MULTI-USER CAPACITY OF M-ARY PPM ULTRA-WIDEBAND COMMUNICATIONS

Li Zhao and Alexander M. Haimovich

Department of Electrical and Computer Engineering
New Jersey Institute of Technology, Newark, NJ 07102.
Tel: (973)-596-3534 Fax: (973)-596-8473
E-mail: li.zhao@njit.edu, haimovic@njit.edu.

ABSTRACT

In this paper we study the information theoretic capacity of M-ary pulse position modulation (PPM) ultra-wideband (UWB) communications for the multi-user channels. Under certain assumptions, the multiple-access interference component at the receiver is modeled as Gaussian. An expression for the signal-to-noise ratio is developed for UWB utilizing rectangular pulses. The information theoretic capacity of the UWB system is studied as a function of number of users. A closed form expression and a lower bound are found for the case of binary PPM. An upper bound on the aggregate capacity is found for the case of large number of users.

1. INTRODUCTION

Ultra-wideband (UWB) is a new technology that has the potential to revolutionize wireless communications by delivering high data rates with very low power densities. UWB short duration waveforms are relatively immune to multipath fading effects as observed in other wireless applications. In addition, very low power and high processing gain will enable overlay and ensure only minimal mutual interference between UWB and other applications. Pulse position modulation (PPM) has been mentioned as a modulation suitable to UWB communications. With PPM, the data modulates the position of the transmitted pulse within an assigned window in time [1].

With bandwidth restrictions effectively removed, UWB promises to speed up wireless data transfer rates. In a previous paper [2], the information theoretic capacity of M-ary PPM UWB communications over an additive white Gaussian noise (AWGN) channel was computed for a single-user. The computation took into account UWB specific constraints such as the power spectrum density limitation under FCC Part 15 rules and the spreading ratio required to achieve a specified jam resistance level. For example, we have shown that under these constraints, a 32-ary PPM UWB system with bandwidth 1 GHz and spreading ratio $\beta = 100$, has a communications range of up to 60 meters for a data rate of 50 Mbits/s or up to 260 meters for a data rate of 10 Mbits/s.

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In this paper, we extend the information theoretic capacity analysis to the multi-user channel. In this case, multiple access interference (MAI) is the factor limiting performance. Previous published work has addressed the issue of how many users can the UWB channel support [3], [4], [5]. This paper extends previous work on multi-user UWB in several ways: it computes the multi-user capacity based on PPM single-user capacity rather than on the Shannon expression ($C = W \log_2(1 + \text{SNR})$) with continuous-valued inputs and outputs; it evaluates the capacity for transmissions using a rectangular pulse.

The multi-user capacity is derived from signal-to-noise ratio (SNR) considerations assuming noncooperative users. Cross-correlation between the pulses of different users causes the noise floor to raise with the number of users. The analysis for rectangular pulses can be extended to other practical waveforms.

The reminder of this paper is organized as follows. Section 2 introduces the signal model. Analysis of the UWB M-ary PPM capacity for multiple-access channels is carried out in section 3. Conclusions are provided in section 4.

2. SIGNAL MODEL

Consider a multi-user UWB communications system. Let the number of users in the system be N_u. Then the ν-th ($1 \leq \nu \leq N_u$) user's transmitted signal has the form:

$$S^{(\nu)}(t) = \sum_{j=-\infty}^{\infty} A^{(\nu)}\, p(t - jT_f - c_j^{(\nu)} T_c - d_j^{(\nu)}), \quad (1)$$

where $p(t)$ is the UWB pulse of duration T_p, E_p is the energy per pulse. To simplify the analysis of UWB multi-user capacity, we assume a rectangular pulse. This analysis can be extended to other waveforms. The pulse repetition interval, referred to as *frame*, is T_f, $A^{(\nu)} = \sqrt{E_p^{(\nu)}}$, $c_j^{(\nu)}$ and $d_j^{(\nu)}$ are respectively, the amplitude, time-hopping and modulation for user ν. The PPM time shift is $d_j^{(\nu)} \in \{\delta_1, \ldots, \delta_M\}$, where we assume $\delta_1 = 0$, and $\delta_1 < \delta_2 < \ldots < \delta_M < T_f$. For a fixed T_f, the symbol rate $R_s = 1/(N_p T_f)$ determines N_p, the number of pulses that form a symbol. The symbol duration is then $T_s = N_p T_f$.

