Target Tracking in MIMO Radar Systems: Techniques and Performance Analysis

Hana Godrich°, Vlad M. Chiriac°, Alexander M. Haimovich°, and Rick S. Blum†
°New Jersey Institute of Technology, Newark, NJ 07102, email: [hg44,vc43,haimovic]@njit.edu
†Lehigh University, Bethlehem, PA 18015-3084, email: rblum@eecs.lehigh.edu

***Abstract*—In this paper, moving target tracking performance in multiple input multiple output (MIMO) radar systems with distributed antennas and non-coherent processing is studied. Due to the use of multiple, widely distributed antennas, MIMO radar architectures support both centralized and decentralized tracking techniques. Each receiving radar may contribute to central processing by providing either raw data or partially/fully processed data. Estimation performance of centralized and decentralized tracking is analyzed through the Bayesian Cramer-Rao bound (BCRB). The BCRB offers insight into the effect of the radars geometric layout, the target location, and propagation path losses on tracking accuracies. It is shown that, with different propagation path loss, the manner in which decentralized estimations are combined in the fusion center effects the overall estimation performance. Two tracking algorithms are proposed, corresponding to respectively, a centralized and decentralized modes of operation. It is demonstrated that communication requirements and processing load may be reduced at a relatively low performance cost. Based on mission needs, the system may use either approach: centralized for high accuracy or decentralized for resource-aware tracking.**

***Index Terms*—MIMO radars, BCRB, target tracking.**

I. Introduction

Elements of multiple input multiple output (MIMO) radar have the ability to transmit diverse waveforms, ranging from independent to fully correlated, by utilizing multiple transmit and receive sensors, widely distributed over an area [1]- [8]. Widely distributed MIMO radar systems enjoy a diversity gain manifested in metrics such as the probability of missed detections and target localization accuracy [3], [7]. These systems improve radar performance by exploiting radar cross section (RCS) diversity [2]. The study of MIMO radar systems has shown improvements in target localization accuracy performance over traditional radar systems [1]. The Cramer-Rao bound (CRB) [9] for target localization accuracy, developed for MIMO radar systems with coherent and non-coherent processing, was shown to be inversely proportional to the signal effective bandwidth in the non-coherent case, but approximately inversely proportional to the carrier frequency in the coherent case. It was further proven that optimization over the sensors' positions lowers the CRB by a factor equal to the product of the number of transmitting and receiving sensors in both non-coherent and coherent processing [7]. Applications of MIMO radar systems for velocity estimation have been studied in [6].

Target tracking is an essential requirement for surveillance systems. Over the years, significant research has been performed in this area [10]- [13]. Lower bounds, such as the Bayesian CRB (BCRB), set performance limits on target tracking, providing useful tools for evaluating the effect of system parameters on estimation accuracies [14]- [15]. A comparative study of the performance of various tracking algorithms for non-coherent MIMO radars is given in [16]. The BCRB is derived for multi-static radar systems, an architecture based on one transmit radar and multiple receive radars in [15]. Target location accuracy is demonstrated for a given target path. Study of tracking capabilities of a moving target is offered through the application of the BCRB for the estimation of both target location and velocity. In [17], it is shown that increasing the number of transmitting and receiving radars provides better tracking performance in terms of higher accuracy target location and velocity estimation. The performance gain is shown to be proportional to the increase in the product of the number of transmitting and receiving radars. Wider spread of the radars results in better accuracies.

In this paper, tracking algorithms for MIMO radar systems with widely separated antennas are proposed, exploiting their inherent ability to utilize centralized and decentralized tracking techniques. Centralized tracking refers to a mode of operation where each receiver sends raw data, or partially processed data, to a central fusion center. Estimates of the targets' location and velocity are obtained through joint processing of the received data. In the decentralized case, each receiver performs its own local estimates of the targets' parameters (location and velocity). These local estimates are then sent to a central fusion center where they are combined to a joint estimate. Herein, the BCRB is formulated to support centralized and decentralized tracking performance analysis. The effect of the propagation path loss on target tracking performance is presented.

The paper is organized as follows: the system signal model is introduced in Section II. The BCRB on target location and velocity is derived in Section III. The contribution of target reflectivity and path loss to tracking performance is evaluated in Section IV. Two tracking algorithms are proposed and analyzed in Section V. Finally, Section VI concludes the paper.

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II. System Model

Assume M transmitting and N receiving radars, widely distributed over an area and time synchronized. The receiving radars could be collocated or separated from the transmitting radars. The radars are located in a two dimensional plane. Consider a single extended target moving in the plane containing the radar sites. An extended target has an RCS that varies as a function of aspect angle. The target's initial location is (x_o, y_o) and its initial velocity is $(\dot{x}_o, \dot{y}_o)$. At state n, defined at the time interval $n\Delta t$, where Δt is the duration of the observation interval, the target is located at coordinates (x_n, y_n) and moves with velocity $(\dot{x}_n, \dot{y}_n)$. The radars coordinate the transmission of a set of orthogonal waveforms, with the lowpass equivalent $s_k(t)$, $k = 1, \ldots, M$. The power of the transmitted waveforms is normalized such that the aggregate power transmitted by the sensors is constant, irrespective of the number of transmit sensors. Let all transmitted waveforms be narrowband signals with individual effective bandwidth β_k defined as $\beta_k^2 = \left[\left(\int_{W_k} f^2 \left|S_k(f)\right|^2 df\right) / \left(\int_{W_k} \left|S_k(f)\right|^2 df\right)\right]$, and an effective time duration T_{b_k} defined as $T_{b_k}^2 = \left[\left(\int_{T_k} t^2 \left|s_k(t)\right|^2 dt\right) / \left(\int_{T_k} \left|S_k(t)\right|^2 dt\right)\right]$, where the integration is over the range of frequencies with non-zero signal content W_k [10]. The signals are narrowband in the sense that for a carrier frequency of f_c, the narrowband signal assumption implies $\beta_k^2 / \ f_c^2 \ll 1$.

