

Low Complexity Turbo MIMO Systems

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Abstract—Various schemes have been recently proposed for multiple-input multiple-output (MIMO) systems with a large number of transmit antennas. In this paper we propose a layered turbo-coded space-time scheme on the basis of performance analysis. It is shown that optimizing coding gain is preferred to optimizing transmit diversity. The performance of the proposed scheme is within 2 dB range of the outage probability and has lower complexity than other schemes with comparable performance.

I. INTRODUCTION

Recent research in information theory has shown that large gains in capacity and reliability of communications over wireless channels are achievable by exploiting the spatial diversity made possible by multiple antennas at the transmitter and at the receiver. Two approaches have emerged to implementing spatial diversity. In the first approach, space-time codes combine the benefits of spatial diversity and channel coding [1]. Space-time coding schemes however, are too complex to support large number of transmit/receive antennas. A second approach, known as BLAST, features a layered architecture, which can achieve massive parallel transmission and very high data rates by using a large number of antennas at the transmitter and at the receiver [2]. The BLAST approach has reasonable complexity, however its performance is not optimized for diversity and coding gain.

The signals at the output port of each receive antenna consist of the superposition of the transmitted signals. Separation of these signals into their original streams is one of the main challenges of multiple-input multiple-output (MIMO) communication systems. Applying iterative interference cancellation techniques [3], [4], the signals from different transmit antennas can be

nearly fully separated without sacrificing receive diversity. With the improved diversity afforded by a large number of receive antennas, the system usually works at moderate to low signal to noise ratio (SNR).

In this paper, we show that at low SNR, even with multiple transmit antenna, the effects of transmit diversity are limited and the performance of the system is dominated by the Euclidean distance between codewords. From this analysis, we conclude that turbo codes are good candidates for implementation as part of MIMO systems.

Analysis and simulation show that turbo decoding also provides a natural mechanism to reduce the effects of residual interference caused by non-ideal cancellation. We show that these effects are further reduced by grouping the transmit antennas to lower the number of layers involved in the transmission. As a result, additional processing, such as minimum mean-square error filtering [5], [3], is not necessary to suppress the residual interference.

With the insights gained from the analysis of MIMO systems, we propose a new *layered turbo space-time coding* (LTST) scheme. LTST is an extension of our previous work on turbo space-time codes [6]. We show that the proposed scheme achieves similar or superior performance to other MIMO schemes such as threaded space-time architecture (TST) proposed in [5] and turbo-greedy coding proposed in [7], with significantly less complexity. While the complexity of these two mentioned schemes increases polynomially with number of transmit/receive antennas, the complexity of the proposed scheme is almost flat.

The rest of this paper is organized as follows. In Section II we present the LTST scheme. The performance analysis of the proposed system and code selection criteria, the algorithm of iterative interference cancellation and decoding is presented in Section III. Simulation re-

sults are presented in Section IV. Section V draws the conclusions.

II. SYSTEM MODEL

Consider a MIMO communication system with n_T transmit and n_R receive antennas operating over a block fading channel. The output of this MIMO channel is represented by $n_T \times 1$ vectors $\mathbf{r}_t \in \mathbb{C}^{n_R}$ that can be expressed

$$\mathbf{r}_t = \sqrt{E_s} \mathbf{H} \mathbf{c}_t + \boldsymbol{\eta}_t, \quad (1)$$

where E_s is the energy of each transmitted symbol and the $n_R \times n_T$ matrix \mathbf{H} consists of the channel coefficients. The i, j element of \mathbf{H} represents the path gain from transmit antenna j to receive antenna i . The channel is assumed flat, Rayleigh and block-fading. This means that the matrix \mathbf{H} consists of complex-valued scalars h_{ij} modeled as zero-mean, mutually independent, identically distributed Gaussian random variables with unity variance such that $E[h_{ij}h_{pq}^*] = \delta_{ip}\delta_{jq}$, where δ is the Kronecker symbol $\delta_{ip} = 1$ for $i = p$ and $\delta_{ip} = 0$ otherwise. The $n_T \times 1$ vector \mathbf{c}_t consists of the symbols transmitted at time t . The noise term $\boldsymbol{\eta}_t$ is modeled by zero-mean, additive white Gaussian noise (AWGN) with variance $N_0/2$ per dimension.

