# Outage Probability of Cellular CDMA Systems with Space Diversity, Rayleigh Fading, and Power Control Error

Weichen Ye and Alexander M. Haimovich, *Senior Member, IEEE*

***Abstract*—It is well known that the capacity of code-division multiple-access (CDMA) systems degrades rapidly with the increase in power control error. Capacity is also affected by small-scale fading such as Rayleigh fading and by the voice activity. The main contribution of this letter is to provide a new simple approximation to the outage probability of the uplink of a CDMA system utilizing an antenna array for space diversity, taking into account stochastic models of the power control error, small-scale fading, and voice activity.**

***Index Terms*—CDMA, power control, Rayleigh fading.**

## I. Introduction

FURTHER improvement in the performance of code-division multiple-access (CDMA) communications, particularly over fading channels, can be obtained by the use of adaptive arrays which provide space diversity. It is well known that CDMA uplink system capacity (in terms of number of users per cell) is maximized if all users' signals are received at the base station with the same power. Power control loops are incorporated in the system to compensate for the path loss between mobile and base. Wireless propagation models traditionally distinguish between *large-scale* and *small-scale* fading. The former characterizes the signal strength over large distances and is commonly modeled by the log-normal distribution, while the latter refers to the rapid fluctuations due to the summing of multipath components with random phase, and is modeled by Rayleigh and other distributions. Power control mitigates the effects of large-scale fading, but practical considerations limit the accuracy of the control loops. Small-scale fading is mitigated by diversity mechanisms such as space diversity provided by an antenna array at the base station. The total interference to any given user is also affected by the *voice activity*. While an accurate analysis of system performance needs to take into account all these factors, most of the published research contains analytical results only for subsets of the factors. For example, large-scale fading and power control are analyzed in [1] and [2], while the effects of Rayleigh fading and diversity are analyzed in numerous other publications. A recent publication has attempted to incorporate both large and small-scale fading in the analysis, however, closed-form expressions are not provided [3]. The contribution of this letter is to provide an accurate approximation of the outage probability in the uplink of a CDMA system taking into account space diversity, Rayleigh fading, power control error (PCE), and voice activity.

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The authors are with the Center for Communications and Signal Processing, ECE Department, New Jersey Institute of Technology, Newark, NJ 07102 USA (e-mail: wxy3490@megahertz.njit.edu; haimovic@megahertz.njit.edu).

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## II. System Model

The system model is of the uplink of a CDMA system, which serves $K_u$ users, and uses a base station with an $M$-element antenna array. The received signals are assumed to undergo independent Rayleigh fading. It is further assumed that the fading is flat and slowly varying such that the low-pass equivalent channel seen by each antenna can be characterized by a complex-valued scalar. The system is assumed interference limited such that outages are caused mainly by cochannel interference (CCI) and the effect of thermal noise is negligible. The complex envelope of the signal received at the base station is then expressed by the following $M$-dimensional vector:

$$\mathbf{x}(t) = \sqrt{\lambda_1} s_1(t-\tau_1) u_1(t-\tau_1)\mathbf{c}_1(t) + \sum_{k=2}^{K_u} \epsilon_k \sqrt{\lambda_k} s_k(t-\tau_k) u_k(t-\tau_k)\mathbf{c}_k(t) \tag{1}$$