The estimate of the propagation time of a signal transmitted by the k-th transmitting radar located at coordinates $T_k = (x_{tk}, y_{tk})$, reflected by a target located at (x_n, y_n) and received by a radar located at $R_\ell = (x_{r\ell}, y_{r\ell})$ can be expressed:

$$\hat{\tau}_{n_{\ell k}} = \tau_{n_{\ell k}} + \varepsilon_{\tau_n}, \tag{1}$$

where $k = 1, .., M$, $\ell = 1, .., N$, the true propagation time $\tau_{n_{\ell k}}$, is the sum of the time delays from radar k to the target and from the target to radar ℓ in state n:

$$\begin{aligned} \tau_{n_{\ell k}} = & \frac{1}{c}\left(\sqrt{\left(x_{tk} - x_n\right)^2 + \left(y_{tk} - y_n\right)^2}\right. \\ & \left. + \sqrt{\left(x_{r\ell} - x_n\right)^2 + \left(y_{r\ell} - y_n\right)^2}\right), \end{aligned} \tag{2}$$

and ε_{τ_n} is the estimation error. The speed of light is denoted by c.

The Doppler shift estimate of a signal transmitted on the ℓk-th path is expressed:

$$\hat{\omega}_{n_{\ell k}} = \omega_{n_{\ell k}} + \varepsilon_{\omega_n}, \tag{3}$$

where $\omega_{n_{\ell k}}$ is the true Doppler shift on the ℓk-th path due to the target velocity at state n:

$$\omega_{n_{\ell k}} = -\frac{2\pi}{\lambda}\left[\cos\left(\phi_{k_n} + \varphi_{\ell_n}\right)\dot{x}_n + \sin\left(\phi_{k_n} + \varphi_{\ell_n}\right)\dot{y}_n\right], \tag{4}$$

and ε_{ω_n} is the estimation error. The term $\dot{x}_n$ stands for the target velocity in direction x, and $\dot{y}_n$for the target velocity in direction y, at the state n. The phase ϕ_{k_n} is the bearing angle of the target at transmitting sensor k with respect to the x axis; the phase φ_{ℓ_n} is the bearing angle of the target at receiving radar ℓ measured with respect to the x axis.

Consider the case of a baseband representation of the signal observed at sensor ℓ due to a transmission from sensor k and reflection from the target at coordinates $X = (x_n, y_n)$, is given by:

$$r_{n_\ell}(t) = \sum_{k=1}^{M} \alpha_{n_{\ell k}} s_{n_k}\left(t - \tau_{n_{\ell k}}\right) e^{-j\omega_{n_{\ell k}} t} + w_{n_\ell}(t), \tag{5}$$

where $\alpha_{n_{\ell k}}$ represents the combined effect of path loss, target reflectivity in direction ℓk-th, and phase shift along the path, the noise $w_{n_\ell}(t)$ is circularly symmetric, zero-mean, complex Gaussian, spatially and temporally white with autocorrelation function $\sigma_w^2 \delta(\tau)$. The model does not include clutter. The observation vector $\mathbf{r}_n$ at state n is:

$$\mathbf{r}_n = \left[r_{n_1}(t), \ldots, r_{n_N}(t)\right]^T. \tag{6}$$

The signal received at each sensor is a mixture of the transmitted signals reflected by the target. The mixture of signals is separated at the receiver end by exploiting the orthogonality between the transmitted waveforms.

The state vector $\mathbf{x}_n$, treated as unknown random, represents the target location and velocity at state n, is:

$$\mathbf{x}_n = \left[x_n, y_n, \dot{x}_n, \dot{y}_n\right]^T. \tag{7}$$

The state model is a linear motion model [12] and [13], represented as:

$$\mathbf{x}_n = \mathbf{F}\mathbf{x}_{n-1} + \mathbf{v}_{n-1}, \tag{8}$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9}$$

and $\mathbf{v}_n$ is the plant noise, modeled as white Gaussian noise process with covariance matrix $\mathbf{Q_v}$ of the form:

$$\mathbf{Q_v} = \begin{bmatrix} \frac{1}{3}\Delta t^3 \mathbf{I} & \frac{1}{2}\Delta t^2 \mathbf{I} \\ \frac{1}{2}\Delta t^2 \mathbf{I} & \Delta t \mathbf{I} \end{bmatrix}. \tag{10}$$

The observation however, is a nonlinear function of the vector of unknown parameters $\boldsymbol{\psi}_n = \left[\boldsymbol{\tau}_n^T, \boldsymbol{\omega}_n^T\right]^T$:

$$\mathbf{z}_n = \mathbf{d}\left(\boldsymbol{\psi}_n\left(\mathbf{x}_n\right)\right) + \mathbf{w}_n, \tag{11}$$

where $\mathbf{d}$ stands for the observation. In the MIMO radar case, $\mathbf{d}$ is the set of signals observed at the receiving radars, expressed by (5) as a nonlinear function of a vector of unknown parameters $\boldsymbol{\psi}_n$, $\boldsymbol{\tau}_n = \left[\tau_{n_{11}}, \tau_{n_{12}}, ...\tau_{n_{MN}}\right]^T$, and $\boldsymbol{\omega}_n = \left[\omega_{n_{11}}, \omega_{n_{12}}, ...\omega_{n_{MN}}\right]^T$.

