Performance of Cellular CDMA Systems with Space Diversity, Fading, and Imperfect Power Control

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Abstract— **It is well known that capacity of code-division multiple-access (CDMA) systems degrades rapidly with the increase in power control error. Capacity is also affected by small-scale fading such as Rayleigh fading and by the voice activity. In this paper we analyze the outage probability of the uplink of a CDMA system. Closed form approximations are provided for two different definitions of the outage encountered in literature. These expressions provide a simple tool to analyze system performance as function of diversity, fading, power control error, and voice activity.**

I. Introduction

Wireless propagation models traditionally distinguish between *large-scale* and *small-scale* fading. The former characterizes the signal strength over large distances and is often modeled by the log-normal distribution, while the latter refers to the rapid fluctuations due to the summing of multipath components with random phase, and is modeled by Rayleigh and other distributions. In addition to fading effects, received signals are also affected by the path loss between transmitter and receiver. It is well known that code-division multiple-access (CDMA) uplink system capacity (in terms of number of users per cell) is maximized if all users' signals are received at the base station with the same power. Power control loops are incorporated in the system to compensate for the path loss between mobile and base. Power control also mitigates the effects of large-scale fading within an accuracy dictated by practical considerations. Adaptive arrays may be used to provide space diversity to mitigate small-scale fading effects in mobile communications. In an uplink of a CDMA system, the total interference to any given user is also by the *voice activity* - the number of users that actually use the channel at a given time. While an accurate analysis of system performance needs to take into account all these factors, most of the published research contains analytical results accounting only for part of the them. For example, large-scale fading and power control are analyzed in [1], [2], while the effects of Rayleigh fading and diversity are analyzed in numerous other publications. Some recent publication attempts to incorporate both large and small-scale fading in the analysis, however the results obtained are not in closed-form [3].

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This paper provides accurate approximations of the outage probability of the uplink of a CDMA system taking into account space diversity, Rayleigh fading, power control error (PCE), and voice activity. Expressions are developed for two different definitions of the outage, one suited for data outage - the other for voice outage.

II. System Model

The system model is of the uplink of a CDMA system. For simplicity, a single cell is assumed which serves K_u users, and uses a base station with an M-element antenna array. The received signals are assumed to undergo independent Rayleigh fading. It is further assumed that the fading is flat and slowly varying such that the lowpass equivalent channel seen by each antenna can be characterized by a complex-valued scalar. The system is assumed interference limited such that outages are caused mainly by cochannel interference (CCI) and the effect of thermal noise is negligible. The CDMA reverse link receiver model is shown in Fig. 1. The complex envelope of the signal received at the base station is then expressed by the M-dimensional vector:

$$\mathbf{x}(t) = \sqrt{\lambda_1}\, s_1(t-\tau_1)u_1(t-\tau_1)\mathbf{c}_1(t) + \sum_{k=2}^{K_u} \epsilon_k \sqrt{\lambda_k}\, s_k(t-\tau_k)u_k(t-\tau_k)\mathbf{c}_k(t), \quad (1)$$

where the first and second terms respectively represent the desired signal and the CCI, λ_k $(k = 1, \cdots, K_u)$ are the powers of the received signals, $\mathbf{c}_k(t)$ are normalized complex Gaussian channel vectors with $E\left[\mathbf{c}_k(t)\mathbf{c}_k^H(t)\right] = \mathbf{I}$, $\mathbf{I}$ is the $M \times M$ identity matrix, the superscript denotes transpose and complex conjugate, $s_k(t)$ are NRZ waveforms of the users' data, $u_k(t)$ are the spreading sequences, ϵ_k are binary random variables indicating the users' voice activity, τ_k are the users' delays. Let $s_k(t) = \sum_i s_k(i)\, g(t-iT_s)$, where $g(t)$ is the basic pulse shape, T_s is the symbol interval, and $s_k \in \{-1, 1\}$ are the users' binary data with $E[s_k(i)] = 0$, and $E[s_k(i)\, s_l(j)] = \delta_{kl}\delta_{ij}$, where $\delta_{ij} = 1$ for $i = j$, and $\delta_{ij} = 0$ otherwise. The signature waveforms are normalized to unit energy over the symbol interval. For convenience, $\tau_1 = 0$. In a system with perfect

