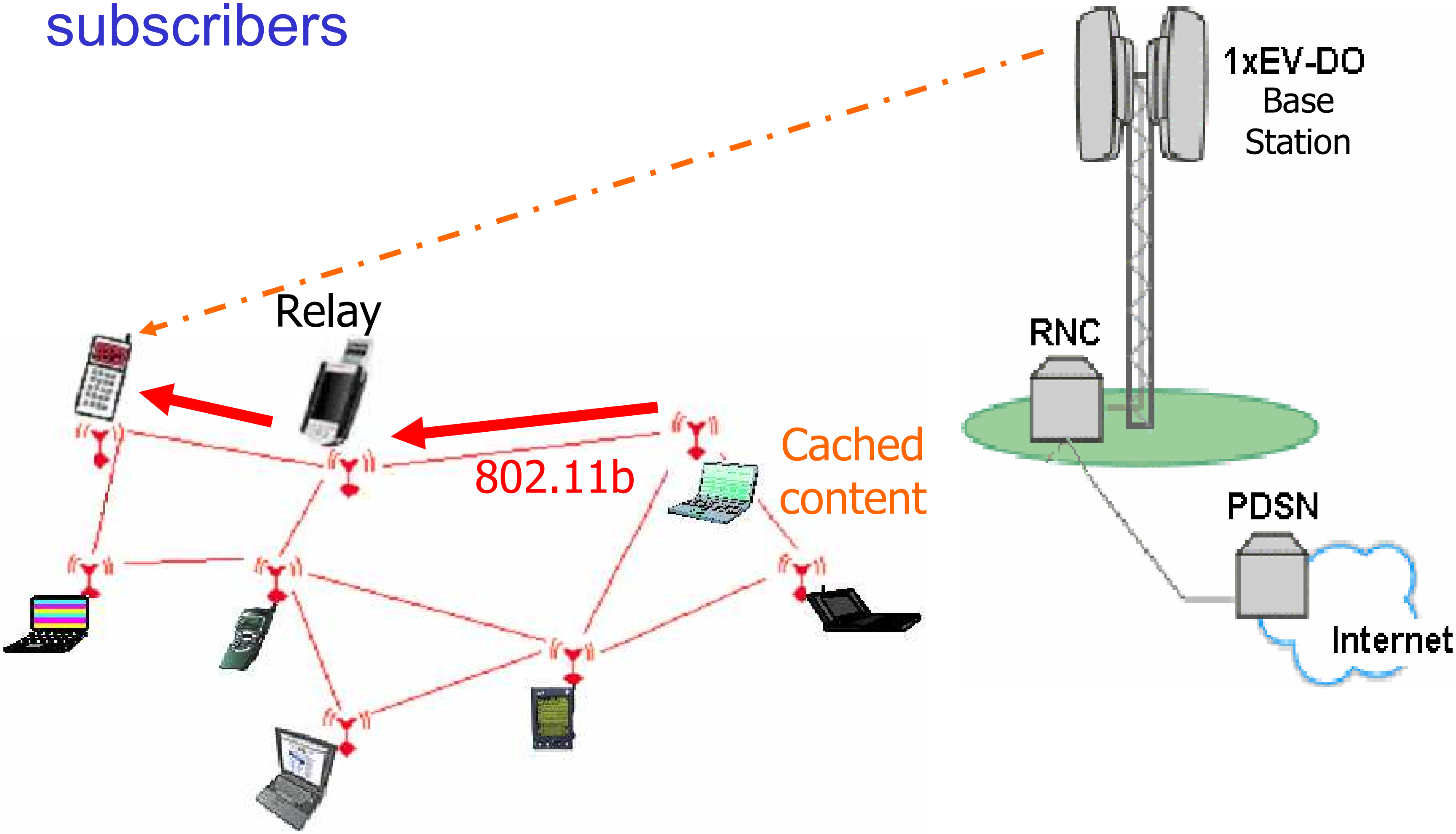
- Improving 802.11 spectrum efficiency through bargaining
  - M. Halldorsson, J. Halpern, L. Li, and V. Mirrokni,”on Spectrum sharing games”, PODC’04

Part I: Improving 3G spectrum access using selfish agents

- Architecture and Protocol
- Incentive and Security Mechanisms
- Game Theoretic Analysis
  - Price of Anarchy
  - Convergence to Pure Strategy Nash Equilibrium
- Evaluation
- Related Work
- Summary

Architecture and Protocol

- Resident subscribers cache popular items from the 3G service provider
- Transit subscribers are serviced by resident subscribers