The Kalman filter (KF) provides an efficient recursive mean to estimate the state of a process, in a way that minimizes the estimation MSE. The KF addresses the problem of estimating state $\mathbf{x}_n$ of a discrete-time controlled process described by the linear stochastic difference equation, given (8), with a measurement $\mathbf{z}_n$, given by (11). When the state and the observation models are linear, the KF is give in [12]. We first define the covariance matrix at state n as:

$$\mathbf{P}_{n|n} \triangleq E\left\{\left(\mathbf{x}_n - \hat{\mathbf{x}}_n\right)\left(\mathbf{x}_n - \hat{\mathbf{x}}_n\right)^T\right\}, \tag{12}$$

where $\hat{\mathbf{x}}_n$ denotes the estimate of $\mathbf{x}_n$. The estimator equation for $\hat{\mathbf{x}}_{n|n-1}$, i.e., the estimated value of $\hat{\mathbf{x}}_n$ given $\hat{\mathbf{x}}_{n-1}$is

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{F}\hat{\mathbf{x}}_{n-1|n-1}, \tag{13}$$

and finally the estimate for state n is

$$\hat{\mathbf{x}}_{n|n} = \hat{\mathbf{x}}_{n|n-1} + \mathbf{G}_n\left[\mathbf{z}_n - \mathbf{H}_n\hat{\mathbf{x}}_{n|n-1}\right]. \tag{14}$$

where $\mathbf{H}_n = \mathbf{d}\left(\boldsymbol{\psi}_n\left(\hat{\mathbf{x}}_{n|n-1}\right)\right)$ and the gain matrix $\mathbf{G}_n$ is $\mathbf{G}_n = \mathbf{P}_{n|n-1}\mathbf{H}_n^T\left[\mathbf{R}_n + \mathbf{H}_n\mathbf{P}_{n|n-1}\mathbf{H}_n^T\right]^{-1}$. The MSE equation is $\mathbf{P}_{n|n-1} = \mathbf{F}_n\mathbf{P}_{n-1|n-1}\mathbf{F}_n^T + \mathbf{Q}_\mathbf{v}$.

In case the process to be estimated and (or) the measurement relationship to the process is non-linear, a KF that linearizes about the current mean and covariance is referred to as an *extended Kalman filter* or EKF. The recursive equations for the EKF differ incorporate a linearization of the relation in (11) where matrix $\mathbf{H}_n$ is $\mathbf{H}_n = \left[\nabla_{\hat{\mathbf{x}}_n}\mathbf{d}\left(\boldsymbol{\psi}_n\left(\mathbf{x}_n\right)\right)\right]^T_{\mathbf{x}_n = \hat{\mathbf{x}}_{n|n-1}}$. Both KF and EKF will be later applied for the centralized and decentralized tracking algorithms.

III. The Bayesian Cramer-Rao Bound (BCRB)

In [14], a recursive, multi-dimensional, generalized BCRB is developed. In general, the BCRB for an unknown vector parameter $\boldsymbol{\theta} \in R^{n\times 1}$, estimated using an observation vector $\mathbf{r}$, is of the form:

$$E_{\mathbf{r},\boldsymbol{\theta}}\left\{\left[\hat{\boldsymbol{\theta}}\left(\mathbf{r}\right) - \boldsymbol{\theta}\right]\left[\hat{\boldsymbol{\theta}}\left(\mathbf{r}\right) - \boldsymbol{\theta}\right]^T\right\} \geq \mathbf{J}_B^{-1} = \mathbf{C}_B, \tag{15}$$

where $\mathbf{J}_B$ is the Bayesian information matrix (BIM), and $\mathbf{C}_B$ is the BCRB matrix. The BIM is calculated using the joint probability density function (pdf) $p_{\mathbf{r},\boldsymbol{\theta}}\left(\mathbf{r},\boldsymbol{\theta}\right)$, as follows:

$$\left[\mathbf{J}_B\right]_{i,j} = E\left\{-\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log p_{\mathbf{r},\boldsymbol{\theta}}\left(\mathbf{r},\boldsymbol{\theta}\right)\right\}, \tag{16}$$

where the notation $[\cdot]_{i,j}$ refers to the term on row i and column j of a given matrix. Based on the relation $p_{\mathbf{r},\boldsymbol{\theta}}\left(\mathbf{r},\boldsymbol{\theta}\right) = p_{\mathbf{r}|\boldsymbol{\theta}}\left(\mathbf{r}\left|\boldsymbol{\theta}\right.\right)\cdot p_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}\right)$, it is shown in [15] that the BIM may be expressed as a linear combination of two matrices:

$$\mathbf{J}_B = \mathbf{J}_D + \mathbf{J}_P, \tag{17}$$

where $\mathbf{J}_D$ is the Fisher information matrix of the data, and $\mathbf{J}_P$ is the Fisher information matrix associated with the a priori knowledge about the unknown vector:

$$\left[\mathbf{J}_D\right]_{i,j} = E_{\boldsymbol{\theta}}\left\{-\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log p_{\mathbf{r}|\boldsymbol{\theta}}\left(\mathbf{r}\left|\boldsymbol{\theta}\right.\right)\right\}, \tag{18}$$

$$\left[\mathbf{J}_P\right]_{i,j} = E_{\boldsymbol{\theta}}\left\{-\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log p_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}\right)\right\}. \tag{19}$$