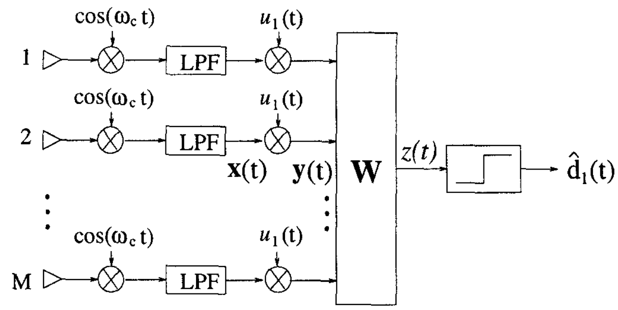

Fig. 1. CDMA reverse link receiver model

power control, all λ_k's are equal. Here, the powers λ_k are modeled as random variables with log-normal distribution. The computation of the outage probability, the main result of this paper, does not require that the λ_k's be identically distributed or even independent. However, the assumption of identical statistics affecting all users is reasonable and it enables to analyze the performance in terms of the system PCE. Henceforth it is assumed that the powers λ_k's are independent and identically distributed. If a random variable λ_k has a log-normal distribution, then $\alpha_k = 10\log_{10}\lambda_k$ is normal. The standard deviation of α_k is referred to as the PCE measured in dB. The voice activity ϵ_k is modeled as a Bernoulli (p) random variable with $\Pr(\epsilon_k = 1) = p$, and $\Pr(\epsilon_k = 0) = 1 - p$, where p is referred to as the *voice activity factor*.

Following spread spectrum demodulation and sampling at the symbol interval, the received signal can be written:

$$\mathbf{y}(i) = \int_{iT_s}^{(i+1)T_s} \mathbf{x}(t)u_1(t)dt = \sqrt{\lambda_1}s_1(i)\mathbf{c}_1 + \sum_{k=2}^{K_u} \epsilon_k\sqrt{\lambda_k}\left(s(i-1)\rho_k^- + s(i)\rho_k^+\right)\mathbf{c}_k, \tag{2}$$

where $\rho_k^- = \int_{iT_s}^{iT_s+\tau_k} u_k(t-\tau_k)u_1(t)dt$, $\rho_k^+ = \int_{iT_s+\tau_k}^{(i+1)T_s} u_k(t-\tau_k)u_1(t)dt$ are the correlations between user signatures. User signatures may be supplied by a long code such as in IS-95, however it is assumed that the correlations above are independent of the symbol interval index i.

The antenna array outputs $\mathbf{y}(i)$ are combined using the method of maximal ratio combining. The array weight vector $\mathbf{w}$ is then acting as a matched filter to the channel, $\mathbf{w} = \mathbf{c}_1$. The array output is expressed $z(i) = \mathbf{w}^H\mathbf{y}(i) = \mathbf{c}_1^H\mathbf{y}(i)$. In the next section, the outage probability is computed based on this signal model.

III. Computation of Outage Probability

The outage probability is defined as the probability that the output signal-to-interference (SIR) falls below a prescribed level. The outage probability is computed for two different definitions of the outage. In one case, the outage is of the *instantaneous* SIR; this definition is fitting for data transmission, since even a brief outage may affect performance. In the other case, the outage is defined with respect to the *average* SIR over Rayleigh fading. This approach is suitable to the computation of voice outage where very short outages are normally not detected.

In the first case, the goal is to compute the outage averaged over Rayleigh fading, PCE, and voice activity. The instantaneous output SIR γ is defined as the ratio of the signal and interference powers expectations taken over the data bits. Following the data independence assumptions, it is easy to obtain:

$$\gamma = \frac{\lambda_1\left|\mathbf{c}_1^H\mathbf{c}_1\right|^2}{\left|\mathbf{c}_1^H\sum_{k=2}^{K_u}\epsilon_k\eta_k\lambda_k\mathbf{c}_k\right|^2}, \tag{3}$$

where $\eta_k = \left(\rho_k^-\right)^2 + \left(\rho_k^+\right)^2$ and $\epsilon_k^2 \equiv \epsilon_k$. The cumulative distribution function of γ conditioned on the input SIR is given in [4], [5]

$$F(\gamma \mid \mu) = \frac{(\gamma/\mu)^M}{(1+\gamma/\mu)^M}, \tag{4}$$

where μ is the input SIR

$$\mu = \frac{\lambda_1}{\sum_{k=2}^{K_u}\epsilon_k\eta_k\lambda_k}. \tag{5}$$

The quantity μ is a function of the large-scale fading parameters λ_k and the voice activity ϵ_k and hence it is a random variable. To fully characterize the outage probability, it is required to obtain the density function of μ. Let $\mathcal{I}$ denote the interference power, then from (5)