The proposed LTST structure is shown in Fig 1. The information data is encoded by a symbol based, parallel concatenated space-time turbo encoder as we proposed in [8], [6]. In short, this is a turbo encoder which generates two dimensional (2-D) codewords to be transmitted from multiple (in our case two) transmit antennas. So the codeword generated by the turbo encoder is a $2 \times N$ symbol matrix, where N is the length of the 2-D codeword. One row of the codeword matrix contains systematic symbols and the other row contains parity symbols. The codeword is then interleaved such that columns in the codeword matrix are preserved. Following interleaving, the codeword is separated into several substreams (layers), which are modulated and transmitted from groups of antennas. In our scheme, each group contains two transmit antennas. The function of the interleaver is to decorrelate adjacent codeword symbols and avoid possible burst errors after interference cancellation at the receiver.

The receiver has two stages. First, the signals received from different layers are separated by cancelling the estimated signals from other layers, which are regarded as interference. Then, the separated signals of each layer are re-assembled, de-interleaved to the original order, and passed to the turbo decoder. Soft decisions of the

transmitted symbols are fed back to improve the interference cancellation. This process repeats for several iterations. The algorithm used at the receiver is briefly described in the next section. The data rate of the proposed scheme is $rn_T \log_2 q$ bits/s/Hz, where q is the cardinality of the symbol constellation and r is the rate of the turbo encoder.

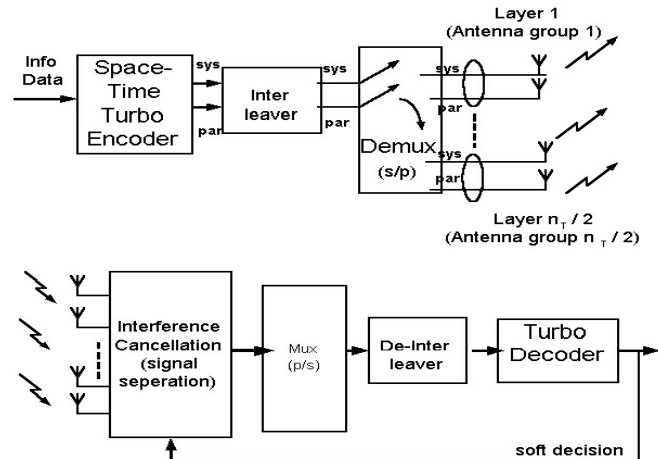


Fig 1. Structure of layered turbo space-time coding scheme

III. CODE DESIGN AND LTST RECEIVER

A. Code design criterion

For the system model described above, where $n_T, n_R \geq 4$, because nearly full receive diversity can be achieved by iterative interference cancellation, the system operates at low SNR of $E_T/N_0 \approx 6$ dB or less, where $E_T = n_T \cdot E_s$ is the total energy of symbols transmitted from all transmit antennas. In this case, assuming interference from other layers is fully cancelled, then each layer is a subsystem with two transmit and n_R receive antennas ($2Tn_RR$). The pairwise error probability (PEP) that a codeword \mathbf{c} is erroneously received as \mathbf{e} is:

$$\begin{aligned} P(\mathbf{c} \rightarrow \mathbf{e}) &\leq \exp(-E_s/4N_0 \|\mathbf{c} - \mathbf{e}\|^2) \quad (2) \\ &= \exp(-E_s/4N_0 \sum_{l=1}^L \|\mathbf{c}_l - \mathbf{e}_l\|^2) \\ &= \prod_{l=1}^L \exp(-E_s/4N_0 \cdot \|\mathbf{c}_l - \mathbf{e}_l\|^2) \\ &= \prod_{l=1}^L P(\mathbf{c}_l \rightarrow \mathbf{e}_l) \end{aligned}$$

where $\|\mathbf{c} - \mathbf{e}\|^2$ is the Euclidean distance between whole codewords (containing all layers); $\|\mathbf{c}_l - \mathbf{e}_l\|^2$, $l = 1, \dots, L$, is the Euclidean distance between the codewords