The spreading ratio is defined $\beta = T_f/T_p$. In previous work, we studied the jam resistance of UWB signals [6] and have shown that the spreading ratio pays an important role in determining the jam resistance of an UWB communication system. The capacity analysis in this paper is carried out for a prescribed value of the spreading ratio.

For simplicity, and without loss of generality, we assume that a single UWB pulse is transmitted for each data symbol, $N_p = 1$. At the expense of capacity, $N_p > 1$ can be used to limit the transmitted peak power. This assumption implies that the symbol interval and the frame interval are the same, $T_s = T_f$, and that the energy per information symbol E_s equals the energy per pulse E_p, $E_s = E_p$. With this assumption, and to simplify notation, in the context of UWB symbols, we will use T_f and E_p for the symbol interval and the symbol energy, respectively.

The received signal $R(t)$ from all users is given by:

$$R(t) = \sum_{\nu=1}^{N_u} S^{(\nu)}(t - \tau^{(\nu)}) + n(t), \qquad (2)$$

where $\tau^{(\nu)}$ is the time delay associated with user ν and $n(t)$ is zero-mean AWGN with power spectral density $N_0/2$.

Without loss generality, we assume the desired user is $\nu = 1$. The single-user optimal receiver is the M-ary pulse correlation receiver followed by a detector. We assume the receiver is perfectly synchronized to user 1, i.e., $\tau^{(1)}$ is known. Furthermore, the time hopping sequence $c_j^{(1)}$ is known at the receiver. The M-ary correlation receiver for user 1 consists of M filters matched to the basis functions $\phi_i^{(1)}(t)$ defined as:

$$\phi_i^{(1)}(t) = p(t - \delta_i - \tau^{(1)}), \;\; i = 1, .., M. \qquad (3)$$

At sample time $t = jT_f$, the output of each filter, y_i, $i = 1, .., M$, is:

$$y_i = \int_{(j-1)T_f}^{jT_f} R(t)\phi_i^{(1)}(t - jT_f - c_j^{(1)})dt, \qquad (4)$$

which can be put in the form:

$$y_i = \begin{cases} A^{(1)} + N_I + N & \text{signal} \\ N_I + N & \text{no signal}, \end{cases} \qquad (5)$$

where N_I and N are respectively, the MAI and AWGN. We have

$$N_I = \sum_{\nu=2}^{N_u} \int_{(j-1)T_f}^{jT_f} A^{(\nu)} p(t - jT_f - c_j^{(\nu)}T_c - \delta_j^{(\nu)} - \tau^{(\nu)}) \cdot \phi_i^{(1)}(t - jT_f - c_j^{(1)})dt, \qquad (6)$$

and

$$N = \int_{(j-1)T_f}^{jT_f} n(t)\phi_i^{(1)}(t - jT_f - c_j^{(1)})dt. \qquad (7)$$

For any pulse $p(t)$ with duration T_p, we define the correlation $h(\Delta)$:

$$h(\Delta) = \int_0^{T_f} p(t)p(t - \Delta)dt. \qquad (8)$$

Then the MAI component in (6) can be written

$$N_I = \sum_{\nu=2}^{N_u} A^{(\nu)} h(\Delta), \qquad (9)$$

where Δ is the time difference between users 1 and ν. The time difference Δ can be expressed:

$$\Delta = (c_j^{(1)} - c_j^{(\nu)})T_c - (d_j^{(1)} - d_j^{(\nu)}) - (\tau^{(1)} - \tau^{(\nu)}). \qquad (10)$$

To provide a statistical characterization of Δ, the following assumption are made:

1. The time-hopping code elements $c_j^{(\nu)}$ are random, independent, identically distributed (i.i.d.) with a uniform distribution over the frame interval T_f, for $\nu = 1, ..., N_u$ and for all j (j is the frame index).
2. Each user ν has a uniformly distributed data source, such that the probability of any M-ary PPM symbol is $1/M$. We further assume that the data sequences corresponding to all users consist of independent symbols. Hence the corresponding time shifts $d_j^{(\nu)}$ are also i.i.d. random variables.
3. The time delays $\tau^{(\nu)}$ are also assumed random, i.i.d. uniformly distributed over the frame interval.

