A Self-Correcting Loop for Joint Estimation-Calibration in Adaptive Radar

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Abstract. It is well known that pointing and random angle errors degrade the performance of adaptive radar quickly. A joint adaptive estimation-calibration (JEC) algorithm is proposed to provide on-line look direction corrections and array calibration. The new algorithm is presented in the context of a generalized sidelobe canceler (GSC). The proposed algorithm uses the array output to generate a control signal for steering vector corrections, which in turn yield higher signal-to-noise and interference ratio (SNIR) at the array output. Doppler technique is used to provide cleaner reference of target signal, which results in better correction of the steering vector. A simple case is investigated to explain how the scheme works and steady state condition analysis is provided. For an 8-element antenna array, the new algorithm is demonstrated by simulations, which show that the output SNIR is improved by the correction of pointing and random angle errors.

I. Introduction

Linearly constrained adaptive processing is a well known technique applicable to adaptive radar. The generalized sidelobe canceler (GSC) is an implementation of the linearly constrained processor [1]. The GSC was studied in [2], [3], [4] and its operation can be explained by referring to Fig. 1. The signals received by an M-element array are beamformed by the $M \times 1$ steering vector $\mathbf{s}$, which provides the necessary phase shifts to co-phase signals impinging from the look direction. The $(M-1) \times M$ matrix transformation $\mathbf{A}$ is designed to block signals represented by the vector $\mathbf{s}$. Thus, when the steering vector points in the direction of arrival (DOA) of the target signal, the $(M-1) \times 1$ vector $\mathbf{z}$ is target signal free. An adaptive algorithm is used to control the $(M-1) \times 1$ weight vector $\mathbf{w}$ to provide an estimate of the interference ya, which is subsequently subtracted from the beamformed signal yc. Higher SNIR thus can be obtained at the array output y. The performance of this system degrades when, due to pointing or random angle errors, $\mathbf{s}$ does not faithfully represent the target signal [5]. The residual target signal contributions in $\mathbf{z}$ are interpreted by the array as interference, and the array proceeds to cancel them. This form of target cancellation is well known and

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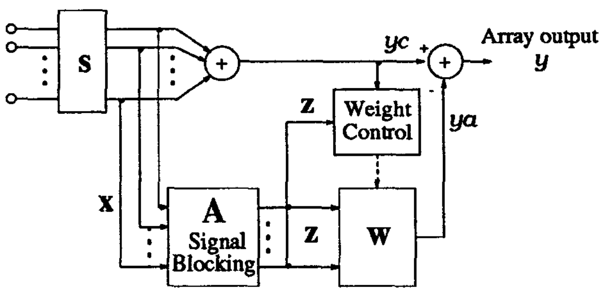

Fig. 1. Narrow-band generalized sidelobe canceler.

various remedies have been suggested. In particular, Bar-Ness and Haber suggested a self-correcting canceler to improve performance of a two-element array using a hybrid microwave component [6]. Their approach was tailored for a spread spectrum signal. It used the high signal-to-noise ratio (SNR) demodulated spread spectrum signal to provide a pure reference, which in turn was applied to control the phase of one of the inputs to the hybrid. In this work we generalize the self-correcting canceler in [6] to the multiple-element case, apply it to the GSC scheme, and evaluate performance with a radar application when Doppler processing is available. The new algorithm is illustrated by a three-element array operating in the presence of single jammer. However, the scheme given here could be extended to the case with multiple jammers.

The sequence of this paper is as follows: the system model is contained in section II, we propose the JEC algorithm with self-correcting loop of steering vector in section III. Our simulation results are presented and discussed in section IV. Section V gives the conclusions.

II. System Model

Consider an M-element antenna array with omnidirectional elements. The following assumptions are adopted:

- the target signal is in the far field and is narrowband, its direction of arrival (DOA) is used as a parameter in our analysis.
- a single far-field narrowband jammer is present.