Incentive and Security Mechanisms

- Incentives
  - Serving a query of item i gets a reward R_i
  - Forwarding a query, total reward f_i, $f_i << R_i$
  - Serviced by the 3G network $C_s(i)$
  - Serviced by resident subscribers $C_0(i)$, $C_0(i) << C_s(i)$

- Security mechanisms
  - Each subscriber has a shared key with service provider
    - authenticate routes
  - Session key
    - Encrypt item by sender during transmission
    - Decrypt item by receiver when session completes
  - Forwarding nodes are informed of the item size, and sample packets and report to the service provider to prevent various cheating behaviors

Incentive and Security Mechanisms (cont'd)

- Cheating behaviors are prevented or discouraged
  - Stealing rewards from forwarding nodes
  - Refusing to pay by the receiver
  - Impersonating the sender
  - Packet dropping
  - Free riding
  - Suboptimal routes

Game Theoretic Model

- Need to answer three questions:
  - Do stable solutions exist?
  - How fast can the players converge to one of them?
  - How far is a stable solution from optimal?

- Assumptions
  - Each player j has a storage space B_j
  - Each item i has a query rate q_i, and size C_i
  - A player's payoff is the sum of the payoff from each item
  - A player may not be interested in all items due to locality of popular content
  - The payoff of an item is equally divided among players who cached the item

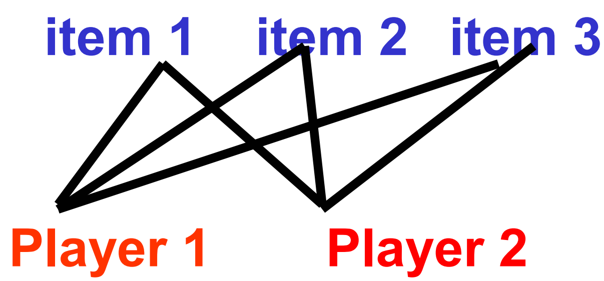
Game Theoretic Model (cont'd)

- Market sharing game
  - A bipartite graph $G=(H \cup U, E)$
    - H is the set of popular items
    - U is the set of players
    - An edge exists between agent j and item i if i is of interest to j
  - A player's action is to choose which set of items to cache
  - A player's payoff is the sum of the payoff from each item
    - An item i has a payoff q_i/n_i if n_i players cache item i

Inefficiency of Non-Cooperation

- Social function: the total queries satisfied by the ad-hoc network
- Price of anarchy: the ratio between the social optimal and the outcome of the selfish behavior of players
  - The social function is a submodular set function and satisfies other properties of valid games, so it is a valid-utility game and the price of anarchy is at most 2.
  - Zipf distribution: 1.45 for complete bipartite graph; 2 for non-complete bipartite graph

Query rate(Payoff) of Items: 10,4,3



- ➢**Both player will cache item 1 and get a payoff of 5**
- ➢**Price of anarchy: 14/10**

Nash Equilibrium (Existence and Finding)

- Pure strategy Nash equilibrium exists.
  - It is a congestion game and we can define a potential function.
- Pure strategy Nash equilibrium for uniform-size items can be found in polynomial time.
  - We find a best-response path of length $O(n^2)$.
- Computing Pure strategy Nash equilibrium in general is NP-hard.
  - need to solve a knapsack problem.

Behavior Analysis and Convergence

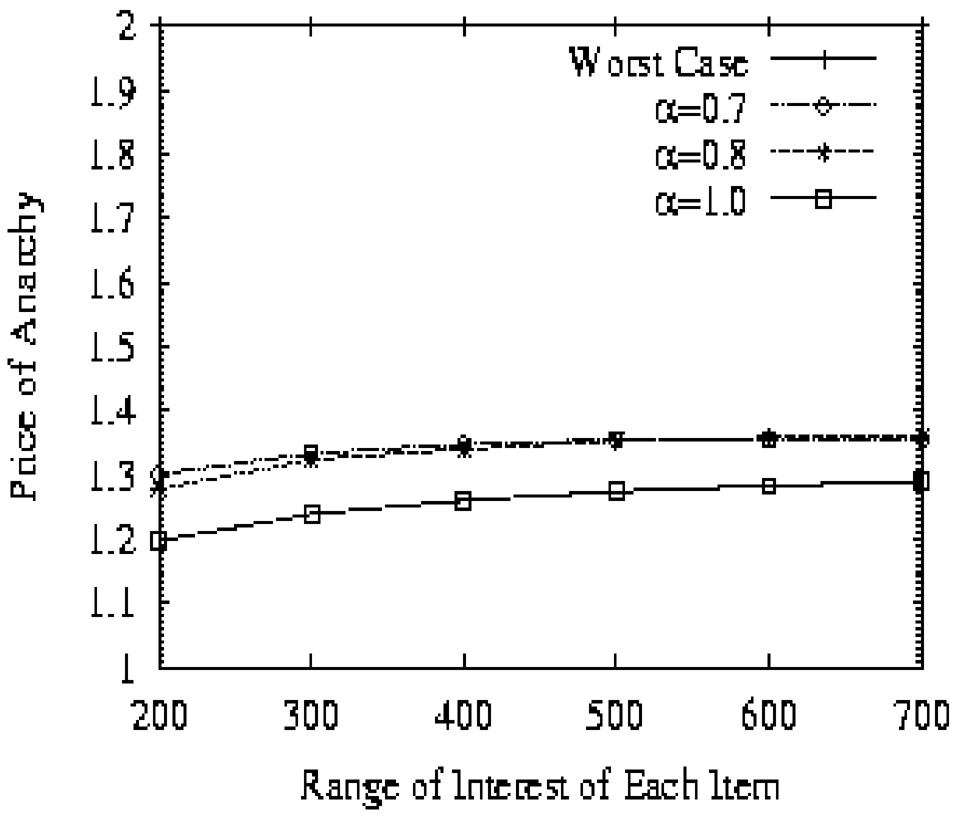
- Each player uses a β-approximation algorithm to compute its approximate best response
- Players will converge to a β-approximate Nash equilibrium
- After one round of best response the social value of the assignment is within log(n) factor of the optimal