where the first and second terms represent, respectively, the desired signal and the CCI; $\lambda_k$ $(k = 1, \cdots, K_u)$ are the powers of the received signals; $\mathbf{c}_k(t)$ are normalized complex Gaussian channel vectors (Rayleigh fading) with $E[\mathbf{c}_k(t)\mathbf{c}_k^H(t)] = \mathbf{I}$; $\mathbf{I}$ is the $M \times M$ identity matrix; the superscript denotes transpose and complex conjugate; $s_k(t)$ are NRZ waveforms of the users' data; $u_k(t)$ are the spreading sequences; $\epsilon_k$ are binary random variables indicating the users' voice activity; and $\tau_k$ are the users' delays. Let $s_k(t) = \sum_i s_k(i) g(t - iT_s)$, where $g(t)$ is the basic pulse shape, $T_s$ is the symbol interval, and $s_k \in \{-1, 1\}$ are the users' binary data with $E[s_k(i)] = 0$, and $E[s_k(i)s_k(j)] = \delta_{ij}$, $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ otherwise, and $E[s_k(i)s_l(j)] = 0$ for $k \neq l$. The signature waveforms are normalized to unit energy over the symbol interval. For convenience, $\tau_1 = 0$. In a system with perfect power control, all $\lambda_k$'s are equal. Here, the powers $\lambda_k$ are modeled as random variables with log-normal distribution, but the values of $\lambda_k$ change slowly compared to Rayleigh fading. The system model presented above incorporates effects of shadowing/power control ($\lambda_k$), Rayleigh fading ($\mathbf{c}_k$), and voice activity ($\epsilon_k$). The main result of this letter, computation of the outage probability, does not require that the $\lambda_k$'s be identically distributed or even independent. However, the assumption of shadowing with the same statistics affecting all users is reasonable and it enables to analyze the performance in terms of the system PCE. Henceforth it is assumed that the powers $\lambda_k$'s are independent and identically distributed. If a random variable $\lambda_k$ has a log-normal distribution, then $\alpha_k = 10\log_{10}\lambda_k$ is normal. The standard deviation of $\alpha_k$ is referred to as the PCE measured in decibels. The voice activity $\epsilon_k$ is modeled as a Bernoulli $(p)$ random variable with $\Pr(\epsilon_k = 1) = p$, and $\Pr(\epsilon_k = 0) = 1 - p$, where $p$ is referred to as the *voice activity factor*. After spread spectrum demodulation and sampling at the symbol interval, the received signal can be written

$$\mathbf{y}(i) = \int_{iT_s}^{(i+1)T_s} \mathbf{x}(t)u_1(t)\,dt = \sqrt{\lambda_1}s_1(i)\mathbf{c}_1 + \sum_{k=2}^{K_u}\epsilon_k\sqrt{\lambda_k}(s(i-1)\rho_k^- + s(i)\rho_k^+)\mathbf{c}_k$$

where $\rho_k^- = \int_{iT_s}^{iT_s+\tau_k} u_k(t-\tau_k)u_1(t)\,dt$ and $\rho_k^+ = \int_{iT_s+\tau_k}^{(i+1)T_s} u_k \cdot(t-\tau_k)u_1(t)\,dt$ are the correlations between user signatures. User signatures may be supplied by a long code such as in IS-95, however it is assumed that the correlations above are independent of the symbol interval index $i$.

The antenna array outputs $\mathbf{y}(i)$ are combined using the method of maximal ratio combining. The array weight vector $\mathbf{w}$ is then acting as a matched filter to the channel, $\mathbf{w} = \mathbf{c}_1$. The array output is expressed $r(i) = \mathbf{w}^H\mathbf{y}(i) = \mathbf{c}_1^H\mathbf{y}(i)$. In the next section, the outage probability is computed based on this signal model.

## III. Computation of Outage Probability

The outage probability is defined as the probability that the output signal-to-interference (SIR) falls below a prescribed level. The goal is to compute the outage averaged over Rayleigh fading, PCE, and voice activity. The instantaneous output SIR $\gamma$ is defined as the ratio of the signal and interference powers expectations taken over the data bits. For a given bit duration, it can be shown [4]

$$\gamma = \frac{\lambda_1|\mathbf{c}_1^H\mathbf{c}_1|^2}{\left|\mathbf{c}_1^H\sum_{k=2}^{K_u}\epsilon_k\sqrt{\eta_k\lambda_k}\mathbf{c}_k\right|^2} \tag{2}$$

where $\eta_k$ is a gain factor incorporating the effect of cross correlation with the desired user's signal and $\epsilon_k^2 \equiv \epsilon_k$. The cumulative distribution function of $\gamma$ conditioned on the input SIR is given in [4] and [5]

$$F(\gamma|\mu) = \frac{(\gamma/\mu)^M}{(1+\gamma/\mu)^M} \tag{3}$$

where $\mu$ is the input SIR

$$\mu = \lambda_1 \Big/ \sum_{k=2}^{K_u}\epsilon_k\eta_k\lambda_k. \tag{4}$$

The quantity $\mu$ is a function of the large-scale fading parameters $\lambda_k$ and the voice activity $\epsilon_k$ and hence it is a random variable. To fully characterize the outage probability, it is required to obtain the density function of $\mu$. Let $I$ denote the interference power, then from (4)

$$I = \sum_{k=2}^{K_u}\epsilon_k\eta_k\lambda_k. \tag{5}$$

Since $\epsilon_k \in \{0, 1\}$, $I$ is the sum of log-normal random variables. The number of elements in the sum is determined by $\epsilon_k$.