\mathbf{c}_l and \mathbf{e}_l of each layer; and L is the total number of layers; $P(\mathbf{c}_l \rightarrow \mathbf{e}_l) \triangleq \exp(E_s/4N_0 \cdot \|\mathbf{c}_l - \mathbf{e}_l\|^2)$ is the pairwise error probability for each layer. As previously mentioned $\frac{E_T}{4N_0} \approx 6$ dB, it follows that $\frac{E_s}{4N_0} \approx -6$ dB $\ll 1$. For such case of low SNR, the PEP of each layer can be approximated as[9]:

$$P(\mathbf{c}_l \rightarrow \mathbf{e}_l) \leq \left(\frac{1}{1 + \frac{E_s}{4N_0} \cdot \|\mathbf{c}_l - \mathbf{e}_l\|^2} \right)^{n_R} \quad (3)$$

Then the pairwise error probability of the whole codeword can be shown to be:

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left(\frac{1}{1 + \frac{E_s}{4N_0} \|\mathbf{c} - \mathbf{e}\|^2} \right)^{n_R} \quad (4)$$

From (4), we note that the PEP is independent of the number of transmit antennas n_T , i.e., the effect of transmit diversity vanishes. It is obvious that the code selection criterion for LTST should be to select codes with good Euclidean distance properties, such as turbo codes.

B. Decoder

The turbo decoder used in our scheme is similar to the MAP decoder proposed in [8], with some modifications to take into account the effect of residual interference as described in the next subsection. With LTST, it is also necessary to compute *a posteriori probabilities* (APP's) for both systematic and parity symbols of a codeword. In short, the MAP decoder outputs for each codeword symbol c_i , a set of APP's given the observations for the entire codeword. The APP's are denoted $P(c_i = s_j | \text{obs})$, $j = 1, \dots, q$, where $\{s_j, j = 1, \dots, q\}$ is the modulation constellation set. The soft decision of the codeword symbol c_i is computed as

$$\tilde{c}_i = \sum_{j=1}^q s_j P(c_i = s_j | \text{obs}) \quad (5)$$

Soft decisions computed in (5) are used for interference cancellation at the next iteration.

C. Interference Cancellation

The signals from different layers are separated by cancelling estimated signals from other layers. At time interval t , suppose the signal from the k 'th layer is the desired signal. Then (1) can be written as

$$\mathbf{r}_t = \mathbf{H}_k \mathbf{c}_{k,t} + \mathbf{H}_k \tilde{\mathbf{c}}_{k,t} + \boldsymbol{\eta}_t, \quad (6)$$

where \mathbf{H}_k is composed of columns of \mathbf{H} pertaining to layer k (channel matrix for layer k) and \mathbf{H}_k is composed of the rest of the columns of \mathbf{H} ; $\mathbf{c}_{k,t}$ is the vector of code-word symbols from layer k and $\tilde{\mathbf{c}}_{k,t}$ is the vector of code-word symbols from other layers. Denote $\tilde{\mathbf{c}}_{k,t}$ a previous soft decision of $\mathbf{c}_{k,t}$. Interference cancellation is carried out by subtracting from the received vector \mathbf{r}_t the estimated contribution of the interference $\mathbf{H}_k \tilde{\mathbf{c}}_{k,t}$:

$$\begin{aligned} \tilde{\mathbf{r}}_{k,t} &= \mathbf{r}_t - \mathbf{H}_k \tilde{\mathbf{c}}_{k,t} \\ &= \mathbf{H}_k \mathbf{c}_{k,t} + \mathbf{v}_t + \boldsymbol{\eta}_t, \end{aligned} \quad (7)$$

where $\tilde{\mathbf{r}}_{k,t}$ is the received signal observation vector from layer k after interference cancellation and $\mathbf{v}_t = \mathbf{H}_k(\mathbf{c}_{k,t} - \tilde{\mathbf{c}}_{k,t})$ is the residual interference. In the first iteration, when no soft decision from the decoder is available, $\tilde{\mathbf{c}}_{k,t}$ is set to zero.