Under the assumptions listed above, and noting that MAI pulses of interest fall within the same UWB frame, the time difference Δ is a random variable uniformly distributed over the interval $[-T_f, T_f]$. This interval allows to accommodate also Δ with negative sign. At the output of user 1's receiver, the distribution of the MAI component N_I depends on the statistical characteristics of Δ. Strictly speaking, the multiple-access interference N_I is not Gaussian. However, by assuming perfect power control, we make the Gaussian assumption for the MAI N_I by invoking the central limit theorem for a sufficiently large number of users N_u.

From (5), the output of the sampler at user 1's receiver consists of independent Gaussian random variables y_i distributed as follows:

$$\begin{array}{lll} y_i & \text{is} & \mathcal{N}(A^{(1)} + E[N_I], \;\; \sigma_I^2 + N_0/2) \quad \text{signal} \\ y_i & \text{is} & \mathcal{N}(E[N_I], \;\; \sigma_I^2 + N_0/2) \quad \text{no signal}, \end{array}$$

where the symbol $\mathcal{N}(a, 1)$ denotes the Gaussian distribution with mean a and unity variance, $E[N_I]$ and σ_I^2 are respectively, the mean and variance of N_I.

3. CAPACITY OF MULTI-USER UWB

Here, the Gaussian model above is used to evaluate the individual user capacity in the presence of MAI contributed by the other users. We also develop a closed-form lower bound for the individual capacity for binary PPM. Finally, we consider the aggregate capacity of all users and develop an upper bound for it.

Consider a UWB system utilizing a rectangular waveform. The UWB pulse $p(t)$ is expressed:

$$p(t) = \sqrt{\frac{1}{T_p}}, \quad 0 \leq t \leq T_p. \tag{11}$$

The correlation function for $p(t)$ in (11) is:

$$h(\tau) = 1 - \frac{|\tau|}{T_p}, \quad 0 \leq |\tau| \leq T_p. \tag{12}$$

As justified earlier, the time shift difference between users Δ defined in (10) is modeled with a uniform distribution over the interval $[-T_f, T_f]$. It follows that the probabilities $P(-T_p \leq \Delta \leq 0) = P(0 \leq \Delta \leq T_p) = T_p/(2T_f) = 1/(2\beta)$. It is noted that the spreading ratio β serves in the measurement of the probability of Δ falling in the interval $[-T_p, T_p]$. From (12), the mean value of $h(\Delta)$ can be calculated as follows:

$$\begin{aligned} E[h(\Delta)] &= E\left[h(\Delta)| - T_p \leq \Delta \leq 0\right] P(-T_p \leq \Delta \leq 0) \\ &\quad + E[h(\Delta)|0 \leq \Delta \leq T_p] P(0 \leq \Delta \leq T_p) \\ &= \frac{1}{2\beta}. \end{aligned} \tag{13}$$

The variance of $h(\Delta)$ is denoted σ_h^2. Since $E[h(\Delta)] = 1/(2\beta)$, we have

$$\begin{aligned} \sigma_h^2 &= E[h^2(\Delta)] - \left(\frac{1}{2\beta}\right)^2 \\ &= E[h^2(\Delta)| - T_p \leq \Delta \leq 0] P(-T_p \leq \Delta \leq 0) \\ &\quad + E[h^2(\Delta)|0 \leq \Delta \leq T_p] P(0 \leq \Delta \leq T_p) - \frac{1}{4\beta^2} \\ &= \frac{1}{3\beta} - \frac{1}{4\beta^2}. \end{aligned} \tag{14}$$

For typical values of $\beta > 100$, we can approximate

$$\sigma_h^2 \approx \frac{1}{3\beta}. \tag{15}$$

Finally, the variance of the MAI term N_I is:

$$\sigma_I^2 = \sum_{\nu=2}^{N_u} {A^{(\nu)}}^2 \sigma_h^2 = \frac{\sum_{\nu=2}^{N_u} {A^{(\nu)}}^2}{3\beta}. \tag{16}$$

Taking into account the AWGN, the SNR per symbol at the output of the correlation receiver is given by

$$\rho_I = \frac{{A^{(1)}}^2}{\sigma_I^2 + N_0/2} = \frac{{A^{(1)}}^2}{\left(\sum_{\nu=2}^{N_u} {A^{(\nu)}}^2/3\beta\right) + N_0/2}. \tag{17}$$

With perfect power control, $A^{(1)} = A^{(\nu)} = \sqrt{E_p}$, and ρ_I becomes:

$$\rho_I = \frac{3\beta}{(N_u - 1) + 3\beta/\rho_0}, \tag{18}$$

where $\rho_0 = 2E_p/N_0$. From this expression, it can be observed that for a low number of users, the performance is noise limited. Conversely, for a number of users $N_u > 3\beta/\rho_0$, the performance increasingly becomes interference limited.

