

ADAPTIVE ARRAYS FOR INCREASED PERFORMANCE IN MOBILE COMMUNICATIONS

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ABSTRACT

Adaptive antenna arrays are applied at the base station to improve performance of a mobile communication system operating in a flat slowly fading Rayleigh channel. Analytical expressions are derived for the average probability of error of the SMI and an eigenanalysis-based method for the case of a single co-channel interference. It is shown that the eigenanalysis-method is superior to the SMI method when the array covariance matrix is estimated from small data sets. The analysis is supported by numerical results obtained from simulations.

1. INTRODUCTION

Antenna arrays provide diversity paths to combat multipath fading and are capable of suppressing spatial interference sources. Under consideration is a narrowband multiple access system such as TDMA. The channel is assumed characterized by flat fading. Within a cell, signals are orthogonal, but cell splitting, unavoidably, increases the chances of co-channel interference from one of the other cells using the same frequency channels. Antenna arrays are suggested could provide a cost efficient means to increase the capacity of wireless mobile communication networks.

The adaptive array theory is well understood and documented [1]. Applications, though, were slow to exploit this body of knowledge, due to technological limitations and high demands on the real-time computing power usually associated with array processing. Nevertheless, conditions may be right for the use of adaptive antennas in mobile communications. Technological advancement lowered the cost of hardware and computing power to the point where adaptive antennas are viable alternatives to other network expansion techniques, such as microcells. The cost of the added complexity may well be offset by increased revenues from expanding traffic and customer base. Furthermore, significant improvement can be achieved even with relatively few

antennas, without undue increase in complexity.

In contrast to traditional array processing where antennas are typically placed at half-wavelength intervals, space diversity is achieved by spacing the antennas at larger intervals to provide for independent signal paths. Optimum signal-to-noise (SNR) ratio is then provided by *maximal ratio combining* whereby each antenna output is weighted proportionally to the signal strength in that channel [2]. Maximal ratio combining optimizes output SNR, under the white Gaussian noise assumption, but ignores directional interferences. *Optimum combining* has been shown to provide both signal enhancement and interference cancellation [3]. Among the important results in this field is the that under the assumption of slow independent Rayleigh fading, an N -element array can null out L spatial interferer and provide $N - L$ diversity paths [4].

In mobile communications the signal environment varies with the motion of the mobiles. Thus fast adaptive techniques capable of tracking the changing signals are of particular interest [5]. In reference [6] we proposed an eigenanalysis based filter, referred to as *eigencanceler*, as an alternative to the Sample Matrix Inversion method (SMI) for small data sets. In this paper we extend our previous results and develop analytical expressions for the average probability of error of the SMI and the eigencanceler methods for the case of single interference and a flat slowly fading Rayleigh channel.

2. PROBLEM FORMULATION

This section defines the data model and provides concise reviews of optimum combining, the SMI and the eigenanalysis-based method.

Data Model

Consider a mobile communication system in which mobiles communicate with the base station. The base sta-

tion utilizes an N -element antenna array. The signals are assumed narrowband with respect to the carrier frequency, hence they can be represented by samples of their complex envelopes. It is further assumed that the channel is characterized by flat Rayleigh fading independent between the antennas. The fading is assumed fixed over the processing interval and independent between processing intervals. Multipath delay spread larger than a symbol duration may cause intersymbol interference and can be equalized using a tap-delay line filter at each antenna. However, in this paper we emphasize spatial processing, hence we will assume that the delay spread is small and that a single weight at each antenna is sufficient. Let $\mathbf{x}(k)$ be the $N \times 1$ data vector of antenna outputs at time $t = kT$, where T is the symbol interval. The data vector consists of contributions of the signal of interest (SOI) $\mathbf{x}_s(k)$, co-channel interferences due to the other users' signals $\mathbf{x}_j(k)$, $j = 1, \dots, L$, and samples of white complex gaussian noise $\mathbf{v}(k)$, with zero mean and variance equal to σ^2 :

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \sum_{j=1}^L \mathbf{x}_j(k) + \mathbf{v}(k) \quad (1)$$