The joint conditional pdf of $\mathbf{r}$ is $p_{\mathbf{r}|\boldsymbol{\theta}}\left(\mathbf{r}\left|\boldsymbol{\theta}\right.\right)$ and $p_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}\right)$ is the pdf of $\boldsymbol{\theta}$. The BCRB sets the lower bound on the estimation MSE for the unknown vector parameter $\mathbf{x}_n = [x_n, y_n, \dot{x}_n, \dot{y}_n]$

$$E_{\mathbf{x}_n}\left\{\left(\left[\hat{\mathbf{x}}_n\right]_i - \left[\mathbf{x}_n\right]_i\right)\left(\left[\hat{\mathbf{x}}_n\right]_i - \left[\mathbf{x}_n\right]_i\right)^T\right\} \geq \mathbf{J}_{B_n}^{-1}\left(\mathbf{x}_n\right),$$

where "$\widehat{()}$" denotes estimated quantities and $\mathbf{J}_{B_n}$ is the BIM of the system at state n, defined in (16). The notation $\left[\mathbf{x}_n\right]_i$ refer to the i-th element of the vector $\mathbf{x}_n$. The recursive BCRB is of the form [14]:

$$\mathbf{J}_B\left(\mathbf{x}_n\right) = \left[\mathbf{Q}_\mathbf{v} + \mathbf{F}\mathbf{J}_B^{-1}\left(\mathbf{x}_{n-1}\right)\mathbf{F}^T\right]^{-1} + E_{\mathbf{x}_n}\left[\mathbf{J}_D\left(\mathbf{x}_n\right)\right], \tag{20}$$

where $\mathbf{J}_D\left(\mathbf{x}_n\right)$ is the FIM of the system at state $n+1$, defined in (18), and $E_{\mathbf{x}_n}[\cdot]$ is the expectation with respect to the joint pdf of the state vector $\mathbf{x}_n$. The FIM $\mathbf{J}_D\left(\mathbf{x}_n\right)$ is derived using the conditional pdf:

$$p\left(\mathbf{r}_n|\boldsymbol{\psi}\right) \propto \exp\left\{-\frac{1}{\sigma_w^2}\sum_{\ell=1}^{N}\int_T\left|r_{n_\ell}(t) - \right.\right. \tag{21}$$

$$\left.\left.\sum_{k=1}^{M}\alpha_{n_{\ell k}}s_{n_k}\left(t - \tau_{n_{\ell k}}\right)e^{-j\omega_{n_{\ell k}}t}\right|^2 dt\right\} \tag{22}$$

and by applying the chain rule [18]:

$$\mathbf{J}_D\left(\mathbf{x}_n\right) = \mathbf{H}_n\cdot\mathbf{J}_D\left(\boldsymbol{\psi}_n\right)\cdot\mathbf{H}_n^T, \tag{23}$$

where matrix $\mathbf{H}_n = \begin{bmatrix}\mathbf{H}_{n_{1,\tau}} & \mathbf{H}_{n_{1,\omega}}\\ \mathbf{H}_{n_{2,\tau}} & \mathbf{H}_{n_{2,\omega}}\end{bmatrix}$, the components of $\mathbf{H}_n$ are defined below, and matrix $\mathbf{J}_D\left(\boldsymbol{\psi}_n\right)$ is the FIM of the unknown vector $\boldsymbol{\psi}_n$, derived following the method in [7]. It can be shown that the FIM is given by,

$$\mathbf{J}_D\left(\boldsymbol{\psi}_n\right) = \text{diag}\ \left(\mathbf{J}_{\tau_n}, \mathbf{J}_{\omega_n}\right), \tag{24}$$

where matrix $\mathbf{J}_{\tau_n}$ is

$$\mathbf{J}_{\tau_n} = 8\pi^2\text{diag}\ \left(\frac{\left|\alpha_{n_{\ell k}}\right|^2}{\sigma_w^2}\beta_k^2\right), \tag{25}$$

and matrix $\mathbf{J}_{\omega_n}$ follows,

$$\mathbf{J}_{\omega_n} = 8\pi^2\text{diag}\ \left(\frac{\left|\alpha_{n_{\ell k}}\right|^2}{\sigma_w^2}T_{b_k}^2\right). \tag{26}$$

Elements of matrices $\mathbf{H}_n$ are found from the derivative of the expressions in (2) and (4) with respect to the state vector in (8). It can be shown that:

$$\begin{aligned}\left[\mathbf{H}_{n_{1,\tau}}\right]_{\ell k} &= \frac{1}{c}\begin{bmatrix}\cos\phi_{n_k} + \cos\varphi_{n_\ell}\\ \sin\phi_{n_k} + \sin\varphi_{n_\ell}\end{bmatrix},\\ \left[\mathbf{H}_{n_{2,\tau}}\right]_{\ell k} &= \mathbf{0},\\ \left[\mathbf{H}_{n_{2,\omega}}\right]_{\ell k} &= -\frac{2\pi c}{\lambda}\left[\mathbf{H}_{n_{1,\tau}}\right]_{\ell k},\end{aligned} \tag{27}$$

and

$$\left[\mathbf{H}_{n_{1,\omega}}\right]_{\ell k} = -\frac{2\pi}{\lambda}\begin{bmatrix}\left[\dot{x}_{n_{\ell k}}\left(\frac{\sin^2\phi_{n_k}}{R_{tk_n}} + \frac{\sin\varphi_{n_\ell}}{R_{r\ell_n}}\right)\right.\\ \left.-\dot{y}_{n_{\ell k}}\left(\frac{\sin 2\phi_{n_k}}{R_{tk_n}} + \frac{\sin 2\varphi_{n_\ell}}{R_{r\ell_n}}\right)\right]\\ \left[\dot{y}_{n_{\ell k}}\left(\frac{\cos^2\phi_{n_k}}{R_{tk_n}} + \frac{\cos^2\varphi_{n_\ell}}{R_{r\ell_n}}\right)\right.\\ \left.-\dot{x}_{n_{\ell k}}\left(\frac{\sin 2\phi_{n_k}}{R_{tk_n}} + \frac{\sin 2\varphi_{n_\ell}}{R_{r\ell_n}}\right)\right]\end{bmatrix}. \tag{28}$$