The signal observation vector $\tilde{\mathbf{r}}_{k,t}$ is passed to the turbo decoder. In turbo decoding, it is necessary to take into account the effect of the residual interference. To simplify the computation, we ignore the correlation between the residual interference from different transmit antennas and model the residual interference \mathbf{v}_t as a vector of uncorrelated zero mean Gaussian random variables. Then the covariance matrix of $\mathbf{v}_t + \boldsymbol{\eta}_t$, denoted as \mathbf{R} , is approximated as:

$$\begin{aligned} \mathbf{R} &= E[(\mathbf{v}_t + \boldsymbol{\eta}_t)(\mathbf{v}_t + \boldsymbol{\eta}_t)^H] \\ &= E[\mathbf{v}_t \mathbf{v}_t^H] + E[\boldsymbol{\eta}_t \boldsymbol{\eta}_t^H] \\ &= N'_0 \mathbf{I}, \end{aligned} \quad (8)$$

where N'_0 is the variance of noise plus residual interference at each receive antenna. In general, the elements of \mathbf{v}_t are correlated. Fully or partially exploring these correlations could lead to performance improvement but with high complexity. In the proposed scheme, the residual interference is treated as uncorrelated. In this case, it is easy to show that

$$N'_0 = \begin{cases} \frac{n_T-2}{n_T} E_T + N_0 & \text{the first iteration} \\ \alpha \cdot \frac{n_T-2}{n_T} E_T + N_0 & \text{the second iteration} \\ N_0 & \text{other iterations} \end{cases} \quad (9)$$

The basic fact underlying the derivation of (8) and (9) is that in the first iteration there is no soft decision from the decoder available and the interference is not cancelled at all; in the second iteration the constant α in (9) is introduced to reflect the scale of the residual interference (related to the symbol error rate after the first

iteration) and can be set to 0.1 in most cases; after the second iteration the variance of the residual interference is small compared to the variance of the noise and it is neglected. The variance of noise plus residual interference N'_0 is then used in the turbo decoding algorithm instead of the variance of pure noise variance N_0 . This is the main modification of the turbo decoding algorithm as we proposed in [8].

D. Complexity comparison

In the proposed receiver, no extra effort is used to suppress the residual interference and the complexity lies mainly in the turbo decoding effort. In other layered MIMO schemes such as TST [5] and turbo-greedy coding [7] either a MMSE filter or a modified joint demodulation and decoding algorithm is used to combat the interference from other layers, the complexity increases polynomially with the number of transmit/receive antennas. We attribute the low complexity, but good performance, of the proposed scheme to the use of turbo code and proper design of the structure of the transmitter.

Since the turbo decoder consists of two component decoders, and the soft decision of each symbol depends on both the signal observation and the *a priori* probability obtained from the other component decoder, the effect of inaccurate signal observation (containing residual interference) is limited. This keeps the decoder from diverging too far away from the correct decisions, especially in the first several iterations. In other schemes, such as TST [5] where a conventional trellis code is used, extra efforts have to be used to ensure the convergence of the algorithm.

The unique structure of the proposed transmitter, where two antennas are grouped in one layer instead of using one antenna in each layer, also plays an important role in limiting the effects of residual interference. This antenna grouping reduces the number of total layers, or the parties involved in the interference cancellation to $\frac{n_T}{2}$ and significantly reduces the scale of error propagation. Simulations show significant performance difference between systems with two antennas per layer and one antenna per layer.

The complexity comparison of LTST and other two schemes with comparable performances, TST [5] and turbo-greedy coding [7], is demonstrated in Table I. The entries in the table denote the number of complex multiplications needed. The second column shows the complexity of interference cancellation/decoding for one data symbol per iteration as a function of number of transmit/ receive antennas. The third and fourth columns

are evaluations of the second column for 4 transmit 4 receive antenna (4T4R) case and 8 transmit 8 receive antenna (8T8R) case, respectively. The fifth column lists the iterations performed for the performance achieved in the simulation shown in the next section. By comparison, the complexity of LTST is significantly less than the other two schemes. Typically, the complexity of the proposed LTST schemes does not increase with number of transmit/receive antennas while that of other two schemes increases polynomially with the number of antennas.

IV. NUMERICAL RESULTS

Numerical results for the proposed LTST scheme are presented for 4T4R and for 8T8R configurations. The space-time turbo encoder used in our schemes is composed of two identical parallel concatenated 8 state recursive systematic space-time encoders as in [8]. For 4-PSK modulation, the data rate for the 4T4R system is 4 bits/s/Hz and for the 8T8R system it is 8 bits/s/Hz. The channel is assumed to be constant over blocks of 130 channel uses. Consequently, the frame length for 4T4R system is 260 symbols (130 channel uses \times 2 layers) and for the 8T8R system is 520 symbols. Numerical results are shown after 10 iterations. The SNR is defined E_T/N_0 , where E_T is the total energy of transmitted symbols from all antennas.