Consistent with the assumed fading model, each source (SOI or interference) may be represented by a propagation vector \mathbf{u} , whose components are realizations of independent complex gaussian random variables with zero mean, and unity variance. This model implies that the array elements are sufficiently separated to allow independent reception between antennas. The vector $\mathbf{x}(k)$, can be written:

$$\mathbf{x}(k) = m_s(k)A_s\mathbf{u}_s + \sum_{j=1}^L m_j(k)A_j\mathbf{u}_j + \mathbf{v}(k) \quad (2)$$

where $m_s(k)$, $m_j(k)$ are assumed equally probable binary symbols, $m_s(k), m_j(k) \in \{-1, 1\}$, A_s, A_j are the amplitudes, and $\mathbf{u}_s, \mathbf{u}_j$ are the propagation vectors of the SOI and interferences, respectively. With slow fading, it is assumed that the interference and noise correlation matrix, conditioned on the channel, is stationary over the processing time,

$$\mathbf{R}_n = \text{E}[\mathbf{x}_n(k)\mathbf{x}_n^H(k) | \mathbf{u}_j] \quad (3)$$

where the superscript $(\cdot)^H$ denotes hermitian and $\mathbf{x}_n(k) = \mathbf{x}(k) - \mathbf{x}_s(k)$ is the interference-plus-noise (I+N) vector. The weight vector that maximizes the array output SNIR is given by the well known relation:

$$\mathbf{w}_o = \mathbf{R}_n^{-1}\mathbf{r}_{sx} \quad (4)$$

where \mathbf{r}_{sx} is the conditioned cross-correlation vector, defined as $\mathbf{r}_{sx} = \text{E}[m_s(k)\mathbf{x}(k) | \mathbf{u}_s]$. Note that

$$\mathbf{r}_{sx} = \text{E}[m_s(k)\mathbf{x}(k) | \mathbf{u}_s] = A_s\mathbf{u}_s \quad (5)$$

The optimal linear processor in eq. (4) is referred to as the optimum combiner.

The statistics of the received signals are typically not known, and furthermore, will vary over time, hence adaptive methods are required to track the signal environment. Methods for estimating the weight vector when the statistics of the data are not known, are commonly classified as either block processing or adaptive processing. With block processing the correlation matrix and cross-correlation vector are estimated from a block of received data. With adaptive processing the new data is used directly to update the weight vector. The weight vector calculation based on the substitution of \mathbf{R}_n in eq. (4) with its maximum likelihood estimate, $\hat{\mathbf{R}}_n = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_n(k)\mathbf{x}_n^H(k)$, is known as the Sample Matrix Inversion (SMI) method [7]:

$$\mathbf{w} = \hat{\mathbf{R}}_n^{-1}\hat{\mathbf{r}}_{sx} \quad (6)$$

where the estimated covariance matrix and the cross-correlation vector, $\hat{\mathbf{r}}_{sx} = \frac{1}{K} \sum_{k=1}^K r(k)\mathbf{x}(k)$, are obtained from averages over K data vectors. The reference $r(k)$ is required to be correlated with the SOI but uncorrelated with the interference. Such a signal can be obtained by a reference loop that subtracts from the received signal, the received signal processed by the sequence of operations despreading-lowpass filtering-spreading. The covariance matrix estimation results in a performance loss with respect to the case of known statistics. It has been shown that this loss is within 3 dB when the block size $K \geq 2N$ [7].

A different approach is suggested from the eigenanalysis of the interference/noise data. Consider the (I+N) correlation matrix \mathbf{R}_n . When the number of interferences L is less than N , the number of antennas in the array, the propagation vectors \mathbf{u}_j , $j = 1, \dots, L$, are linearly independent (with probability one) and span a signal subspace, referred to as the *interference subspace*. Let $\lambda_1 \geq \dots \geq \lambda_L \geq \lambda_{L+1} = \dots = \lambda_N = \sigma^2$ be the eigenvalues of \mathbf{R}_n . The L largest λ_i are referred to as *principal eigenvalues* and their associated eigenvectors are referred to as *principal eigenvectors*. The interference subspace is spanned by the principal eigenvectors. The *noise subspace* is defined as the signal subspace associated with the equal $(N - L + 1)$ eigenvalues. Let \mathbf{Q}_c and \mathbf{Q}_v be matrices whose columns are the dominant and noise eigenvectors, respectively. Then in [6] the eigencanceler is defined as the optimal weight vector constrained to the noise subspace,

$$\begin{aligned} \mathbf{w}_e &= \mathbf{Q}_v\mathbf{Q}_v^H\mathbf{r}_{sx} \\ &= (\mathbf{I} - \mathbf{Q}_c\mathbf{Q}_c^H)\mathbf{r}_{sx} \end{aligned} \quad (7)$$