Using (24) and (27) in (23), yields:

$$\mathbf{J}_D\left(\mathbf{x}_n\right) = \mathbf{H}_n\mathbf{J}_D\left(\boldsymbol{\psi}_n\right)\mathbf{H}_n^T. \tag{29}$$

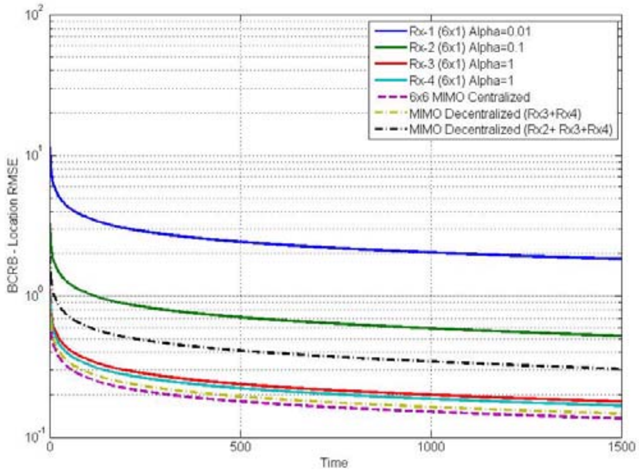

Fig. 1. BCRB on target location tracking for the systems in Figure 1 with receivers one and two experiencing different levels of path loss.

The lower bound on the state vector estimation error in state n, is found by substituting (29) in (20). Next, a numerical analysis of the BCRB in (20) is provided, leading to a better understanding of the tracking performance in MIMO radar systems.

IV. Numerical Analysis

It is apparent from (20) and (29) that the performance of the BCRB depends the on the geometric layout of the MIMO radar system, the track of the target, and the propagation path loss coefficients $|\alpha_{n_{\ell k}}|^2$. In practice, each propagation path between a set of transmitting and receiving radars has different characteristics, depending on path loss, target reflectivity and phase errors. To assess the effect of the propagation paths on tracking performance, different path propagation coefficients are set.

It can be seen in Figure 1 that receivers with increasing path loss show worse estimation capabilities, as expected. The joint MIMO radar tracking with path loss (6×4 MIMO with $\alpha_{k1} = 0.01$, $\alpha_{k2} = 0.1$, and $\alpha_{k3} = \alpha_{k4} = 1$, $\forall k = 1, ..., M$) experience some performance degradation, when compared with the case of no path loss (6×4 MIMO with $\alpha_{k1} = \alpha_{k2} = \alpha_{k3} = \alpha_{k4} = 1$, $\forall k = 1, ..., M$). The advantage of MIMO radar systems over MISO (multiple-input single-output) systems is also demonstrated in Figure 1 by comparing the 6×1 MISO system Rx3 or Rx4 with the 6×4 MIMO with $\alpha_{k1} = \alpha_{k2} = \alpha_{k3} = \alpha_{k4} = 1$, $\forall k = 1, ..., M$. The performance advantage of the decentralized MIMO case over the MISO one is manifested as well. The manner in which the MISO estimations are combined to generate a decentralized MIMO system effects performance. It is apparent that by discarding estimates from receivers that undergo significant path loss (decentralized, combining Rx3 and Rx4), estimation performance are getting closer to the centralized MIMO. Based on these insight, two tracking algorithms are proposed in what follows.

V. Tracking Algorithms

As established in [17], MIMO radar systems provide tracking accuracy advantages that grow proportionally with the number of transmitting and receiving radars. However, increasing the number of transmitting and receiving radars leads to increased communication needs and computational load. These depend on the specific tracker employed. Target position and velocity may be tracked based on a centralized or a decentralized approach. In a centralized tracking approach, the observations are jointly measured in a fusion center to produce the target's location and velocity estimates. The decentralized approach takes advantage of observations obtained at a receiver from signals provided by the M transmitting radars, and generates a local estimate of the target location and velocity. These estimates are then sent to a fusion center to be fused based on a local cost function. MIMO radar systems with at least three transmitters support decentralized target position and velocity estimation, as each receiver may act as a MISO radar system. Thus, processing in a MIMO radar system may be distributed among N MISO systems, each with an $M \times 1$ structure. It is expected that the individual subsystems will provide reduced performance, when compared to the MIMO system (as illustrated in Figure 1). Choice of an adequate fusion algorithm, for which the separate estimates are combined effectively, may overcome the degradation due to the weaker propagation paths.

A. Centralized Tracking

Centralized tracking may use direct or indirect estimation techniques. The multiple propagation paths, created by multiple transmitted waveforms and echoes from scatterers received at distributed antennas, support target parameters estimation, such as location and velocity, through either direct or indirect estimation. With direct estimation, the observations collected by the sensors are jointly processed to produce target location and/or velocity estimates. With indirect estimation, the TOAs and Doppler shifts are estimated first, and target location and/or velocity are subsequently estimated based on the relations given in (2) and 3. The advantage of using direct estimation is in the estimation lower MSE, while the indirect estimation technique offers data compression.

Indirect techniques require a preliminary stage where TOAs and Doppler frequencies are first estimated at the receiving radars, and then transmitted to the fusion center, where localization is subsequently estimated by multilateration. This estimation approach incorporates an intermediate step of estimating the unknown parameter vector as follows,

$$\hat{\boldsymbol{\psi}}_n = \arg \max_{\hat{\boldsymbol{\tau}}_n^T, \hat{\boldsymbol{\omega}}_n^T} \left[\sum_{k=1}^{M} \sum_{\ell=1}^{N} \int_T r_\ell(t) s_k^* \left(t - \hat{\tau}_{n_{\ell k}}\right) e^{-j\hat{\omega}_{n_{\ell k}} t} dt \right], \tag{30}$$

were $\hat{\boldsymbol{\psi}}_n = \left[\hat{\boldsymbol{\tau}}_n^T, \hat{\boldsymbol{\omega}}_n^T\right]^T$.