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- following complex demodulation, the target signal, jammer, and noise are represented by complex zero-mean, wide-sense stationary random processes, which are mutually independent and have independent real and imaginary parts.

Assume the array elements are equally spaced, let $d/\lambda = b$, where d is the interelement spacing and λ is the wavelengh. The signal time delay at m-th element with respect to the first is:

$$\tau_m = \frac{m\,d}{c} \cdot \sin\theta_a, \quad m = 0, \cdots, M-1,$$

where c represents wave speed in the propagation medium, and θ_a is the DOA of signal. Here the signal could be target signal or jammer. Then the k-th sample of the target signal at the m-th element can be written:

$$r_s(m,k) = a_s\, e^{j(2mb\pi \sin\theta_s + \psi_k)}, \quad m = 0, \cdots, M-1, \quad (1)$$

and the k-th sample of the jammer at the m-th element:

$$r_i(m,k) = a_i\, e^{j(2mb\pi \sin\theta_i + \phi_k)}, \quad m = 0, \cdots, M-1. \quad (2)$$

The quantities a_s and a_i are the voltage amplitudes of the target signal and jammer respectively, ψ_k and ϕ_k are independent and uniformly distributed in $(-\pi, \pi)$, θ_s is the DOA of target signal, and θ_i is the DOA of jammer. The overall input at the m-th element is given by

$$r(m,k) = r_s(m,k) + r_i(m,k) + n(m,k), \quad (3)$$

where $n(m,k)$ is additive white Gaussian noise. The GSC's adaptive weight vector $\mathbf{w}$ can be updated by several methods, such as LMS, RLS or sample matrix inversion (SMI).

In (1) and (2), the signal (target signal or jammer) at the first element is taken as a reference, which is unweighted. The steering vector is then defined as

$$\mathbf{s} = \begin{bmatrix} 1 \\ \mathbf{s}' \end{bmatrix},$$

where $\mathbf{s}'$ is a $(M-1) \times 1$ steering vector.

In the context, matrices and vectors will be denoted by bold uppercase and bold lowercase letters, respectively.

III. Joint Estimation-Calibration

We propose to augment the GSC with a self-correcting loop for joint estimation-calibration of the steering vector. The JEC system comprising the GSC and the self-correcting loop is shown in Fig. 2. A Doppler filter bank is inserted in the signal path to provide a cleaner reference of the target signal. For an M-element antenna array, the signal blocking matrix $\mathbf{A}$ is restricted by $\mathbf{A}\,\mathbf{1} = \mathbf{0}$, where the $M \times 1$ vector $\mathbf{1} = [1, \cdots, 1]^T$, and the $(M-1) \times 1$ vector $\mathbf{0} = [0, \cdots, 0]^T$. The superscript "$T$" represents the transpose. If the steering beam does not accurately point to the DOA of target signal, then the output $\mathbf{z}$ of the signal blocking matrix contains a target signal residual. In this case, the output of the Doppler filter bank g, which is used as the reference of target signal, is correlated with the vector $\mathbf{z}$. The correlation between $\mathbf{z}$ and g is used by the correcting loop to control the steering vector $\mathbf{s}$, such that the cross-correlation between $\mathbf{z}$ and g is minimized. The JEC algorithm is formulated as follows:

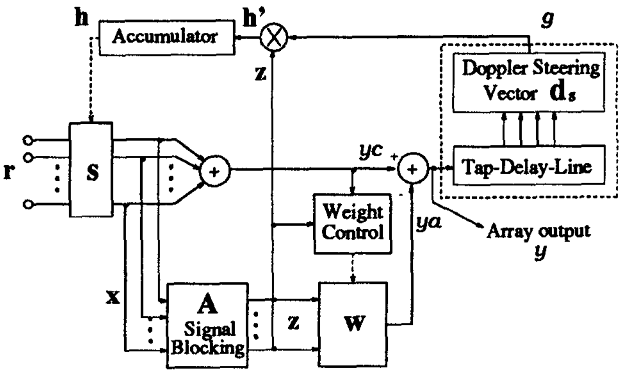

Fig. 2. Joint estimation-calibration system of generalized sidelobe canceler.