This method is referred to in other publications as low rank adaptive processing [8]. The eigencanceler's weight

vector is obviously sub-optimal to the weight vector in eq. (4).

When the correlation matrix is estimated from the data, the eigencanceler's weight vector is obtained by substituting the interference subspace with its estimate:

$$\mathbf{w}_e = (\mathbf{I} - \hat{\mathbf{Q}}_c \hat{\mathbf{Q}}_c^H) \hat{\mathbf{r}}_{sx} \quad (8)$$

3. PERFORMANCE ANALYSIS

In this section analytical expressions are developed for the probability of error of the SMI and the eigenanalysis-based methods for the case of single interference.

The output SNIR of the optimum combiner, given by

$$\gamma = |A_s|^2 \mathbf{u}_s^H \mathbf{R}_n^{-1} \mathbf{u}_s \quad (9)$$

is a random variable conditioned on the propagation vectors \mathbf{u}_s and \mathbf{u}_j , $j = 1, \dots, L$. For a given SNIR, BPSK modulation, and making the Gaussian assumption, the conditioned probability of error is given by $P(e | \gamma) = Q(2\gamma)$, where $Q(\cdot)$ is the Gaussian tail function. A performance measure independent of the specific fading is the probability of error averaged over all fading conditions.

$$P_e = \int_0^\infty P(e | \gamma) f_\gamma(\gamma) d\gamma \quad (10)$$

where $f(\gamma)$ is the probability density function. Analytical computation of P_e in the general case is unwieldy, but for one interference the average probability of error of the was computed in [3]. The performance of the optimum combiner, where the covariance matrix is assumed known, provides an upper bound with respect to SMI or eigenanalysis processing. To simplify the analysis, we initially assume that \mathbf{R}_n is estimated, but \mathbf{r}_{sx} is known. The output SNIR for the SMI method is given by:

$$\hat{\gamma} = \frac{(\mathbf{u}_s^H \hat{\mathbf{R}}_n^{-1} \mathbf{u}_s)^2}{\mathbf{u}_s^H \hat{\mathbf{R}}_n^{-1} \mathbf{R}_n \hat{\mathbf{R}}_n^{-1} \mathbf{u}_s} \quad (11)$$

The SNIR normalized with respect to the SNIR of the optimum combiner is given by:

$$\rho = \frac{(\mathbf{u}_s^H \hat{\mathbf{R}}_n^{-1} \mathbf{u}_s)^2}{\mathbf{u}_s^H \hat{\mathbf{R}}_n^{-1} \mathbf{R}_n \hat{\mathbf{R}}_n^{-1} \mathbf{u}_s, \mathbf{u}_s^H \mathbf{R}_n^{-1} \mathbf{u}_s} \quad (12)$$

The random variable ρ is conditioned on both the channel and the covariance matrix estimate, and is bounded $0 \leq \rho \leq 1$. It is known that the density of ρ conditioned on the channel, is actually independent on the covariance matrix and is given by the expression [7]:

$$f_\rho(\rho | \gamma) = \frac{K!}{(N-2)!(K+1-N)!} (1-\rho)^{N-2} \rho^{K+1-N} \quad (13)$$

Since $\hat{\gamma} = \rho\gamma$ and $f_\gamma(\gamma)$ and $f_\rho(\rho)$ are known, the density of $\hat{\gamma}$ can be found from the expression:

$$f_{\hat{\gamma}}(\hat{\gamma}) = \int_0^\infty \frac{1}{\gamma} f_\rho\left(\frac{\hat{\gamma}}{\gamma}\right) f_\gamma(\gamma) d\gamma \quad (14)$$