$$\mathcal{I} = \sum_{k=2}^{K_u}\epsilon_k\eta_k\lambda_k. \tag{6}$$

Since $\epsilon_k \in \{0,1\}$, $\mathcal{I}$ is the sum of log-normal random variables. The number of elements in the sum is determined by ϵ_k. A method based on cumulant matching has been used to approximate the distribution of a sum of log-normal random variables [6], [2]. The method assumes that $\mathcal{I}$ is log-normal; it then proceeds to match $E[\mathcal{I}]$ and $E[\mathcal{I}^2]$ computed from (6) with the corresponding cumulants of the log-normal distribution. Consistent with the assumption that $\mathcal{I}$ is log-normal, it can be expressed $\mathcal{I} = e^b$, where b has a normal distribution, $b \sim \mathcal{N}\left[m_b, \sigma_b^2\right]$. It follows that

$$E[\mathcal{I}] = E\left[e^b\right] = e^{m_b+\sigma_b^2/2}, \tag{7}$$

$$E\left[\mathcal{I}^2\right] = E\left[e^{2b}\right] = e^{2m_b+2\sigma_b^2}. \tag{8}$$

Let the user powers λ_k be expressed in terms of the normal random variables $a_k \sim \mathcal{N}\left[m_a, \sigma_a^2\right]$, $\lambda_k = e^{a_k}$. The random variable a_k is related to the previously defined quantity α_k, by

$$a_k = \alpha_k(\ln 10)/10. \tag{9}$$

From (6), and taking the expectation over ϵ_k and a_k,

$$\begin{aligned} E[\mathcal{I}] &= E\left[\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{a_k}\right] = \sum_{k=2}^{K_u}\eta_k E[\epsilon_k]E[e^{a_k}] \\ &= pe^{m_a+\sigma_a^2/2}\sum_{k=2}^{K_u}\eta_k \equiv e^{\xi_1}. \end{aligned} \quad (10)$$

Similarly,

$$\begin{aligned} E[\mathcal{I}^2] &= E\left[\left(\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{a_k}\right)^2\right] \\ &= \sum_{k=2}^{K_u}\eta_k^2 E\left[\epsilon_k^2 e^{2a_k}\right] + 2\sum_{k=2}^{K_u}\sum_{j=k+1}^{K_u}\eta_k\eta_j E\left[\epsilon_k e^{a_k}\epsilon_j e^{a_j}\right] \\ &= pe^{2m_a+2\sigma_a^2}\left(\sum_{k=2}^{K_u}\eta_k\right)^2 + p^2 e^{2m_a+\sigma_a^2}\sum_{k=2}^{K_u}\sum_{j\neq k}\eta_k\eta_j \\ &\equiv e^{\xi_2}. \end{aligned} \quad (11)$$

By equating (7) with (10) and (8) with (11), the unknown quantities m_b, σ_b^2 can be expressed

$$m_b = 2\xi_1 - \frac{1}{2}\xi_2 \quad (12)$$

$$\sigma_b^2 = \xi_2 - 2\xi_1. \quad (13)$$

Now, from (5) and the log-normality of λ_1 and $\mathcal{I}$, it follows that μ is also log-normal. Indeed,

$$g = \ln\mu = \ln\frac{\lambda_1}{\mathcal{I}} = a_1 - b, \quad (14)$$

is normal, and hence $\mu = e^g$ is log-normal. The density function of μ is determined from the mean and variance of g, namely, $m_g = m_a - m_b$ and $\sigma_g^2 = \sigma_a^2 + \sigma_b^2$, respectively. Thus all the ingredients required to compute the density function of μ are available.