A method based on cumulant matching has been used to approximate the distribution of a sum of log-normal random variables [2], [6]. The method assumes that $I$ is log-normal; it then proceeds to match $E[I]$ and $E[I^2]$ computed from (5) with the corresponding cumulants of the log-normal distribution. Consistent with the assumption that $I$ is log-normal, it can be expressed $I = e^b$, where $b$ has a normal distribution, $b \sim \mathcal{N}[m_b, \sigma_b^2]$. It follows that

$$E[I] = E[e^b] = e^{m_b+\sigma_b^2/2} \tag{6}$$

$$E[I^2] = E[e^{2b}] = e^{2m_b+2\sigma_b^2}. \tag{7}$$

Let the user powers $\lambda_k$ be expressed in terms of the normal random variables $a_k \sim \mathcal{N}[m_a, \sigma_a^2]$, $\lambda_k = e^{a_k}$. The random variable $a_k$ is related to the previously defined quantity $\alpha_k$, by $a_k = \alpha_k(\ln 10)/10$. From (5), and taking the expectation over $\epsilon_k$ and $a_k$,

$$\begin{aligned} E[I] &= E\left[\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{a_k}\right] \\ &= \sum_{k=2}^{K_u}\eta_k E[\epsilon_k]E[e^{a_k}] \\ &= pe^{m_a+\sigma_a^2/2}\sum_{k=2}^{K_u}\eta_k \equiv e^{\xi_1}. \end{aligned} \tag{8}$$

Similarly,

$$\begin{aligned} E[I^2] &= E\left[\left(\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{a_k}\right)^2\right] \\ &= \sum_{k=2}^{K_u}\eta_k^2 E[\epsilon_k^2 e^{2a_k}] \\ &\quad + 2\sum_{k=2}^{K_u}\sum_{j=k+1}^{K_u}\eta_k\eta_j E[\epsilon_k e^{a_k}\epsilon_j e^{a_j}] \\ &= pe^{2m_a+2\sigma_a^2}\sum_{k=2}^{K_u}\eta_k^2 + p^2e^{2m_a+\sigma_a^2}\sum_{k=2}^{K_u}\sum_{j\neq k}\eta_k\eta_j \\ &\equiv e^{\xi_2}. \end{aligned} \tag{9}$$

By equating (6) with (8) and (7) with (9), the unknown quantities $m_b$, $\sigma_b^2$ can be expressed

$$m_b = 2\xi_1 - \frac{1}{2}\xi_2 \tag{10}$$

$$\sigma_b^2 = \xi_2 - 2\xi_1. \tag{11}$$

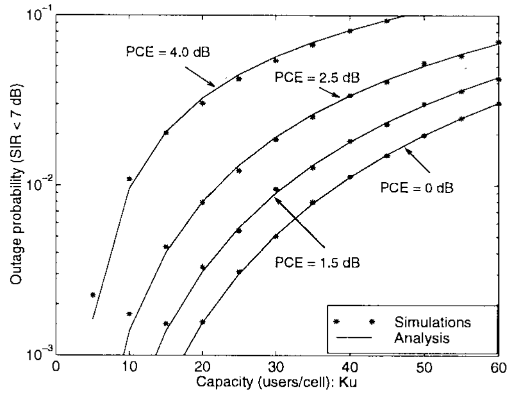


Fig. 1. Outage probability versus capacity. Four antenna elements ($M = 4$), different PCE's, and $p = 3/8$.