Algorithm:

1. Initialization: $\mathbf{s}(0) = [\,1, \quad \mathbf{s}'^T(0)\,]^T$, where $\mathbf{s}'(0) = [\, e^{j2\pi b \sin\theta_0}, \cdots, e^{j2\pi(M-1)b\sin\theta_0} \,]^T$, $b = d/\lambda \leq 0.5$, and θ_0, the target nominal angle, is chosen according to whatever prior target information is available.

For $k = 0, 1, \ldots$

2. $\mathbf{x}(k) = \mathbf{s}^*(k) \circ \mathbf{r}(k)$, where $\mathbf{r}(k) = [r(0,k), \cdots, r(M-1,k)]^T$, the superscript "$*$" denotes complex conjugate, and the symbol "$\circ$" denotes element-by-element multiplication between the components of two vectors.

3. $\mathbf{z}(k) = \mathbf{A}\mathbf{x}(k)$, where the signal blocking matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

4. $yc(k) = \mathbf{1}^T\mathbf{x}(k)$.

5. $ya(k) = \mathbf{w}^H(k)\mathbf{z}(k)$, where the superscript "H" denotes complex conjugate and transpose. As mentioned previously, a variety of algorithms are available for computing the weight vector $\mathbf{w}(k)$. In this work we choose to use sample matrix inversion (SMI), i.e., $\mathbf{w}(k) = \mathbf{R}_z^{-1}\mathbf{f}_{z,yc}$, where $\mathbf{R}_z =$

$\frac{1}{L_1}\sum_{l_1=0}^{L_1-1} \mathbf{z}(k-l_1)\,\mathbf{z}^H(k-l_1)$ and $\mathbf{f}_{z,yc} = \frac{1}{L_2}\sum_{l_2=0}^{L_2-1} \mathbf{z}(k-l_2)yc^*(k-l_2)$.

6. The array output: $y(k) = yc(k) - ya(k)$.

7. Assume that Q coherent samples are available, then at the output of Doppler processing, the reference signal $g(k) = \mathbf{d}_s^H \mathbf{y}(k)$, where $\mathbf{d}_s$ is the $Q \times 1$ steering vector corresponding to the target Doppler bin, and $\mathbf{y}(k) = [y(k), \ldots, y(k+Q-1)]^T$.

8. $\mathbf{h}'(k) = \mathbf{z}(k)\, g^*(k)$.

9. $\mathbf{h}(k) = <\mathbf{h}'(k)> = \frac{1}{L_3}\sum_{l_3=0}^{L_3-1} \mathbf{h}'(k-l_3)$, where the symbol "$< >$" denotes time average, and L_3 is chosen a suitable value to smooth variations in $\mathbf{h}(k)$.

10. $\mathbf{s}'(k+1) = \mathbf{s}'(k) - \frac{\mu}{\|\mathbf{h}(k)\|} \cdot \mathbf{h}(k) \circ \mathbf{s}'(k)$, where μ is the step size, and

$$\mathbf{s}(k+1) = \begin{bmatrix} 1 \\ \mathbf{s}'(k+1) \end{bmatrix}.$$

Let $\hat{\mu}(k) = \frac{\mu}{\|\mathbf{h}(k)\|}$, the JEC actually uses a time-varying step size. High target signal power at array input will result in a large vector $\mathbf{h}(k)$, the time-varying step size in the JEC algorithm can overcome the difficulty of gradient noise amplification, which may be caused by large vector $\mathbf{h}(k)$. Thus the correction applied to the steering vector $\mathbf{s}'(k+1)$ is normalized with respect to the Euclidean norm of vector $\mathbf{h}(k)$. This approach is also used in normalized LMS algorithm [7].