Combining eq. (14) and the conditioned probability of error $P_e(e | \gamma)$, the average probability of error for the SMI method can be written:

$$P_e = \int_0^\infty \frac{1}{\gamma} f_\gamma(\gamma) \left[\int_0^\gamma P_e(e | \gamma) f_\rho\left(\frac{\hat{\gamma}}{\gamma}\right) d\hat{\gamma} \right] d\gamma \quad (15)$$

The normalized SNIR for the eigenanalysis-based method is given by the expression:

$$\rho = \frac{(\mathbf{u}_s^H \hat{\mathbf{Q}}_v \hat{\mathbf{Q}}_v^H \mathbf{u}_s)^2}{\mathbf{u}_s^H \hat{\mathbf{Q}}_v \hat{\mathbf{Q}}_v^H \mathbf{R}_n \hat{\mathbf{Q}}_v \hat{\mathbf{Q}}_v^H \mathbf{u}_s, \mathbf{u}_s^H \mathbf{R}_n^{-1} \mathbf{u}_s} \quad (16)$$

The density of ρ for the eigenanalysis-based method has been determined in [9], and is given by:

$$f_\rho(\rho | \gamma) = \frac{K^L}{2^L \Gamma(L)} (1-\rho)^{L-1} \rho^{-(L+1)} e^{-K(1-\rho)/2\rho} \quad (17)$$

where $\Gamma(\cdot)$ is the standard gamma function. Then, the average probability of error for the eigenanalysis-based method can be found from expression (15) with $f_\rho(\hat{\gamma} | \gamma) = f_\rho(\rho | \gamma)$ given by eq. (17).

For the general case, when both the covariance matrix and the cross-correlation vector are unknown and are estimated from the data, it is very difficult to find a closed form expression for the density of the normalized SNR ρ . However, when the input SNR $\gg 1$, the density of the normalized SNR for the SMI method is represented by the expression for known cross-correlation vector, eq. (13), substituting $N+1$ for N , [1]. Hence, the performance of an N element array with unknown \mathbf{r}_{sx} is approximately same as the performance of an $N-1$ array with known \mathbf{r}_{sx} .

4. NUMERICAL RESULTS

The input SNR at each element is defined as:

$$\text{SNR} = \frac{|A_s|^2 \text{E} [|u_{sn}|^2]}{\sigma^2} \quad n = 1, \dots, N \quad (18)$$

where the expectation is over the components of the random propagation vector \mathbf{u}_s . Similarly, the INR at each element is defined as:

$$\text{INR} = \frac{|A_j|^2 \text{E} [|u_{1n}|^2]}{\sigma^2} \quad n = 1, \dots, N \quad (19)$$

In the simulations used to generate the figures in this section, INR = 2 dB. Figures 1 and 2 compare the theoretical performance of the optimum combining, SMI and eigencanceler methods for array sizes $N = 5$ and $N = 9$. The case of sample size $K = 20$ is shown in Figure 1, and the case of $K = 50$ is shown in Figure 2. The curves were generated using eqs. (10) and (15) with the appropriate density functions. As expected, optimum combining, i.e. known array covariance matrix, provides an upper bound on the performance. The eigenanalysis-based method's performance is very close to the optimal in all cases. The advantage of the eigenanalysis-based method is particularly evident for the lower number of samples and the larger array. Conversely, for a fixed probability of error, the eigenanalysis-based method provides a SNR advantage. For example, when the bit error is 10^{-3} , $N = 9$, and $K = 20$, the eigenanalysis-based method's advantage over the SMI method is approximately 3 dB, and its loss with respect to the optimum is indistinguishable. When the sample size increases to $K = 50$ the differences between the methods are less evident, however, the eigenanalysis-based method still provides BER curves closer to optimum bound than the SMI method. The match between theory and simulations is considered in Figure 3. The curves shown represent the theoretical probability of error, and the probability of error based on simulations, for $N = 9$, $K = 20$, and INR = 2 dB. The simulation probability of error was calculated from $P_e = Q(2\gamma)$, where

$$\gamma = \frac{|A_s|^2 |\mathbf{w}^H \mathbf{u}_s|^2}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}}$$

and the weight vector for each method was provided by simulations.