The multi-user channel is modeled as an AWGN channel with two-sided noise spectral density $\sigma_I^2 + N_0/2$. The expression for the multi-user capacity is obtained by substituting ρ_I for the SNR in the single-user capacity expression [2]:

$$C(\rho_I) = \log_2 M - E_{\mathbf{v}|\mathbf{X}_1} \log_2 \sum_{m=1}^{M} \exp\left[\sqrt{\rho_I}(v_m - v_1)\right], \tag{19}$$

where $\mathbf{X}_1$ is the symbol transmitted by user 1, and the random variables $v_m, m = 1, .., M$, have the following distribution:

$$\begin{aligned} &v_1 && \text{is} \quad \mathcal{N}(\sqrt{\rho_I}, 1) \\ &v_m && \text{is} \quad \mathcal{N}(0, 1) \quad m \neq 1. \end{aligned}$$

Several figures were produced to demonstrate the multi-user capacity expressions obtained in this paper. In generating these figures, we focused on the effect of MAI and thus ignored the effect of thermal noise. Fig. 1 presents the user capacity in bits per PPM symbol of UWB as a function of the number of users for various number of modulation levels M. The curves were obtained by Monte Carlo runs of (19) with spreading ratio $\beta = 10$. From the figure it can be observed that for a low number of users, the UWB user capacity is approximated by $\log_2 M$, the noise-less capacity of M-ary PPM. Fig. 2 contains curves of the user capacity for a spreading ratio $\beta = 100$. From both figures, it is observed that the effect of MAI becomes noticeable for a number of users larger than about 0.3β. The individual user capacity effectively vanishes for a number of users larger than approximately 10β.

For a better understanding of the trade-offs among the variables governing the user capacity, it is of interest to develop a closed form expression for (19). This is possible for the special case of $M = 2$.

From (19), the user capacity in the presence of MAI for binary PPM is given by

$$C(\rho_I) \quad = \quad 1 - E_{\mathbf{v}|\mathbf{X}_1} \log_2\left(1 + \exp\left(\sqrt{\rho_I}(v_2 - v_1)\right)\right)$$

$$= \quad E_{\mathbf{v}|\mathbf{X}_1} \log_2 \frac{2}{1+\exp\left(\sqrt{\rho_I}(v_2 - v_1)\right)}. \quad (20)$$

Computing the expected value using the Gaussian distributions of v_1 and v_2, we have:

$$C(\rho_I) = \frac{1}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left(-\frac{(v_1 - \sqrt{\rho_I})^2}{2}\right) \exp\left(-\frac{{v_2}^2}{2}\right) \cdot \log_2 \frac{2}{1+\exp\left(\sqrt{\rho_I}(v_2 - v_1)\right)} dv_1 dv_2. \quad (21)$$

Set $x = v_2 - v_1$, we obtain the capacity of binary PPM,

$$C(\rho_I) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{(x-\sqrt{\rho_I})^2}{4}\right) \cdot \log_2 \left(\frac{2}{1+\exp\left(-\sqrt{\rho_I}x\right)}\right) dx. \quad (22)$$

This expression is similar in form to [7, eq. (21)]. The expression in the reference was developed for direct sequence CDMA. Due to the similarity between the two expressions, other results from the reference can be of use. Using steps similar to [7, eq. (21)], it can be shown that a lower bound to the user capacity in the MAI regimen (22), is given by

$$C(\rho_I) > 1 - \frac{\sqrt{2}\pi^2(\log_2 e)}{\pi^2 - 8} \left(\exp\left(-\frac{\pi^2-8}{4\pi^2}\rho_I\right) \cdot \operatorname{erfc}\left(\frac{2\sqrt{\rho_I}}{\pi}\right) - \frac{2\sqrt{2}}{\pi}\operatorname{erfc}\left(\frac{\sqrt{\rho_I}}{2}\right)\right). \quad (23)$$

A lower bound is of interest as it indicates the worst case of user capacity in the presence of MAI.