A simple case is used to illustrate the scheme. Consider a 3-element antenna array, assume that the interelement spacing is equal to half wavelength, such that $b = d/\lambda = 0.5$. At time k, the steering vector is given by

$$\mathbf{s}(k) = \begin{bmatrix} 1, & e^{j\pi \sin\theta_k}, & e^{j2\pi\sin\theta_k} \end{bmatrix}^T = \begin{bmatrix} 1, & \mathbf{s}'^T(k) \end{bmatrix}^T,$$

where θ_k is the beam steering direction. In the interest of brevity, we suppress the time index notation.

The total input to the array can be expressed as:

$$\begin{aligned} \mathbf{r} &= a_s\, e^{j\psi} \cdot \begin{bmatrix} 1 \\ e^{j\pi\sin\theta_s} \\ e^{j2\pi\sin\theta_s} \end{bmatrix} \\ &\quad + a_i\, e^{j\phi} \cdot \begin{bmatrix} 1 \\ e^{j\pi\sin\theta_i} \\ e^{j2\pi\sin\theta_i} \end{bmatrix} + \mathbf{n}_0, \end{aligned} \tag{4}$$

where $\mathbf{n}_0$ is the noise vector. The steering vector shifts the phase angle of $\mathbf{r}$ (except the first element) to generate the vector $\mathbf{x}$, which is

$$\begin{aligned} \mathbf{x} &= \mathbf{s}^* \circ \mathbf{r} \\ &= a_s\, e^{j\psi} \cdot \begin{bmatrix} 1 \\ e^{j\pi(\sin\theta_s - \sin\theta)} \\ e^{j2\pi(\sin\theta_s-\sin\theta)} \end{bmatrix} \\ &\quad + a_i\, e^{j\phi} \cdot \begin{bmatrix} 1 \\ e^{j\pi(\sin\theta_i-\sin\theta)} \\ e^{j2\pi(\sin\theta_i-\sin\theta)} \end{bmatrix} + \mathbf{n}_1, \end{aligned} \tag{5}$$

where $\mathbf{n}_1$ is the noise vector. The signal blocking matrix is given by

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

Then the output of signal blocking matrix is:

$$\begin{aligned} \mathbf{z} &= \mathbf{A}\,\mathbf{x} \\ &= a_s\, e^{j\psi} \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_s - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_s-\sin\theta)} \end{bmatrix} \\ &+ a_i\, e^{j\phi} \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_i - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_i-\sin\theta)} \end{bmatrix} + \mathbf{n}_2. \end{aligned} \tag{6}$$

The primary input yc is

$$\begin{aligned} yc &= \mathbf{1}^T \mathbf{x} \\ &= a_s\, e^{j\psi}[1 + e^{j\pi(\sin\theta_s-\sin\theta)} + e^{j2\pi(\sin\theta_s-\sin\theta)}] \\ &+ a_i\, e^{j\phi}[1 + e^{j\pi(\sin\theta_i-\sin\theta)} + e^{j2\pi(\sin\theta_i-\sin\theta)}] \\ &+ \mathbf{1}^T\mathbf{n}_1. \end{aligned} \tag{7}$$

The sidelobe cancelling signal ya is given by

$$\begin{aligned} ya &= \mathbf{w}^H\,\mathbf{z} \\ &= a_s\, e^{j\psi} \cdot \mathbf{w}^H \begin{bmatrix} 1 - e^{j\pi(\sin\theta_s - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_s-\sin\theta)} \end{bmatrix} \\ &\quad + a_i\, e^{j\phi} \cdot \mathbf{w}^H \begin{bmatrix} 1 - e^{j\pi(\sin\theta_i - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_i-\sin\theta)} \end{bmatrix} \\ &\quad + \mathbf{w}^H\mathbf{n}_2, \end{aligned} \tag{8}$$