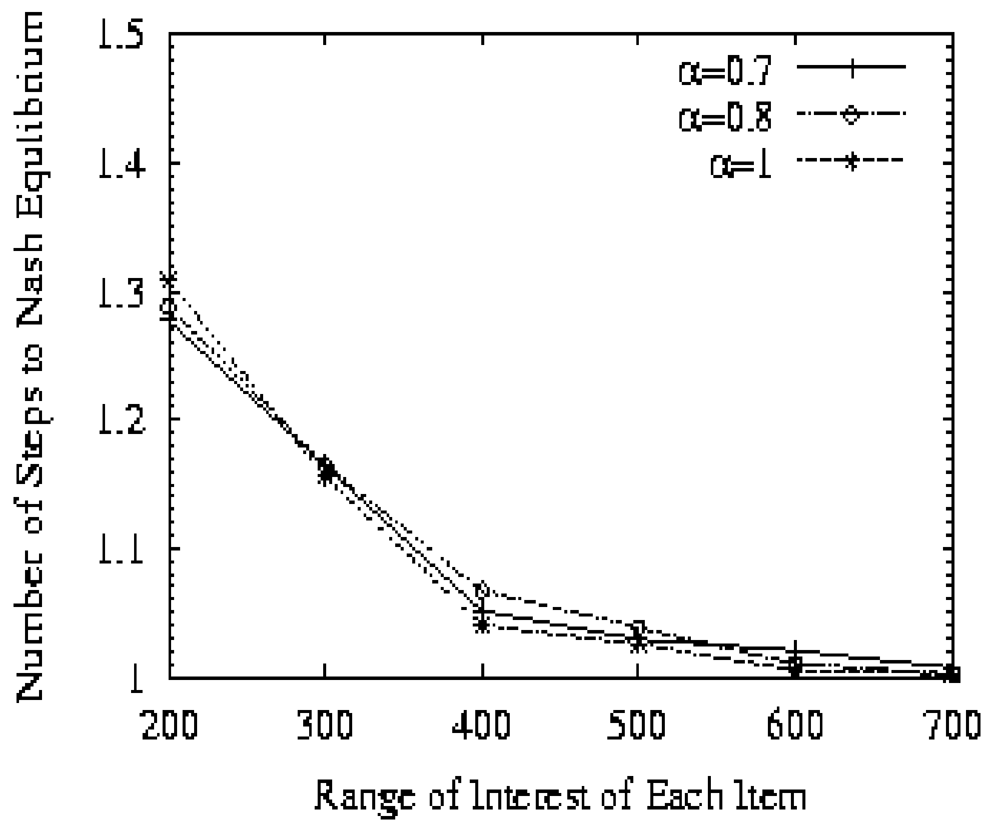
Evaluation

- Setting:
  - 100 resident nodes
  - 800x800 area
  - Each node can cache 5 items in the uniform case, 20 units in the non-uniform case
  - Item sizes for the non-uniform case follow a lognormal distribution
  - Transmission range 115 meters
  - 1000 items with Zipf distribution $1/i^{\alpha}$
  - Vary radius of interest of items and α