Indirect localization enables data compression and reduced complexity while potentially dealing with higher sidelobes.

Following, a centralized tracker with indirect estimation is proposed in Table 1.

Table 1: Centralized Tracking Algorithm

1. Initialization: $\hat{\mathbf{x}}_o$, $\mathbf{P}_{0|0} = \mathbf{J}_D^{-1}(\hat{\mathbf{x}}_o)$
2. Project the state ahead:
$$\begin{cases} \hat{\mathbf{x}}_{n|n-1} = \mathbf{F}\hat{\mathbf{x}}_{n-1|n-1} \\ \mathbf{H}_n = \left\{\nabla_{\mathbf{x}_n}\left[\mathbf{d}(\mathbf{x}_n)\right]\right\}^T\Big|_{\mathbf{x}_n = \hat{\mathbf{x}}_{n|n-1}} \end{cases}$$
3. *Locally at receiver* ℓ :
   - 3.1 Estimate time delays $\hat{\boldsymbol{\tau}}_{n_\ell}$ and Doppler shifts $\hat{\boldsymbol{\omega}}_{n_\ell}$ with 30 search around $\hat{\mathbf{x}}_{n|n-1} = \mathbf{F}\hat{\mathbf{x}}_{n-1|n-1}$
   - 3.2 Calculate $\mathbf{C}_{n_{\boldsymbol{\psi}}}(\ell) = \mathbf{J}_{D_n}^{-1}\left(\hat{\boldsymbol{\psi}}_n\right)\Big|_{\ell,k=1,\dots,M}$
   - 3.3 Transmit to fusion center:
$$\begin{cases} \hat{\boldsymbol{\tau}}_{n_\ell} = \left[\hat{\tau}_{n_{\ell 1}}, ..., \hat{\tau}_{n_{\ell M}}\right]^T, \\ \hat{\boldsymbol{\omega}}_{n_\ell} = \left[\hat{\omega}_{n_{\ell 1}}, ..., \hat{\omega}_{n_{\ell M}}\right]^T, \\ \mathbf{C}_{n_{\boldsymbol{\tau}}}(\ell), \mathbf{C}_{n_{\boldsymbol{\omega}}}(\ell), \end{cases}$$
4. *Central fusion center:* Get all $\hat{\boldsymbol{\tau}}_{n_\ell}$, $\hat{\boldsymbol{\omega}}_{n_\ell}$, $\mathbf{C}_{n_{\boldsymbol{\tau}}}(\ell)$, and $\mathbf{C}_{n_{\boldsymbol{\omega}}}(\ell)$.
5. Join estimations and MSEs:
$$\begin{cases} \hat{\boldsymbol{\psi}}_n = \left[\hat{\boldsymbol{\tau}}_{n_1}^T, ..., \hat{\boldsymbol{\tau}}_{n_N}^T, \hat{\boldsymbol{\omega}}_{n_1}^T, ..., \hat{\boldsymbol{\omega}}_{n_N}^T\right]^T \\ \mathbf{C}_{n_{\boldsymbol{\psi}}} = \text{diag}\left[\mathbf{C}_{n_{\boldsymbol{\tau}}}(1), ..., \mathbf{C}_{n_{\boldsymbol{\tau}}}(N), \right. \\ \left. \mathbf{C}_{n_{\boldsymbol{\omega}}}(1), ..., \mathbf{C}_{n_{\boldsymbol{\omega}}}(N)\right] \end{cases}$$
6. Project the covariance matrix $\mathbf{P}_{n|n-1}$ and gain matrix $\mathbf{G}_n$ for the EKF:
$$\mathbf{P}_{n|n-1} = \mathbf{F}\mathbf{P}_{n-1|n-1}\mathbf{F}^T + \mathbf{Q}_{\mathbf{v}}$$
$$\mathbf{G}_n = \mathbf{P}_{n|n-1}\mathbf{H}_n^T\left(\mathbf{H}_n\mathbf{P}_{n|n-1}\mathbf{H}_n^T + \mathbf{C}_{n_{\boldsymbol{\psi}}}\right)^{-1}$$
7. Update estimator equation $\hat{\mathbf{x}}_{n|n}$:
$$\hat{\mathbf{x}}_{n|n} = \hat{\mathbf{x}}_{n|n-1} + \mathbf{G}_n\left(\hat{\boldsymbol{\psi}}_n - \mathbf{d}\left(\hat{\mathbf{x}}_{n|n-1}\right)\right)$$
$$\mathbf{K} = \mathbf{H}_n\mathbf{P}_{n|n-1}\mathbf{H}_n^T + \mathbf{C}_{n_{\boldsymbol{\psi}}}$$
$$\mathbf{P}_{n|n} = \mathbf{P}_{n|n-1} + \mathbf{G}_n\mathbf{K}\mathbf{G}_n^T$$
8. Send $\hat{\mathbf{x}}_{n|n}$ to all Receiving radars (to be used in the MLE search)
9. Go to step 2.

The extended Kalman filter (EKF) is used for the model given in 8 and 11 at the fusion center. The observation vector (see (11)) in this case is $\mathbf{z}_n = \boldsymbol{\psi}_n$, and $\mathbf{d}(\mathbf{x}_n)$ is defined by the non linear relations in (2) and (4). The initial target position and velocity $\hat{\mathbf{x}}_o = \left[\hat{x}_o, \hat{y}_o, \hat{\dot{x}}_o, \hat{\dot{y}}_o\right]^T$ are chosen based on a preliminary MLE obtained following target detection. The initial pdf $\mathbf{P}_{0|0}$ is determined based on the CRB.