where $\mathbf{w}$ is the SMI solution of the weight vector (see step 5 in the algorithm), we can express it as $\mathbf{w} = [w_1,\ w_2]^T$. Combine (7) and (8), we can get the expression of the array output as follows:

$$\begin{aligned} y &= yc - ya \\ &= a_s y_s e^{j\psi} + a_i y_i e^{j\phi} + v_y, \end{aligned} \tag{9}$$

where the quantities y_s and y_i are given by

$$y_s = 1 + \sum_{m=1}^{2} [(1 + w_m^*)\, e^{jm\pi(\sin\theta_s - \sin\theta)} - w_m^*],$$

$$y_i = 1 + \sum_{m=1}^{2} [(1 + w_m^*)\, e^{jm\pi(\sin\theta_i - \sin\theta)} - w_m^*].$$

The noise $v_y = \mathbf{1}^T\mathbf{n}_1 - \mathbf{w}^H\mathbf{n}_2$. Assume the Doppler processing is performed, Q coherent samples are available, and the phase difference in consecutive samples is γ_s and γ_i for target signal and jammer respectively. We use a vector to represent the array output for Q coherent samples,

$$\mathbf{y} = a_s y_s e^{j\psi}\mathbf{d}_s + a_i y_i e^{j\phi}\mathbf{d}_i + \mathbf{n}_3, \tag{10}$$

where $\mathbf{d}_s = [1, e^{j\gamma_s}, \cdots, e^{j(Q-1)\gamma_s}]^T$, $\mathbf{d}_i = [1, e^{j\gamma_i}, \cdots, e^{j(Q-1)\gamma_i}]^T$ and $\mathbf{n}_3$ is the noise. Then the reference signal g can be written as

$$\begin{aligned} g &= \mathbf{d}_s^H \mathbf{y} \\ &= Q a_s y_s e^{j\psi} + a_i y_i \mathbf{d}_s^H \mathbf{d}_i e^{j\phi} + v_g \\ &= g_s e^{j\psi} + g_i e^{j\phi} + v_g, \end{aligned} \quad (11)$$

where v_g is the noise, $g_s = Q a_s y_s$ and $g_i = a_i y_i \mathbf{d}_s^H \mathbf{d}_i$. Multiply $\mathbf{z}$ by the complex conjugate of g, we get the expression of $\mathbf{h}'$,

$$\mathbf{h}' = \mathbf{z}\, g^*. \quad (12)$$

By investigating (6) and (11), and using the assumption that the target signal, jammer and noise are mutually independent, we take the time average of $\mathbf{h}'$ in order to extract the control signal $\mathbf{h}$ at time k, which is given by

$$\begin{aligned} \mathbf{h}(k) &= <\mathbf{h}'> \\ &= g_s\, a_s \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_s - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_s - \sin\theta)} \end{bmatrix} \\ &\quad + g_i\, a_i \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_i - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_i - \sin\theta)} \end{bmatrix} \\ &\quad + <\mathbf{n}_2 v_g^*>. \end{aligned} \quad (13)$$

Since the noise is zero-mean Gaussian distributed and mutually independent between the antenna elements, the third quantity in (13) is negligible comparing to the first and second quantities in (13), such that:

$$\begin{aligned} \mathbf{h}(k) &\cong g_s\, a_s \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_s - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_s - \sin\theta)} \end{bmatrix} \\ &\quad + g_i\, a_i \cdot \begin{bmatrix} 1 - e^{j\pi(\sin\theta_i - \sin\theta)} \\ 1 - e^{j2\pi(\sin\theta_i - \sin\theta)} \end{bmatrix} \\ &= g_s\, a_s\, \mathbf{s}'^* \circ (\mathbf{s}' - \mathbf{s}'_s) + g_i\, a_i\, \mathbf{s}'^* \circ (\mathbf{s}' - \mathbf{s}'_i) \\ &= \mathbf{s}'^* \circ [g_s\, a_s(\mathbf{s}' - \mathbf{s}'_s) + g_i\, a_i(\mathbf{s}' - \mathbf{s}'_i)], \end{aligned} \quad (14)$$

where $\mathbf{s}'_s = [\ e^{j\pi\sin\theta_s},\ \ e^{j2\pi\sin\theta_s}\ ]^T$ and $\mathbf{s}'_i = [\ e^{j\pi\sin\theta_i},\ e^{j2\pi\sin\theta_i}\ ]^T$.