In Fig. 3, the lower bound (23) is plotted as a function of symbol SNR ρ_I along with the exact binary PPM capacity per (22). The capacity generated by Monte Carlo runs of (19) is also plotted for the comparison. It can be observed that the lower bound given in (23) is very close to the actual capacity, particularly for SNR values of interest $\rho_I \geq 5$ dB.

Our model for multi-user UWB assumes that each user contributes a fraction of the traffic in the channel. The aggregate capacity in the channel is obtained by summing all the user capacities,

$$\mathcal{C} = N_u C. \quad (24)$$

Fig. 4 presents the aggregate capacity for binary PPM UWB as a function of number of users for spreading ratios $\beta = 50$ and $\beta = 100$. The curves were generated using (22) and (24). From the figure it can be observed that when the number of the users is increased without bound, the aggregate capacity converges to a constant value. The asymptotic capacities for $N_u \to \infty$ are also shown in the figure.

The upper bound on the multi-user aggregate capacity can be also proved theoretically. In the appendix it is shown that the individual user capacity for $\rho_I \to 0$ is upper bound by

$$C(\rho_I) < (\log_2 e)\frac{\rho_I}{4}. \quad (25)$$

From (18) and (25), the aggregate capacity defined in (24) has an upper bound for small SNR ρ_I

$$\mathcal{C} < (\log_2 e)\frac{3N_u\beta}{4(N_u-1)+12\beta/\rho_0}. \quad (26)$$

When the number of users increases without bound, the aggregate capacity converges to

$$\lim_{N_u\to\infty} \mathcal{C} < (\log_2 e)\frac{3\beta}{4}. \quad (27)$$

Not surprisingly, this upper bound is proportional to the spreading ratio β.

4. CONCLUSIONS

In this paper, we analyzed the channel capacity of M-ary PPM UWB communications for multiple noncooperative users. Under our assumptions, the MAI component at the receiver was modeled Gaussian. An expression for the SNR after demodulation was developed for UWB utilizing rectangular pulses. The information theoretic capacity per user was expressed as a function of the number of users. A closed form expression was developed for the channel capacity for binary PPM. Using this expression, a lower bound was found for the user channel capacity. An upper bound was found for the aggregate capacity of all users. This upper bound is proportional to the spreading ratio β.

APPENDIX

In this appendix, we prove the upper bound (25) on the individual user capacity of binary PPM with MAI.

The exponential function can be expressed as a power series

$$\exp\left(-\sqrt{\rho_I}x\right) = 1 + \sum_{n=1}^{+\infty} \frac{(-\sqrt{\rho_I}x)^n}{n!}. \quad (28)$$

Using (28) in (22), the logarithm term is expanded in a power series

$$\begin{aligned}
&\log_2\left(\frac{2}{1+\exp\left(-\sqrt{\rho_I}x\right)}\right) \\
&= (-\log_2 e)\ln\left(1+\frac{\sum_{n=1}^{+\infty}\frac{(-\sqrt{\rho_I}x)^n}{n!}}{2}\right) \\
&= (\log_2 e)\sum_{m=1}^{+\infty}(-1)^m \frac{\left(\frac{1}{2}\sum_{n=1}^{+\infty}\frac{(-\sqrt{\rho_I}x)^n}{n!}\right)^m}{m} \\
&= (\log_2 e)\left(\frac{\sqrt{\rho_I}x}{2} - \frac{\rho_I x^2}{8} + \frac{\rho_I^2 x^4}{192} - o\left[\rho_I^3\right]\right), \quad (29)
\end{aligned}$$

where, by definition, if $f(\rho) = o\left[\rho^3\right]$, then $f(\rho)/\rho^3 \to 0$ as $\rho \to 0$. It follows that for ρ sufficiently small, we have the upper bound

$$\log_2\left(\frac{2}{1+\exp\left(-\sqrt{\rho_I}x\right)}\right) < (\log_2 e)\left(\frac{\sqrt{\rho_I}x}{2} - \frac{\rho_I x^2}{8}\right), \quad (30)$$

Substituting (30) into (22), we obtain the upper bound on the multi-user capacity

$$
\begin{aligned}
C(\rho_I) &< \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{+\infty}\exp\left(-\frac{(x-\sqrt{\rho_I})^2}{4}\right)(\log_2 e) \\
&\quad \cdot \left(\frac{\sqrt{\rho_I}x}{2}-\frac{\rho_I x^2}{8}\right)dx \\
&= (\log_2 e)\left(\frac{\rho_I}{4}-\frac{\rho_I^2}{8}\right) \\
&< (\log_2 e)\frac{\rho_I}{4}. \qquad (31)
\end{aligned}
$$