Evaluation: Uniform Case

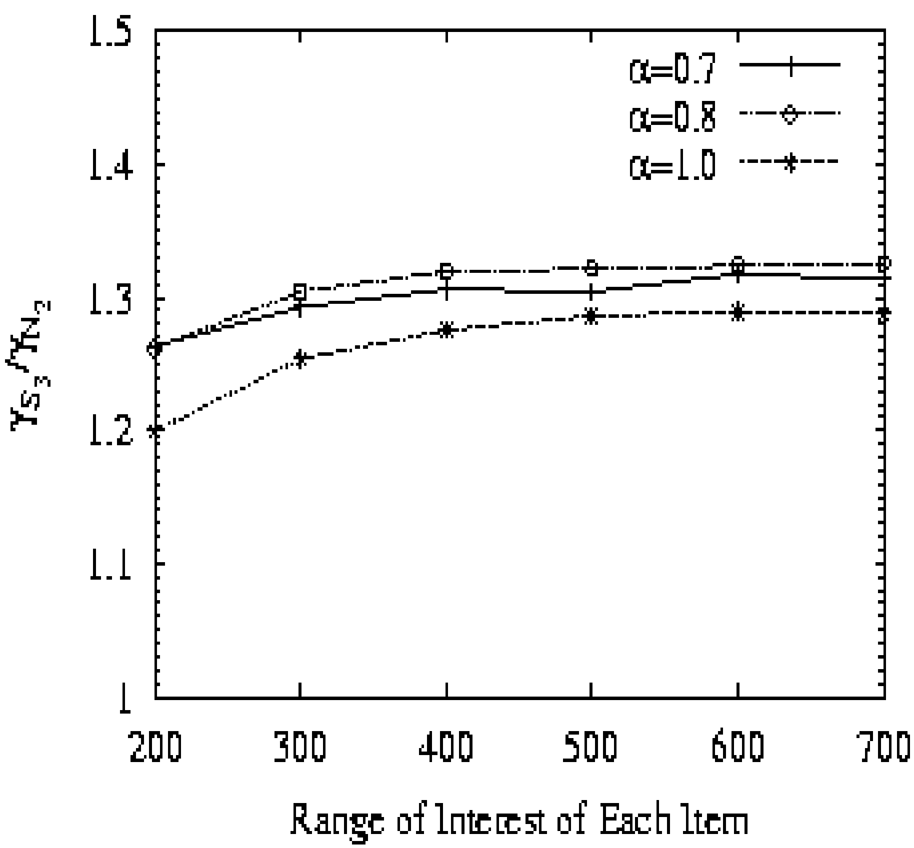
- Price of Anarchy and Convergence
  - Inefficiency due to selfish behavior is small (< 1.36)
  - Greedy behavior quickly converges to Nash equilibrium (1 or 2 rounds)