Fig. 2 shows that the steady state condition is given by $\mathbf{h}(k) = \mathbf{0}$, as $k \to \infty$. From (14) we conclude that

$$\mathbf{s}'(k) - \mathbf{s}'_s = \frac{g_i(k)\, a_i}{g_s(k)\, a_s + g_i(k)\, a_i} \cdot (\mathbf{s}'_i - \mathbf{s}'_s), \quad \text{as } k \to \infty. \quad (15)$$

In a perfect pointing system, the steering vector $\mathbf{s}'$ is equal to $\mathbf{s}'_s$. Therefore the expression at the left-hand side of (15) is the steady state error of the steering vector. From (15) we notice that if $|g_s(k)a_s| \gg |g_i(k)a_i|$, the error $\mathbf{s}'(k) - \mathbf{s}'_s$ is very small. Higher SNIR at array input and cleaner reference signal will then result in smaller steady state error. It is also noticed that when $g_i(k) \to 0$, the error $\mathbf{s}'(k) - \mathbf{s}'_s \to 0$, which is the desired result when a pure reference signal is available.

IV. Numerical results

Numerical results illustrate the performance of an 8-element antenna array using the JEC algorithm. All antenna elements are omnidirectional and equally spaced, the interelement spacing is equal to half wavelength. The direction of arrival (DOA) of target signal is $\theta_s = 0^o$, the DOA of jammer is $\theta_i = 10^o$. At array input, the interference-to-noise ratio INR = 10 dB, SNR values are shown in the figures. Curves shown are averages of 100 Monte Carlo runs. Doppler processing consists of four coherent samples, and the Doppler phase difference between the target signal and jammer is $\gamma_i - \gamma_s = 70^o$.

Two types of errors are investigated: pointing errors and random angle errors. The initial steering vector can be written as $\mathbf{s}'(0) = [e^{j(\pi\sin\theta_0+\beta_1)}, \ldots, e^{j(7\pi\sin\theta_0+\beta_7)}]^T$, where θ_0 is the initial beam direction, and β_m ($m = 1, \cdots, 7$) are the random angle errors, which are zero-mean Gaussian random variable.

Fig. 3 shows the performance improvement with JEC algorithm when the initial beam has a 0.5^o pointing error ($\theta_0 = 0.5^o$), but no random angle error ($\beta_m = 0$). The step size is $\mu = 0.2$. The simulation results show that the higher the input SNR, the larger is the SNIR loss at array output due to same pointing error. For input SNR = 7 and 4 dB, 4 dB and 1.2 dB SNIR improvement are achieved respectively by JEC algorithm, though there is still 0.2 dB loss comparing to the perfect pointing system.

Fig. 4 gives the simulation result for the case of random angle errors in the initial steering vector. We assume that there is no pointing error in this case ($\theta_0 = 0^o$), and that the random angle errors β_m are Gaussian random variable with zero-mean and standard deviation $\sigma_\beta = 3^o$. The step size is $\mu = 0.2$. Fig. 4 shows that the initial random angle errors can also be corrected by the JEC algorithm, and that the SNIR at array output is improved.

For input INR = 20 dB, Fig. 5 and 6 give the results for same pointing and random angle errors as in Fig. 3 and 4, respectively. Both figures show that the JEC algorithm is not affected by high INR environment.

V. Conclusions

In this paper, we proposed the JEC algorithm to correct the pointing and random angle errors in the steering vector of a GSC. A simple case with three antenna elements and single jammer was investigated to illustrate this algorithm. In the steady state, the steering vector pointing error can be substantially reduced by this algorithm. Numerical results illustrated the SNIR improvement at the array output.