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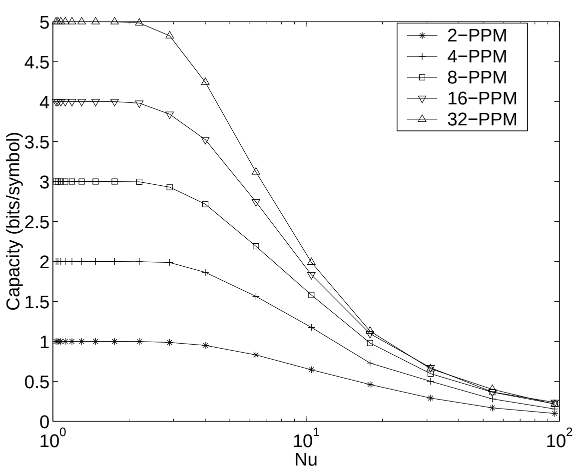

Fig. 1. User capacity for multi-user UWB as a function of number of users N_u for spreading ratio $\beta = 10$.

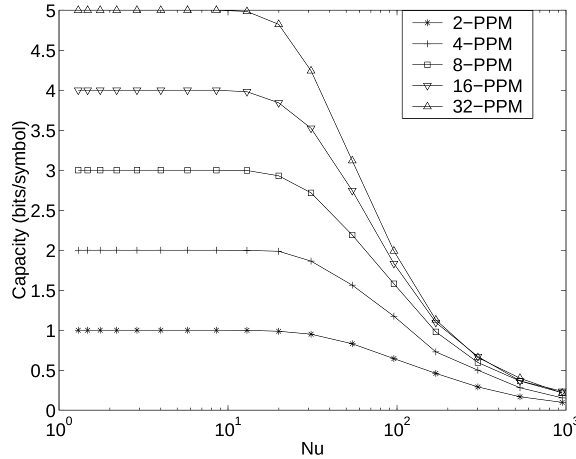

Fig. 2. User capacity for multi-user UWB as a function of number of users N_u for spreading ratio $\beta = 100$.

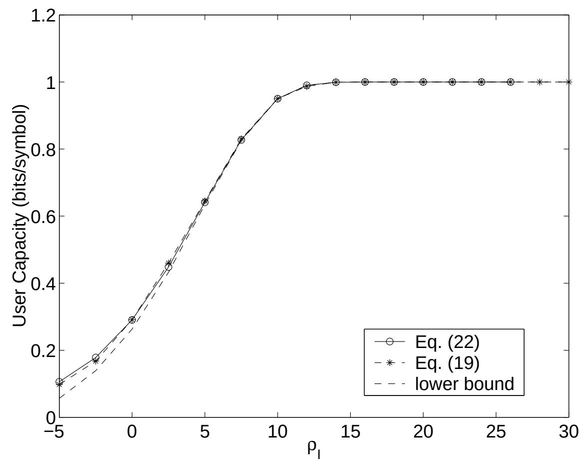

Fig. 3. User capacity of 2-PPM UWB system for spreading ratio $\beta = 100$.

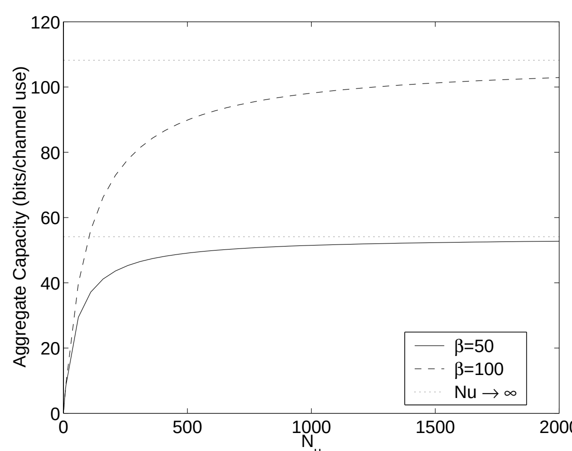

Fig. 4. Aggregate capacity of 2-PPM UWB systems as a function of number of users N_u for $\rho_0 = 10$ dB.