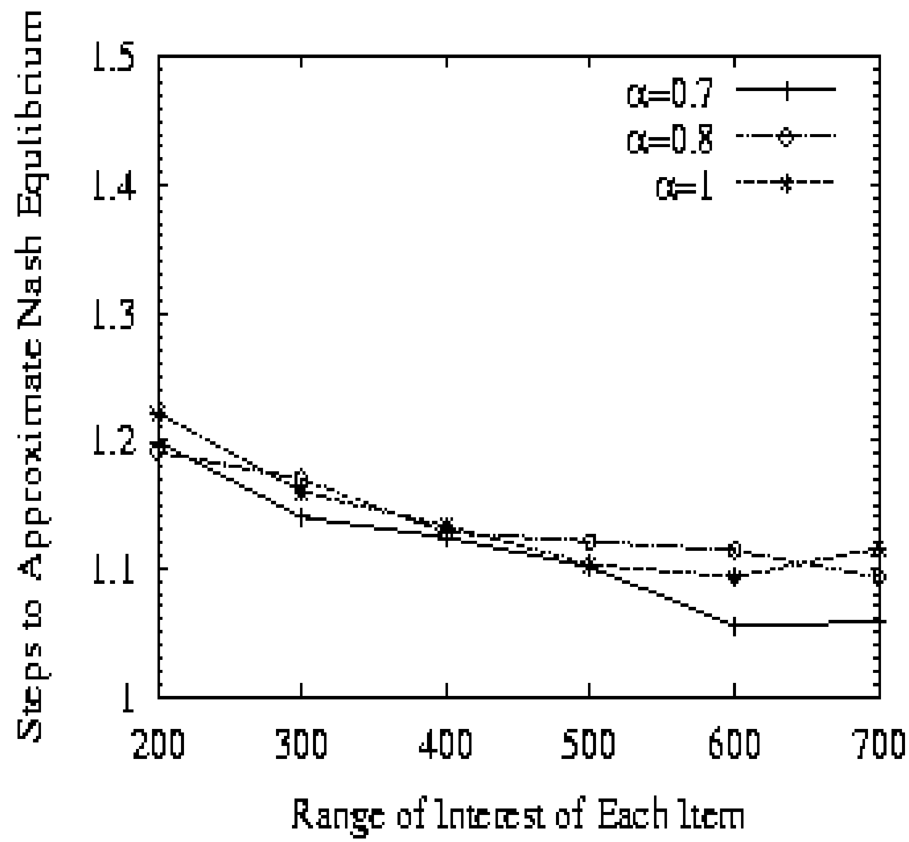




Evaluation: Non-Uniform Case

- Price of Anarchy and Convergence
  - Inefficiency due to selfish behavior is small (<1.33)
  - Greedy behavior quickly converges to approximate Nash equilibrium (1 or 2 rounds)





Related Work

- Incentive and Game Theory in Ad-Hoc Networks
  - Providing forwarding incentives
    - Sprite, CONFIDANT, Ad-Hoc VCG
  - Analyzing incentives to connect and form a network
    - topology-control game (POMC'03)

Summary

- 3G spectrum access efficiency can be improved by offloading content from 3G to ad-hoc networks
  - Inefficiency of selfish behavior is small.
  - Convergence to an approximate solution is fast.

- Open Problems
  - Find approximate Nash Equilibrium in polynomial time.
  - Convergence to constant factor approximation.

Part II: Improving wireless LAN spectrum access through bargaining

- Motivation
- Network Model and Game Theoretic Model
- Price of Anarchy
- Related Work
- Summary

Motivation

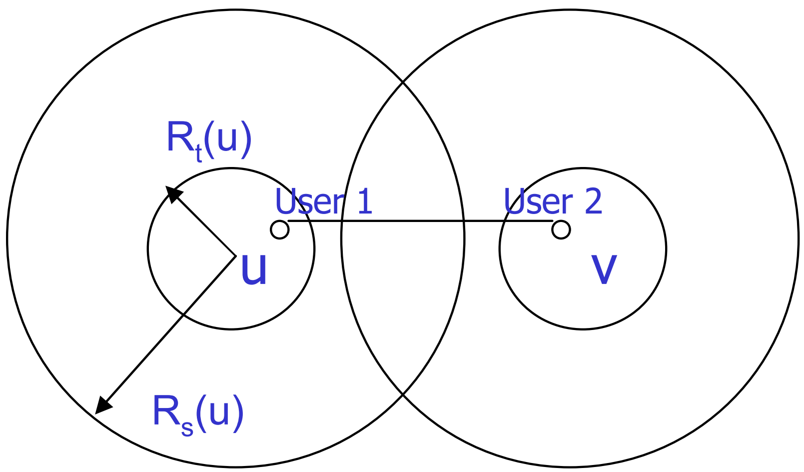
- The Federal Communication Commission (FCC) in the US allocates two types of spectrum
  - Dedicated spectrum: exclusively used by one entity
  - Free spectrum: available for any entity
- Dedicated spectrum allocation is very inefficient.
  - Recent measurements by M. McHenry "Dupont Circle Spectrum Utilization during Peak Hour".
- The question is how efficient free spectrum allocation is compared to the optimal allocation, i.e., what is the price of anarchy?

Network Model

- We study the 802.11 network setting
  - There are a limited number of non-interfering channels, e.g. 3 for 802.11b
  - Each agent owns a set of Access Points (AP)
  - Each AP u
    - must be assigned a channel
    - is set a transmission power P
      - P determines the transmission range $R_t(u)$ and the interference range $R_s(u)$
    - can service any subscriber within $R_t(u)$

Network Model (Cont'd)

- Interference graph G(V,E):
  - V is the set of APs
  - An edge $(u,v) \in E$ if u,v can not be assigned the same channel, i.e., $dist(u,v) < R_t(u) + R_t(v) + \max(R_s(u), R_s(v))$



*Thus, the interference graph is a unit disk graph if each AP uses the same power **P**.*

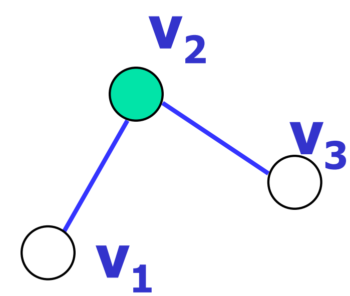
Game Theoretic Model

- The utility of the service provider (agent) for an AP is:
  - the expected number of users in $R_t(u)$ if a channel is assigned
  - 0 if the AP can not be assigned a channel
- An agent can assign channel ***A*** to an AP if:
  - channel ***A*** is available
  - channel ***A*** is obtained through bargaining
- The utility of an agent is the sum of the utilities of all its APs.