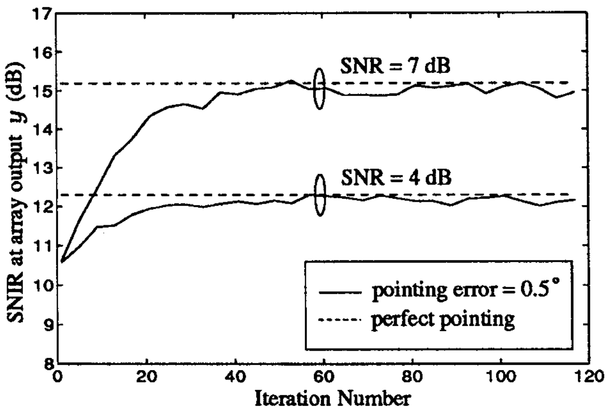

Fig. 3. Output SNIR vs. iteration number with JEC algorithm, input INR = 10 dB.

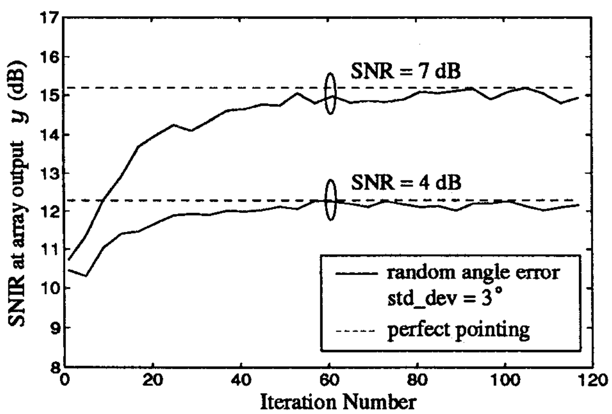

Fig. 4. Output SNIR vs. iteration number with JEC algorithm, input INR = 10 dB.

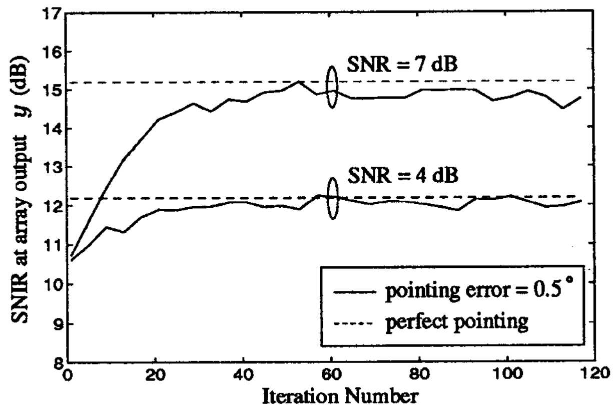

Fig. 5. Output SNIR vs. iteration number with JEC algorithm, input INR = 20 dB.

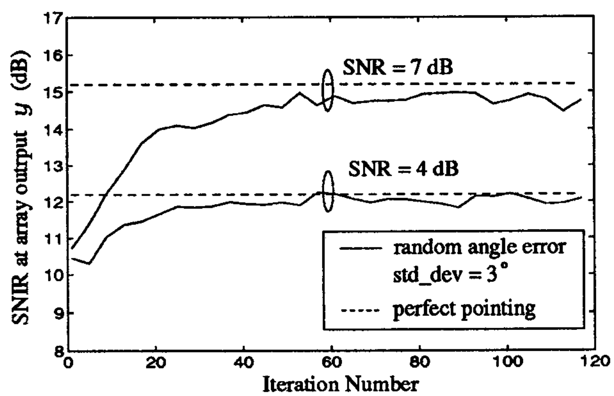

Fig. 6. Output SNIR vs. iteration number with JEC algorithm, input INR = 20 dB.

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