Fig. 2 shows the frame error rate (FER) of 4T4R systems. It can be observed that LTST, TST [5] and turbo-greedy coding [7] curves are approximately parallel indicating same diversity gain. LTST and turbo-greedy coding have almost the same performance outperforming TST by approximately 1 dB. It can also be observed that the performances of both LTST and turbo-greedy coding are within 2 dB range of the outage probability. Also shown in this figure is the FER curve for LTST 4T4R with only one antenna in each layer (labeled as LTST-1 in the figure). This curve diverges from the curve of LTST 4T4R as SNR increases, indicating the effect of error propagation caused by residual interference.

Fig. 3 shows the FER comparison of 8T8R LTST, TST and turbo-greedy systems. It can be observed that LTST and turbo-greedy coding still have similar performance and outperform TST by about 1 dB.

V. CONCLUSION

In this paper, we analyzed the performance of a general MIMO system and presented a new layered approach to optimize performance of such systems. While

other reported layered schemes employ special efforts to suppress the residual interference caused by inaccurate cancellation, we show that by applying turbo coding and by properly designing the structure of the transmitter by grouping the transmit antennas, these efforts can be saved. Simulations show the proposed scheme performs within 2 dB range of the outage probability.

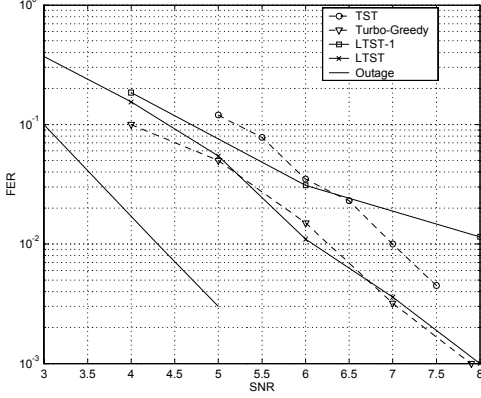


Fig 2 FER comparison of LTST and other schemes, 4T4R 4b/s/Hz

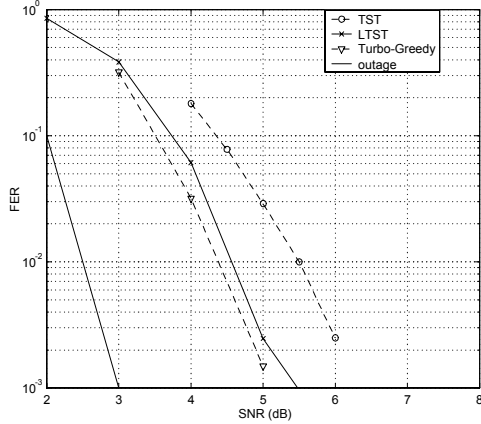


Fig 3 FER comparison of LTST and other schemes, 8T8R 8b/s/Hz

	Complexity /data symbol/ iteration	4T4R	8T8R	Iterations
Turbo-Greedy	$2 \cdot (q^2 + (q^2 - q)(n_T - 2)) \rightarrow$	80 \rightarrow		10
	$2 \cdot L \cdot (q \cdot n_T + (q^2 - 1) \frac{n_T(n_T - 1)}{2})$	424	480 \rightarrow 856	
TST	$n_R^3 + 3n_R^2 + 2^{v+\log q} \cdot 2$	176	736	4
LTST	$n_R + 2^{v/2+\log q} \cdot 2$	68	72	10

Table I Complexity comparison of LTST and other schemes¹

¹In this table, q denotes the modulation constellation size. For the simulation results presented in section IV, $q = 4$ for all three schemes.

The parameter v denotes the number of memory in encoders. As for the simulation results presented in section IV, for LTST $v = 6$ and for TST $v = 2$.

For Turbo-Greedy coding, the complexity is a random variable within the range as presented in the table; L is a design

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parameter to provide performance-complexity trade off. As for the simulation results presented in section IV, $L = 2$ for 4T4R case and $L = 8$ for 8T8R case. [7]