Game Theoretic Model (Cont'd)

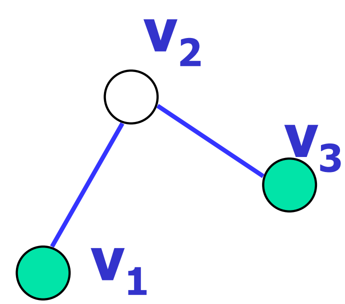
- The correspondence of channel assignment and graph coloring
  - A *social optimal* assignment corresponds to a maximum *k-colorable* sub-graph of the interference graph
  - The assignment of a Nash Equilibrium corresponds to a maximal *k-colored* subset of nodes
  - The set of nodes assigned a given channel forms a maximal independent set (MAX-IS)

Game Theoretic Model (Cont'd)



$w(v_1)+w(v_3)>w(v_2)$

- We consider two easily implementible *local* bargains
  - 2-buyer-1-seller bargain
    - If two APs v_1,v_3 can be colored by un-coloring AP v_2, and $w(v_1)+w(v_3)>w(v_2)$, then the exchange will be made in the equilibrium
  - 1-buyer-multiple-seller bargain
    - If an AP is uncolored, but its weight is greater than the sum of weights of all its neighbors of a particular color, then the AP will be colored by un-coloring the interfering APs through bargaining
- These bargains correspond to local improvement of graph coloring



$w(v_2)>w(v_1)+w(v_3)$

Price of Anarchy

- The ratio between the payoff of social optimal and the total payoffs of the worst case Nash Equilibrium
- We consider the following games:

Game	Transmission power	User distribution
Basic coloring game	Uniform	Uniform
Weighted coloring game	Uniform	Non-uniform
Basic power control game	Non-uniform	Uniform
Weighted power control game	Non-uniform	Non-uniform

- We consider how different types of bargains improve the price of anarchy

Price of Anarchy:
k colors vs. 1 color

- Theorem 1: If the price of anarchy in the game where a certain type of bargaining is allowed and there is one channel is ρ,

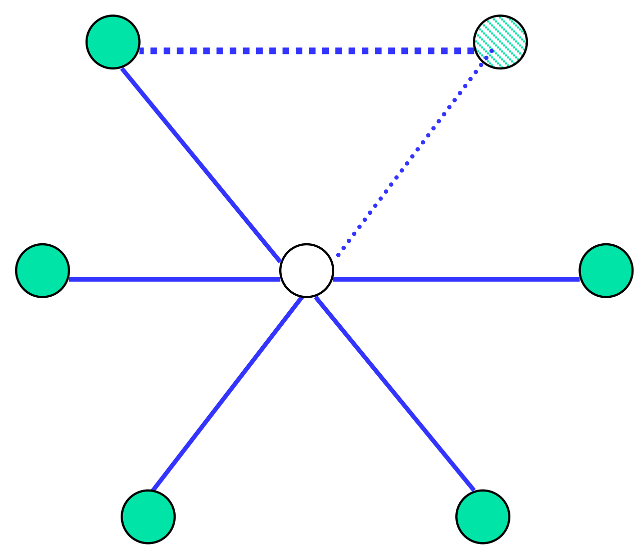
  then, for all k, the price of anarchy for the same game with k channels and the same type of bargaining is at most $\rho+\max(0,1-\rho/k)$ and at least ρ.

Thus, we only need to consider the 1 channel case.

Price of Anarchy: Basic Coloring Game

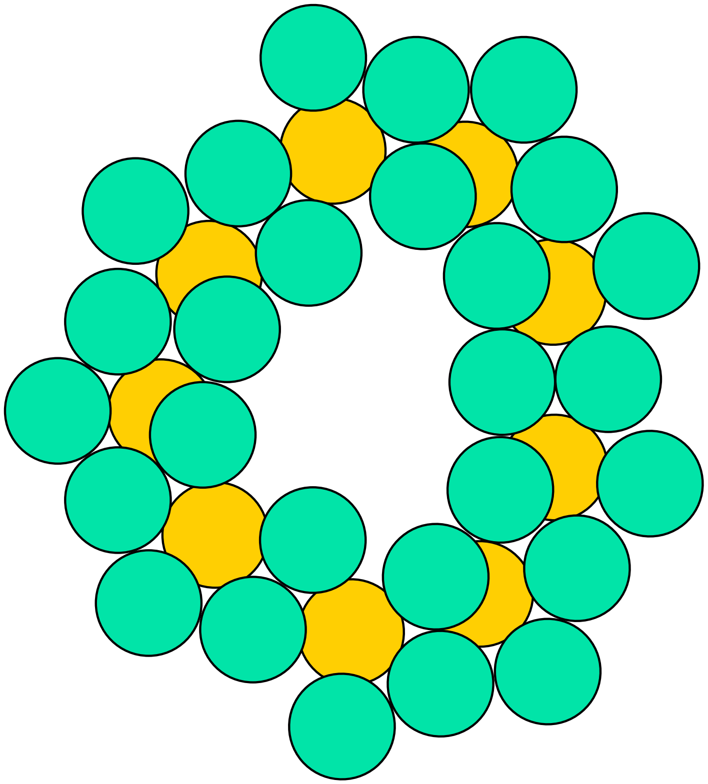
- The utility of an agent is the area covered by its APs which are assigned channels
  - Proportional to the number of “colored” APs
- Theorem 2: The price of anarchy for this case is at most 5+max(0,1-5/k) and at least 5.
  - Follows from Theorem 1 and following example