B. Decentralized Tracking

In decentralized tracking each receiving radars performs local estimates of the target location and velocity, i.e., $\hat{\mathbf{x}}_{n_\ell}$at receive radar ℓ, where $\ell = 1, .., N$, and tracks it using a local KF. Either direct MLE-based estimation or indirect estimation may be used, where in the latter one each receiver radar estimates $\hat{\boldsymbol{\psi}}_{n_\ell} = \left[\hat{\boldsymbol{\tau}}_{n_\ell}, \hat{\boldsymbol{\omega}}_{n_\ell}^T\right]^T$ and then $\hat{\mathbf{x}}_{n_\ell}$ is tracked using a local EKF. The local tracking algorithm for the decentralized case is provided in Table 2. The initial values of $\hat{\mathbf{x}}_{o_\ell}$ and $\mathbf{P}_{0|0_\ell}$ are set based on a preliminary localization process following target detection. At each state n, the receive radar local tracker compares the trace of the local covariance matrix in the previous state $\mathbf{P}_{n-1|n-1_\ell}$ with a weighted value of the trace of the fusion center covariance matrix, corresponding to the final state estimate. For the case where the local estimate at receiver ℓ performance poorly, i.e., $\rho trace\left(\mathbf{P}_{n-1|n-1}\right) < trace\left(\mathbf{P}_{n-1|n-1_\ell}\right)$, the projected value of the current target state utilizes $\hat{\mathbf{x}}_{n-1|n-1}$, provided by the fusion center, instead of $\hat{\mathbf{x}}_{n-1|n-1_\ell}$.

Table 2: Decentralized - Receive radar ℓ

1. Initialization: $\hat{\mathbf{x}}_{o_\ell}$, $\mathbf{P}_{0|0_\ell} = \mathbf{J}_D^{-1}(\hat{\mathbf{x}}_{o_\ell})$
2. Project the state ahead:
   - 2.1 Get from fusion center:
$$\begin{cases} \hat{\mathbf{x}}_{n-1|n-1} \\ \mathbf{P}_{n-1|n-1} \end{cases}$$
   - 2.2 If $\rho trace\left(\mathbf{P}_{n-1|n-1}\right) < trace\left(\mathbf{P}_{n-1|n-1_\ell}\right)$ than $\hat{\mathbf{x}}_{n|n-1_\ell} = \mathbf{F}\hat{\mathbf{x}}_{n-1|n-1}$ else $\hat{\mathbf{x}}_{n|n-1_\ell} = \mathbf{F}\hat{\mathbf{x}}_{n-1|n-1_\ell}$
   - 2.3 Set $\mathbf{H}_{n_\ell} = \left\{\nabla_{\mathbf{x}_{n_\ell}}\left[\mathbf{d}(\mathbf{x}_{n_\ell})\right]\right\}^T\Big|_{\mathbf{x}_{n_\ell} = \hat{\mathbf{x}}_{n|n-1_\ell}}$
3. Estimate time delays $\hat{\boldsymbol{\tau}}_{n_\ell}$ and Doppler $\hat{\boldsymbol{\omega}}_{n_k}$ and set
$$\begin{cases} \hat{\boldsymbol{\psi}}_n = \left[\hat{\boldsymbol{\tau}}_{n_\ell}^T, \hat{\boldsymbol{\omega}}_{n_\ell}^T\right]^T \\ \mathbf{C}_{n_{\boldsymbol{\psi}}}(\ell) = \mathbf{J}_{D_n}^{-1}\left(\hat{\boldsymbol{\psi}}_n\right)\Big|_{\ell,k=1,\dots,M} \end{cases}$$
4. Project the covariance matrix $\mathbf{P}_{n|n-1_\ell}$ and gain matrix $\mathbf{G}_{n_\ell}$ for the EKF:
$$\mathbf{P}_{n|n-1_\ell} = \mathbf{F}\mathbf{P}_{n-1|n-1_\ell}\mathbf{F}^T + \mathbf{Q}_{\mathbf{v}}$$
$$\mathbf{K}_{n_\ell} = \mathbf{H}_{n_\ell}\mathbf{P}_{n|n-1_\ell}\mathbf{H}_{n_\ell}^T + \mathbf{C}_{n_{\boldsymbol{\psi}}}(\ell)$$
$$\mathbf{G}_{n_\ell} = \mathbf{P}_{n|n-1_\ell}\mathbf{H}_{n_\ell}^T\mathbf{K}_{n_\ell}^{-1}$$
5. Update estimator equation $\hat{\mathbf{x}}_{n|n_\ell}$:
$$\hat{\mathbf{x}}_{n|n_\ell} = \hat{\mathbf{x}}_{n|n-1_\ell} + \mathbf{G}_{n_\ell}\left(\hat{\boldsymbol{\psi}}_{n_\ell} - \mathbf{d}\left(\hat{\mathbf{x}}_{n|n-1_\ell}\right)\right)$$
$$\mathbf{P}_{n|n_\ell} = \mathbf{P}_{n|n-1_\ell} + \mathbf{G}_{n_\ell}\mathbf{K}_{\mathbf{n}_\ell}\mathbf{G}_{n_\ell}^T$$
6. Send $\hat{\mathbf{x}}_{n|n_\ell}$ to the fusion center
7. Go to step 2.

By integrating the updated information provided by the fusion center, these receivers keep track of the target state closer than they would otherwise. As path loss condition change or the relative position of the receiver with respect to the target location creates better condition, the local tracker will have a better recovery.