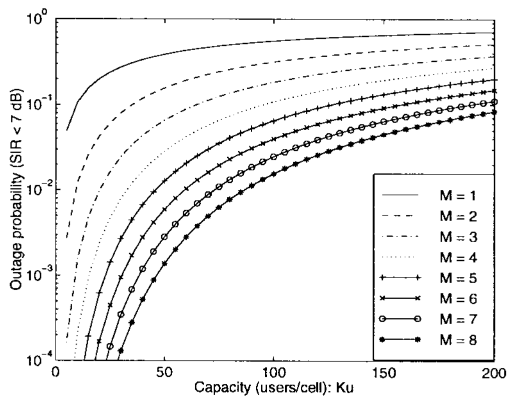


Fig. 2. Outage probability versus capacity. Theoretical results for PCE $= 1.5$ dB, $M = 1$ to 8, and $p = 3/8$.

Now, from (4) and the log-normality of $\lambda_1$ and $I$, it follows that $\mu$ is also log-normal. Indeed, $z = \ln \mu = \ln(\lambda_1/I) = a_1 - b$, is normal, and hence $\mu = e^z$ is log-normal. The density function of $\mu$ is determined from the mean and variance of $z$, $m_z = m_a - m_b$ and $\sigma_z^2 = \sigma_a^2 + \sigma_b^2$, respectively. Thus all the ingredients required to compute the density function of $\mu$ are available.

The outage probability is defined as the probability that $\gamma$, the output SIR, falls below a threshold $t$, $P_o = \Pr(\gamma < t)$. The outage probability conditioned on the input SIR $\mu$ is given by (3),

$$P_o(\mu) = F(\gamma = t) = \frac{(t/\mu)^M}{(1 + t/\mu)^M}. \tag{12}$$

The average outage probability accounting for Rayleigh fading, PCE and voice activity is given by

$$P_o = \int_0^\infty P_o(\mu) f(\mu)\, d\mu \tag{13}$$

or since $\mu = e^z$,

$$P_o = \int_{-\infty}^\infty P_o(z) f(z)\, dz = E_z[P_o(z)] \tag{14}$$

where

$$P_o(z) = \frac{(te^{-z})^M}{(1 + te^{-z})^M}. \tag{15}$$

An exact closed-form expression for the integral in (14) is not available, however for the normally distributed random variable $z$, a simple but accurate approximation exists for $E_z[P_o(z)]$ in terms of the mean $m_z$ and the standard deviation $\sigma_z$, and for an arbitrary function $P_o(z)$ of the random variable $z$ [7]:

$$P_o = E_z[P_o(z)] \cong \frac{2}{3} P_o(m_z) + \frac{1}{6} P_o(m_z + \sqrt{3}\sigma_z) + \tfrac{1}{6} P_o(m_z - \sqrt{3}\sigma_z). \tag{16}$$

The previous relation provides for the closed-form computation of the outage probability as a function of Rayleigh fading, PCE, and voice activity.

Computer simulations were used to verify (16), the main result of this letter. The simulation consisted of a CDMA system employing BPSK modulation and BPSK spreading, with a spreading ratio of 156, and with a voice activity factor of $p = 3/8$. The outage probability is plotted in Fig. 1 as a function of the capacity (number of users per cell) for $M = 4$ antenna elements and with the PCE as a parameter. The outage threshold was set at SIR = 7 dB, corresponding to a bit error rate of $10^{-3}$. Theoretical results are calculated from (16). Simulation curves are based on results compiled from 1 million Monte Carlo runs. For an outage of $10^{-2}$, the system capacity is approximately 38, 32, 22, and 10 users per cell for PCE = 0, 1.5, 2.5, and 4.0 dB, respectively. In a CDMA system with PCE 1.5 to 2.5 dB, the system capacity degrades 16 to 42% compared to the case of perfect power control. The effect of space diversity (theoretical result only) is shown in Fig. 2, for PCE $= 1.5$ dB, $M = 1$ to 8, and all other parameters similar to those used in Fig. 1. With $P_o = 10^{-2}$, eight-branch space diversity provides a capacity of 87 users/cell, while the capacity for two-branch space diversity is only nine users/cell. Without space diversity ($M = 1$), however, it is very difficult to meet $P_o = 10^{-2}$. Figs. 1 and 2 show that capacity of a CDMA system increases with space diversity and decreases with PCE. Therefore, space diversity can be used to compensate for PCE effects in wireless CDMA communications.

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