The outage probability is defined as the probability that γ, the output SIR, falls below a threshold ζ, $P_o = \Pr(\gamma < \zeta)$. The outage probability conditioned on the input SIR μ is given by (4),

$$P_o(\mu) = F(\gamma = \zeta) = \frac{(\zeta/\mu)^M}{(1+\zeta/\mu)^M}, \quad (15)$$

The average outage probability accounting for Rayleigh fading, PCE and voice activity is given by

$$P_o = \int_0^\infty P_o(\mu)f(\mu)d\mu \quad (16)$$

or since $\mu = e^g$,

$$P_o = \int_{-\infty}^\infty P_o(g)f(g)dz = E_g[P_o(g)], \quad (17)$$

where

$$P_o(g) = \frac{(\zeta e^{-g})^M}{(1+\zeta e^{-g})^M}. \quad (18)$$

An exact closed-form expression for the integral in (17) is not available, however for the normally distributed random variable g, a simple, but accurate approximation exists for $E_g[P_o(g)]$ in terms of the mean m_g and the standard deviation σ_g and for an arbitrary function $P_o(g)$ of the random variable g [7]:

$$\begin{aligned} P_o &= E_g[P_o(g)] \cong \frac{2}{3}P_o(m_g) + \frac{1}{6}P_o\left(m_g + \sqrt{3}\,\sigma_g\right) \\ &\quad + \frac{1}{6}P_o\left(m_g - \sqrt{3}\,\sigma_g\right). \end{aligned} \quad (19)$$

The previous relation provides for the closed-form computation of the outage probability as a function of Rayleigh fading, PCE, and voice activity.

The outage probability provides an indicator of how often the communications quality is under a certain acceptable level. The system capacity is generally estimated according to this parameter. When the mobiles transmit voice rather than data, it is more suitable to consider outages over longer durations such that are detectable by the user. A more meaningful approach in this case is to define outage with respect to the SIR averaged over the Rayleigh fading. Define the vector of channel coefficients $\mathbf{c}_1 = [c_{11}, \cdots, c_{1M}]^T$. Consistent with earlier definitions of the channel, c_{1m} $(m = 1, \cdots, M)$ are independent complex Gaussian random variables with zero mean and $E\left[|c_{1m}|^2\right] = 1$. The average signal power at the array output is then given by:

$$\begin{aligned} S_E &= E\left[\lambda_1\left|\mathbf{c}_1^H\mathbf{c}_1\right|^2\right] \\ &= \lambda_1 E\left[\left(\sum_{m=1}^{M}|c_{1m}|^2\right)^2\right]. \end{aligned} \quad (20)$$

It is not difficult to show that

$$S_E = \left(M^2 + M\right)\lambda_1. \quad (21)$$

The average interference power is given by:

$$\mathcal{I}_E = E\left[\left|\mathbf{c}_1^H\left(\sum_{k=2}^{K_u}\epsilon_k\sqrt{\eta_k\lambda_k}\,\mathbf{c}_k\right)\right|^2\right]. \quad (22)$$

The vector $\sum_{k=2}^{K_u}\epsilon_k\sqrt{\eta_k\lambda_k}\mathbf{c}_k$ has a complex Gaussian distribution with zero mean and covariance matrix $\mathcal{I}\,\mathbf{I}$, where $\mathcal{I}$ was defined in (6), and $\mathbf{I}$ is the identity matrix. It follows that one can set

$$\sum_{k=2}^{K_u}\epsilon_k\sqrt{\eta_k\lambda_k}\,\mathbf{c}_k = \sqrt{\mathcal{I}}\;\mathbf{c}_p, \quad (23)$$

where $\mathbf{c}_p = [c_{p1}, \cdots, c_{pM}]^T$ is complex Gaussian with zero mean and $E[\mathbf{c}_p \mathbf{c}_p^H] = \mathbf{I}$. The average interference power can then be written as:

$$\mathcal{I}_E = \mathcal{I} E\left[\left|\mathbf{c}_1^H \mathbf{c}_p\right|^2\right] = M\mathcal{I}. \quad (24)$$

The average SIR can be expressed as:

$$\gamma_E = \frac{S_E}{\mathcal{I}_E} = (M+1)\frac{\lambda_1}{\mathcal{I}} = (M+1)\mu. \quad (25)$$

The outage is now defined as the probability that γ_E falls below a threshold ζ_E: $P_{oE} = \Pr(\gamma_E < \zeta_E)$. Utilizing (25), and the expression $g = \ln \mu$, the outage probability can be written:

$$P_{oE} = \Pr\left(g < \ln \frac{\zeta_E}{M+1}\right). \quad (26)$$

Since g is normally distributed, (26) can be expressed

$$P_{oE} = 1 - \frac{1}{2}\text{erfc}\left(\frac{\ln \frac{\zeta_E}{M+1} - m_g}{\sqrt{2}\,\sigma_g}\right), \quad (27)$$

where m_g and σ_g were defined previously.