The local estimates are sent to a central fusion center, where they are combined based on a predetermined cost function. The estimates are chosen such that path with significant fading or low reflectivity will be either discarded of introduce with very high cost coefficients. If the trace of a given estimate has reached a predetermined performance threshold χ, an associated flag is set to zero ($\kappa_\ell = 0$). Estimates with $\kappa_\ell = 0$ are ignored. All others are combined using a weighted sum of the estimates $\hat{\mathbf{x}}_{n_\ell}$, where the local covariance matrix is used to determine the weights. By doing so, the overall estimation MSE is kept as close as possible to the centralized performance. The fusion center's decentralized algorithm is described in Table 3.

Performance of the centralized and decentralized algorithms are provided in Figure 2. For the simulation, a 6×4 MIMO

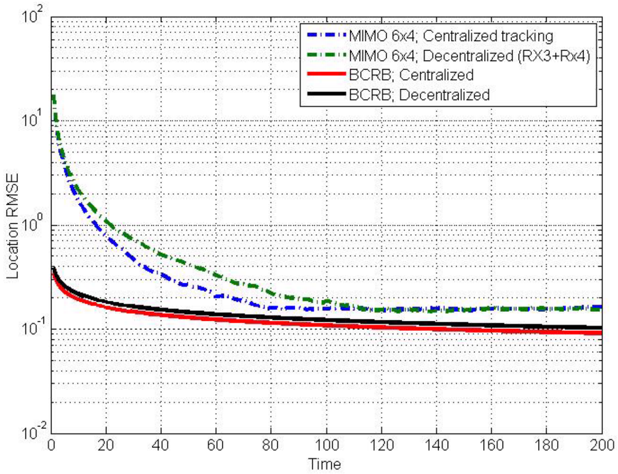

Fig. 2. BCRB on target location tracking for centralized and decentralized tracking performance.

system is considered with $\alpha_{k1} = 0.01$, $\alpha_{k2} = 0.1$, and $\alpha_{k3} = \alpha_{k4} = 1$, $\forall k = 1, ..., M$. Signal to noise ratio is set to $SNR = 15$dB. The decentralized tracker in this case combines the estimates from receiver three and four while discarding the other two, as they do not cross the predetermined threshold χ. The proposed decentralized algorithm achieves accuracies very close to the centralized one.

Table 3: Decentralized - Fusion center	
1.	Initialization: $\hat{\mathbf{x}}_o$, $\mathbf{P}_{0\|0} = \mathbf{J}_D^{-1}(\hat{\mathbf{x}}_o)$
2.	Project the state ahead: $\begin{cases} \hat{\mathbf{x}}_{n\|n-1} = \mathbf{F}\hat{\mathbf{x}}_{n-1\|n-1} \\ \mathbf{H} = \mathbf{I} \end{cases}$
3.	Get estimations and MSEs from Rx radars: $\begin{cases} \hat{\mathbf{x}}_{n_\ell} & \ell = 1, ..., N \\ \mathbf{C}_{n_\mathbf{x}}(\ell) \end{cases}$
4.	Join estimations and MSEs: $\begin{cases} 4.1 & \text{For a predetermined threshold } \chi \\ & \text{If } trace(\mathbf{C}_{n_\mathbf{x}}(\ell)) \le \chi \text{ than } \kappa_\ell = 0 \\ 4.2 & \text{Set } \mathbf{C}_{n_\mathbf{x}} = \left(\sum_{\ell=1,\kappa_\ell \neq 0}^{N} \mathbf{C}_{n_\mathbf{x}}^{-1}(\ell) \right)^{-1} \\ 4.3 & \text{Set } \tilde{\mathbf{x}}_n = \sum_{\ell=1,\kappa_\ell \neq 0}^{N} \Lambda(\ell)\hat{\mathbf{x}}_{n_\ell} \\ & \text{where } \Lambda(\ell) = \mathbf{C}_{n_\mathbf{x}}^{-1}(\ell)\mathbf{C}_{n_\mathbf{x}} \end{cases}$
5.	Project the covariance matrix $\mathbf{P}_{n\|n-1}$ and gain matrix $\mathbf{G}_n$ for the KF: $\mathbf{P}_{n\|n-1} = \mathbf{F}\mathbf{P}_{n-1\|n-1}\mathbf{F}^T + \mathbf{Q}_\mathbf{v}$, $\mathbf{G}_n = \mathbf{P}_{n\|n-1}(\mathbf{P}_{n\|n-1} + \mathbf{C}_{n_\mathbf{x}})^{-1}$
6.	Update estimator equation $\hat{\mathbf{x}}_{n\|n}$: $\hat{\mathbf{x}}_{n\|n} = \hat{\mathbf{x}}_{n\|n-1} + \mathbf{G}_n(\tilde{\mathbf{x}}_n - \hat{\mathbf{x}}_{n\|n-1})$, $\mathbf{P}_{n\|n} = (\mathbf{I} - \mathbf{G}_n)\mathbf{P}_{n\|n-1}$
7.	Send $\hat{\mathbf{x}}_{n\|n}$ and $\mathbf{P}_{n\|n}$ to all Receive radars (to be used in the MLE search and the EKF)
8.	Go to step 2.

VI. Conclusions

Study of moving target tracking capabilities is offered through the use of the BCRB for the estimation of both target location and velocity in non-coherent MIMO radar systems with widely distributed antennas. The MIMO radar architecture supports both centralized and decentralized tracking techniques. Based on this study, two tracking schemes are proposed. The first is a centralized architecture, based on joint processing of partially processed (compressed) data at the fusion center. This approach provides highly accurate target tracking and takes full advantage of the MIMO configuration. The second is a decentralized scheme, based on a hybrid combination of local processing at the receiving radars and joint tracking at the fusion center. The later supports resource aware system operation. Reduced communication requirements and processing load may be achieved with relatively low cost with the proposed decentralized method.

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