unit disk graph is 6-claw free, i.e., the size of a MAX-IS $\geq$ 1/5 $\times$ the size of the largest IS



Price of Anarchy: Basic Coloring Game (Cont'd)

- Theorem 3 (Bargaining can help!)
  If 2-buyer-1-seller bargains are allowed, then the price of anarchy is at most 3+max(0,1-3/k) and at least 3.
  - Upper bound follows from the analysis of local optimization for independent set by Hurkens and Schrijver'89
  - Lower bound follows from the example on the right

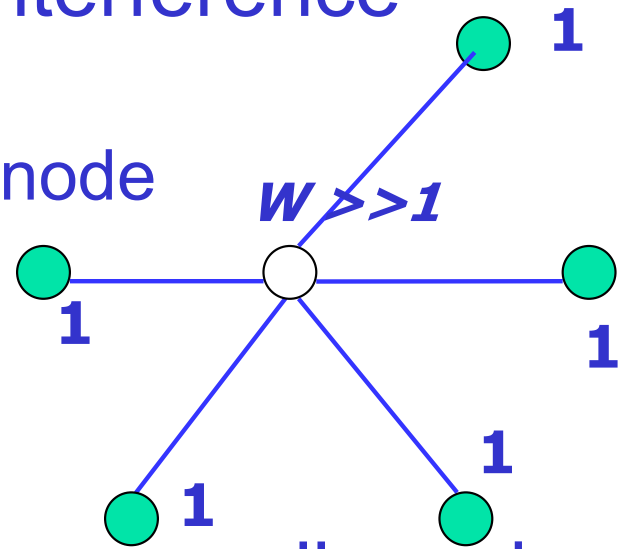


Lower bound of 27/9 = 3

Price of Anarchy: Weighted Coloring Game

- Theorem 4 (Unbounded without bargaining!)

  The price of anarchy is unbounded unless bargains involved at least min(p,τ) where p is the number of players and the interference graph is (τ +1)-claw free.
  - Consider a star where the central node has a large weight and τ leaves of smaller weight



- Theorem 5 (Bargaining Helps!)

  If 1-buyer-multiple-seller bargains are allowed, the price of anarchy for this case is at most 5+max(0,1-5/k) and at least 5.
  - Argument similar to Theorem 2

Price of Anarchy: Basic Power Control Game

- Theorem 6 (Unbounded without bargaining!)

  The price of anarchy is unbounded unless bargains involved at least min(p,τ) agents, where p is the number of players and the interference graph is (τ +1)-claw free.

  - Argument similar to Theorem 4

Price of Anarchy: Basic Power Control Game (Cont'd)

- Theorem 7 (Bargaining Helps!)

  If 1-buyer-multiple-seller bargains are allowed, then the price of anarchy is at most 9 and at least 7-ε, for any ε >0.

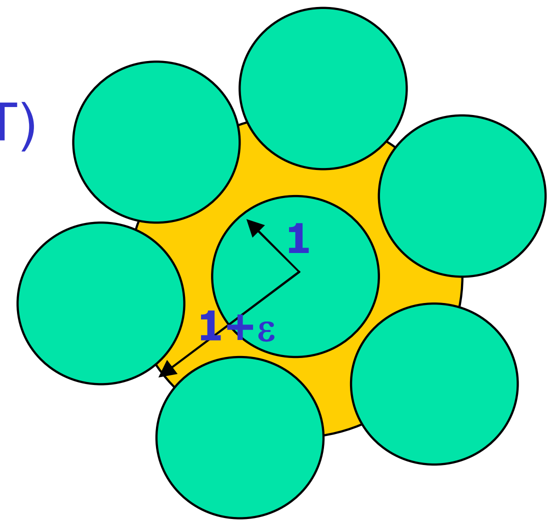
  - Proof Sketch of upper bound
    - Divide vertices in OPT into S(OPT) and L(OPT)

      small: interfere with at least one vertex with greater weight in LOPT;

      large: otherwise.
    - Lemma: for any u in LOPT, let $N_s(u)$($N_L(u)$) be the set of neighbors in S(OPT) (L(OPT)).