Eqs.(19) and (27) are expressions of the outage probability for two different criteria. When the communication link quality is sensitive to the instantaneous SIR, (19) should be used to evaluate the outage probability. Conversely, (27) is to be used when the average SIR determines the performance.

IV. Numerical Results

Computer simulations were used to verify the results in (19) and (27). The simulations consisted of a CDMA system employing BPSK modulation and BPSK spreading, with a spreading ratio of 156, and with a voice activity factor of $p = 3/8$. The outage probability of instantaneous SIR is plotted in Fig. 2 as a function of the capacity (number of users/cell) for $M = 4$ antenna elements and with the PCE as a parameter. The outage threshold is set at $\zeta = 7$ dB, corresponding to a bit error rate of 10^{-3}. Analytical results are calculated from (19). Simulation curves are based on results compiled from one million Monte Carlo runs. For an outage of 10^{-2}, the system capacity is approximately 38, 32, 22, and 10 users/cell for PCE = 0, 1.5, 2.5 and 4.0 dB, respectively. In a CDMA system with PCE 1.5 to 2.5 dB, the system capacity degrades 16% to 42% compared to the case of perfect power control. Fig. 2 shows a good match between analytical results and simulations. To see the effect of PCE on the system capacity clearly, Fig. 3 gives the analytical curve of capacity versus PCE, where $P_o(\gamma < 7\text{ dB}) = 10^{-2}$ and $M = 4$. This result shows that the system capacity degrades with the increase of PCE, but the curve is nonlinear. However, in the range of PCE = 1 to 4 dB, the capacity goes down almost linearly with the increase of PCE, the rate is about 8 users/cell/dB.

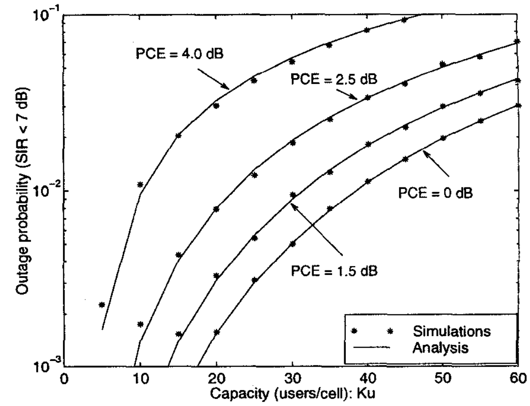

Fig. 2. Outage probability versus capacity. Four antenna elements ($M = 4$) and different PCEs, $p = 3/8$.

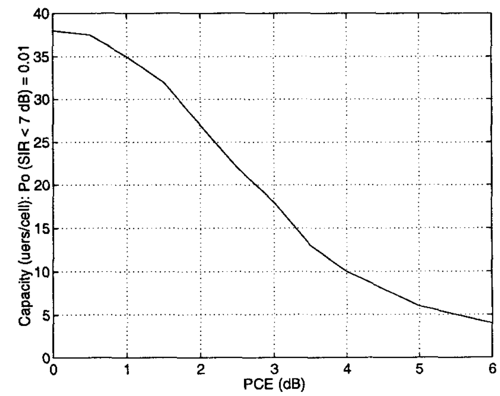

Fig. 3. Capacity (users/cell) versus PCE. Analytical results for $M = 4$, $p = 3/8$ and $P_o(\text{SIR} < 7\text{ dB}) = 10^{-2}$.

When the PCE increases to 6 dB, the system capacity is below 5 users/cell.

The effect of space diversity (analytical results only) is shown in Fig. 4, for PCE = 1.5 dB, $M = 1$ to 8, and all other parameters similar to those used in Fig. 2. With $P_o = 10^{-2}$, 8-branch space diversity provides a capacity of 87 users/cell, while the capacity for two-branch space diversity is only 9 users/cell. Without space diversity ($M = 1$), the system capacity is below 5 users/cell. Utilizing the results in Fig. 4, and setting $P_o = 10^{-2}$, Fig. 5 shows that the capacity increases almost linearly with the degrees of space diversity; the capacity increase for each additional degree of space diversity is approximately 13 users/cell.