5. CONCLUSIONS

An eigenanalysis-based method was suggested for adapting the weights of an adaptive array at the base station of a mobile communication system. Estimation of the array covariance matrix entails a loss in the array output SNR with respect to the optimal case of known covariance matrix. The density function of this loss has been derived elsewhere for both the SMI and the eigenanalysis-based methods, and is used to develop an analytical expression for the average probability of error for each method for the case of single interference. It is shown that the eigenanalysis-based method has performance close to the optimal and superior to the performance of the SMI method. Simulations are used to corroborate the analytical results.

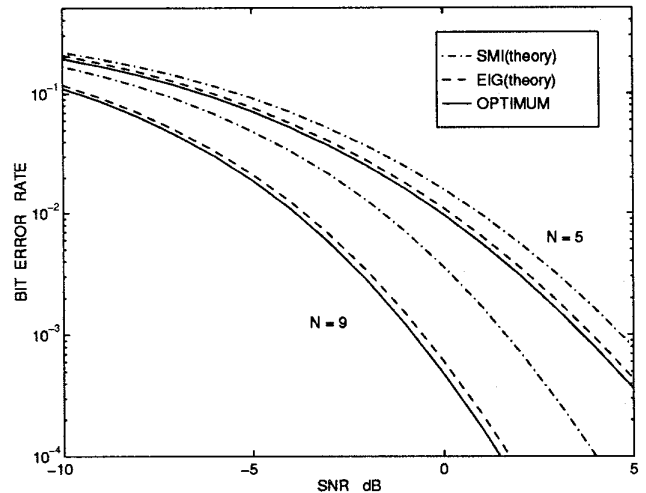


Figure 1: The average BER versus the average received SNR for adaptive array with one interference with $K=20$, INR=2dB. (analytical results)

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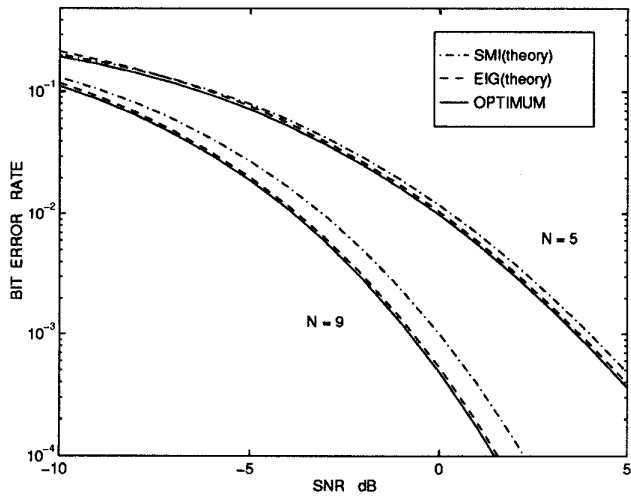


Figure 2: The average BER versus the average received SNR for adaptive array with one interference with $K=50$, $INR=2\text{dB}$. (analytical results)

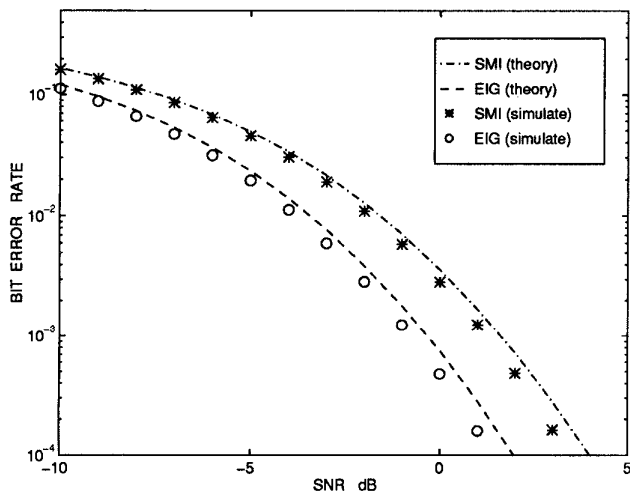


Figure 3: The average BER versus the average received SNR for adaptive array with one interference. (simulation results)