      Then $\Sigma_{v\in N_s(u)} w(v) \leq (9-|N_L(u)|)\Sigma_{v\in N_L(u)} w(v)$
    - $w(L(OPT)) \leq \Sigma_{v\in LOPT}|N_L(v)|\, w(v)$;

      $w(S(OPT)) \leq \Sigma_{v\in LOPT}(9-|N_L(v)|)\, w(v)$
  - Lower bound follows from the example



Price of anarchy > 7/(1+ ε)

Price of Anarchy: Weighted power control game

- Theorem 8 (Unbounded even with k-buyer-m-seller bargaining!)

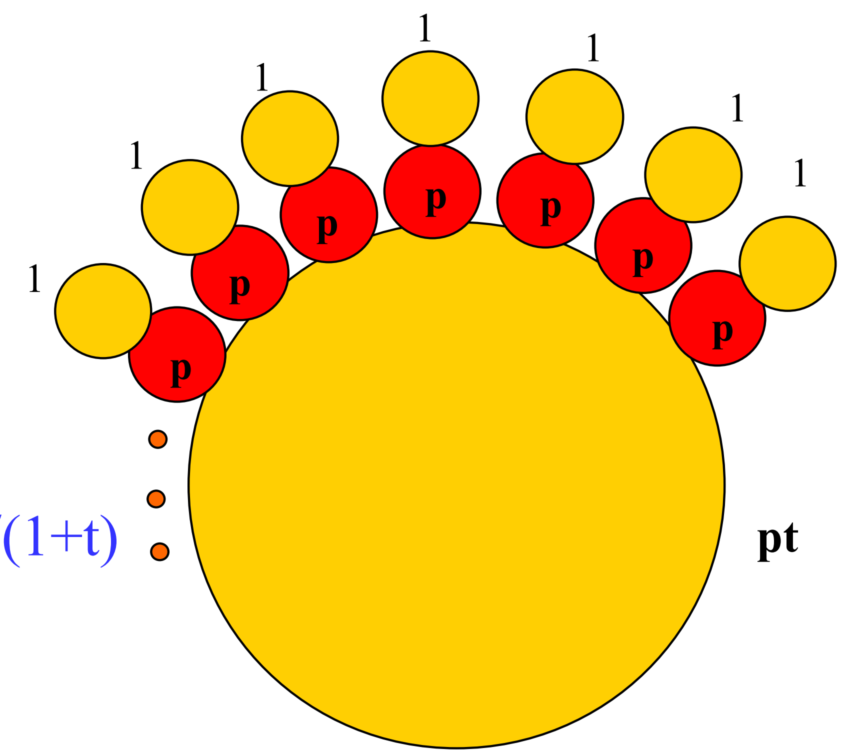
  The price of anarchy is unbounded even if k-buyer-m-seller bargains are allowed.

  - See the example

LOPT=pt+p

OPT=p^2

$\Rightarrow$ Price of anarchy =p/(1+t)



Price of Anarchy: Weighted power control game (Cont'd)

- Theorem 9 (Generalized bargains help!)

  Suppose distances have been normalized:
  - any two vertices with distance>1do not have an edge between them in the interference graph.

  Bargains with arbitrary sets of vertices within distance sqrt(d) are allowed.

  then, the price of anarchy is at most $(d/(d-1))^2$.

Related Work

- Price of anarchy
  - Worst-case equilibria, Koutsoupias and Papdimitriou,1999
  - Selish routing, Roughgarden and Tardos, 2002
  - Facility location game, A. Vetta, 2002
  - Market sharing game, M. Goemans et al., 2004
  - Selfish caching game, B.G. Chun, et al., 2004
- Spectrum allocation
  - Spectrum Etiquette, Satapathy and Peha, 2000
  - Artificial economy, O. Aftab, 2002

Summary

- We modeled spectrum sharing as a game
- If k-buyer-m-seller bargains are allowed, then the price of anarchy is bounded if users are distributed uniformly or every AP uses the same transmission power
- Future directions
  - Further investigate the weighted power control game
  - Investigate the price of anarchy of different types of bargaining procedures
  - Investigate time to convergence of Nash equilibrium under various assumptions about bargaining

Conclusion

- As wireless networks get more and more pervasive and decentralized, wireless networking is bound to cope and exploit selfish agents