Next, we examine the outage probability of average SIR. The outage threshold is set at $\zeta_E = 7$ dB. The analytical curves in Fig. 6 are computed from (27), and the simulation curves are based on ten million Monte Carlo runs. For an outage of 10^{-2}, the system capacity is approximately 357, 170, 90 and 30 users/cell for PCE = 0, 1.5, 2.5 and 4.0 dB,

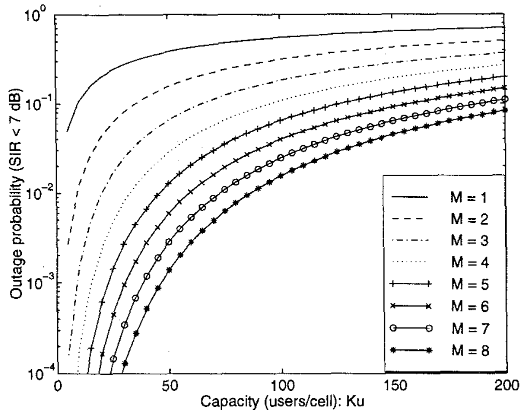

Fig. 4. Outage probability versus capacity. Analytical results for PCE = 1.5 dB and $M = 1$ to 8, $p = 3/8$.

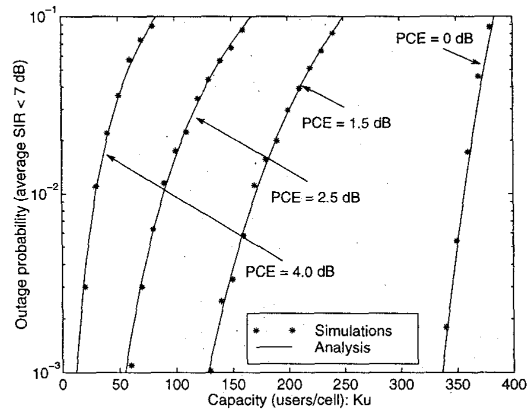

Fig. 6. Outage probability versus capacity (users/cell), consider the avearge SIR. Four antenna elements ($M = 4$) and different PCEs, $p = 3/8$.

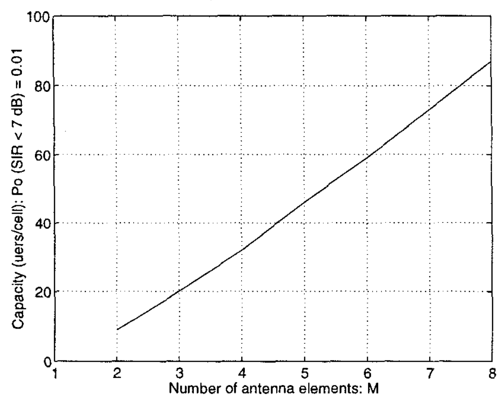

Fig. 5. Capacity (users/cell) versus number of antenna elements. Analytical results for PCE = 1.5 dB, $p = 3/8$ and $P_o(\text{SIR} < 7\text{ dB}) = 10^{-2}$.

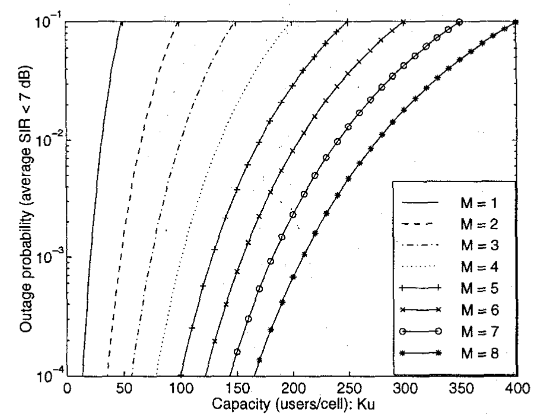

Fig. 7. Outage probability versus capacity (users/cell), consider the avearge SIR. Analytical results for PCE = 1.5 dB and different number of antenna elements, $p = 3/8$.

respectively. Consequently, for PCE = 1.5 to 2.5 dB, the system capacity degrades 52% to 75% compared to the case of perfect power control. Comparing Figs. 2 and 6, we conclude that for same threshold, the system capacity is significantly larger for an outage computed from the average SIR then for the computation based on the instantaneous SIR. However, the average SIR outage degrades faster due to PCE. The effect of space diversity on the outage probability for average SIR is shown in Fig. 7 for parameters similar to those used in Fig. 4. For $P_o = 10^{-2}$, the system capacity is about 31 to 276 users/cell for $M = 1$ to 8, i.e., the average capacity increase for each additional degree of space diversity is about 34 users/cell.

Figs. 2 to 7 show that capacity of a CDMA system increases with space diversity and decreases with PCE. Therefore, it is concluded that space diversity can be used to compensate for PCE effects in wireless CDMA communications.

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