An Analytical Model for Measuring QoS in Ad-Hoc Wireless Networks

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***Abstract*—This paper develops an analytical model for evaluating the Quality of Service (QoS) in wireless ad-hoc networks. In doing so it extends the trunking theory concepts to encompass the effect of co-channel interference. Chosen as the QoS figure of merit, the *transmission blocking probability,* is derived as a function of the number of nodes, the network density and parameters of a Markov chain model for the multiple user channel access protocol. This expression is validated by computer simulations.**

I. INTRODUCTION

In the last few years mobile ad-hoc networks for personal communication services have emerged as an alternative architecture to infrastructure based wireless networks (e.g. cellular networks). The distributed nature of ad-hoc networks makes them more robust to systemic failures, easier to deploy, and more flexible to reconfigure than infrastructure-based networks. The requirement to provide real time multimedia traffic along with voice and data traffic, and the varying priorities of transmission over wireless ad-hoc networks, have led QoS topics to become an active area of current research. Most of the research efforts that have been reported in the literature, emphasize on the development of analysis tools or protocols and mechanisms that facilitate the provisioning of efficient QoS techniques [1],[5]. Substantial effect on the performance of these networks has been demonstrated [9],[10],[11]. In most of the published literature, performance evaluation is carried out through simulations. Few publications provide QoS analysis of particular network functions such as handoff algorithms [6], effects of Rayleigh fading [4], and performance characteristics such as packet completion [3].

With multiple user channel access techniques such as carrier sense multiple access/collision avoidance (CSMA/CA), when omnidirectional antennas are used, neighbors of both the transmitter and the receiver nodes must be blocked from transmitting on the same channel as the transmitter/receiver (TX/RX). Unlike landlines or wireless cellular systems with frequency reuse patterns, in wireless ad-hoc networks the maximum number of simultaneous transmissions (L) supported by the network is also a function of the network density and of the radio range of each node in the network. If neighbors were not blocked, the number of possible transmissions would be equal to the number of nodes in the network (M) divided by two (M / 2). However, since neighbors must be blocked, this quantity is less than M / 2 ($L < M / 2$).

Our goal in this paper is to develop an analytical model that can be used as a tool in the provisioning and evaluation of QoS in ad-hoc networks. In such networks, many users share a limited number of radio frequency (RF) channels. In effect, this is a generalization of the *trunking* concept, whereby a small pool of channels service a large population by providing access to users on demand. When applying trunking to ad-hoc networks, we are not concerned only with channels in use, but also with blocking of channels due to interference from nearby users. One of the main contributions of this paper is to extend trunking theory concepts to encompass the effect of co-channel interference. As in classical trunking theory [2], the probability of obtaining one of these channels with minimum delay is a sensible choice for a figure of merit for quantifying QoS. We define the *transmission blocking probability* (TBP) as the probability that a node is blocked from transmitting (either because itself or its destination are blocked) given that it has a packet to send. In the sequel, we derive analytical expressions for the TBP for ad-hoc networks.

The rest of this paper is organized as follows. First, in Section II, we present the system model that is used in our analysis, and derive the corresponding transition state diagram. In Section III we derive the expression for the TBP, while Section IV describes the results from two computer simulation models that were developed to verify some of the assumptions made and the equations derived. Finally, Section V concludes the paper and highlights some of our current and future research work.

II. SYSTEM MODEL

The system model that is used in this paper derives from the multiple user channel access protocol used in ad-hoc networks. As mentioned previously, a transmission between two nodes results in the blocking of the neighboring nodes, where a *neighbor* is defined as a node found within the transmission range of another node. Our network does not

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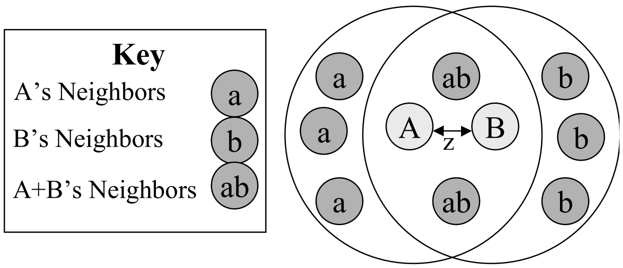

Fig. 1. Transmissions with Omni-Directional Antennas

employ power control, and, without loss of generality, each node's transmission range is normalized to 1 km. To illustrate the concept of neighbor nodes, let us assume that node A (see Fig. 1) would like to send a packet to node B. In order to ensure interference free transmission, all of the RF neighbors of both nodes A and B must be blocked. The total number of blocked nodes is expressed in terms of number of RF neighbors of node A (N_A), number of RF neighbors of node B (N_B), and the number of shared RF neighbors (N_{AB}):

$$\beta_{AB} = N_A + N_B - N_{AB} - 2. \qquad (1)$$

In the example in Fig. 1, $\beta_{AB} = 8$. Note that more and more nodes get blocked with each new transmission. In the following we use a Markov Chain shown in Fig. 2, to model the blocking process of the RF neighbors. Each state *(s)* in our model represents the actual number of current transmissions, and is associated with the following parameters:

- β_s - Incremental number of blocked nodes in state s
- B_s - Total number of blocked nodes in state s
- C_s - Total number of communicating nodes in state *s* is *2s*
- T_s - Total number of involved nodes = $B_s + C_s$
- λ_s - Transmission initiation rate in state s
- μ_s - Transmission service rate in state *s*
- α_s - Destination blocking rate in state s
- L - Number of states (maximum number of transmissions)

In the following we assume that there are *M* nodes in the network at all times and the nodes are distributed uniformly over a 2-dimentional square plane with dimensions *W* X *W*. Traffic is distributed uniformly over all the nodes and both the packet interarrival times and the packet transmission (processing) times are exponentially distributed with the packet arrival and service rates denoted λ and μ respectively. The packet queue size is assumed to be one; that is if a node is blocked from transmitting, the packet gets discarded. Finally, it is assumed that all the nodes in the network share a single RF channel.

III. Derivation of Blocking Prbability

The derivation of the TBP is performed in three steps. First we derive an expression for the average number of blocked nodes (β_1) as a function of network density. Then, we derive an expression for the node blocking probability (NBP) based on the Markov chain system model of the channel access protocol. The NBP is expressed as a function of network size *M*, number of blocked neighbors β_1, and the node packet arrival rate to packet service rate ratio ($\rho = \lambda / \mu$). Finally we use the NBP expression as a stepping stone for deriving the TBP.

A. Average Number of Blocked Neighbors

First, we need to determine the average number of blocked neighbors β_1 for a given TX/RX pair. Since all the nodes in the network share a single channel, the RF neighbors of a TX/RX pair must be blocked from transmitting during the entire transmission sequence (like in CSMA/CA [2]).

In order to study various random network configurations, we first derive an expression for β_1 as a function of network density. We start with the area circumscribed by the two intersecting circles shown in Fig. 1. Denote this area *A(z)*, where *z* is the distance between nodes. It can be shown that

$$A(z) = 2\pi - 4\int_{z/2}^{1} \sqrt{1 - x^2}\, dx \,. \qquad (2)$$

The number of nodes in the intersecting circles is found by multiplying the area *A(z)* by the network density *D* ($D = M / W^2$).

The probability density function (PDF) of the distance *z* between two uniformly distributed nodes can be expressed [7]:

$$f_z(z) = \frac{2z}{W^2}\left(\frac{z^2}{W^2} - \frac{4z}{W} + \pi\right) \quad 0 \le z \le W \qquad (3)$$

The average number of blocked neighbors can be found using the PDF of *z* and the area function *A(z)*. The probability that a node is a RF neighbor (within 1 distance unit range) is obtained by integrating over the PDF:

$$P(z \le 1) = \int_0^1 f_z(z)dz \quad \text{(assuming } W \ge 1\text{)} \qquad (4)$$

The average number of blocked neighbor nodes β_1 can be

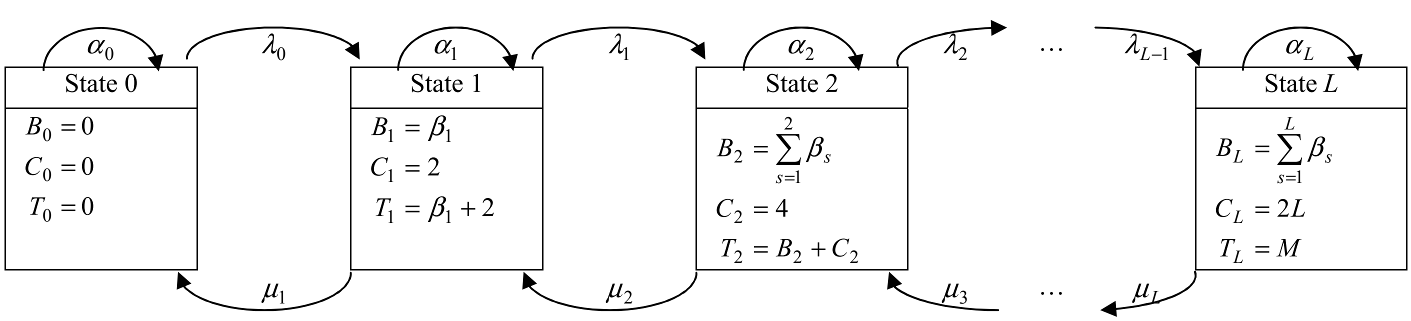

Fig. 2. Markov Chain Model of Multiple User Channel Access

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obtained by averaging over neighbor distance z and multiplying by the network density D, as follows:

$$\begin{aligned}\beta_1 &= D\int_0^1 A(z) f_z(z \mid z \le 1)dz \\ &= D\int_0^1 A(z)\frac{P(z \le 1 \mid \mathbf{z} = z) f_z(z)}{P(z \le 1)} dz \\ &= \frac{D}{P(z \le 1)}\int_0^1 A(z) f_z(z)dz\end{aligned} \qquad (5)$$

With each successive transmission, the incremental number of blocked neighbors β_s becomes smaller. To model the relation between β_s and the state s, we ran computer simulations and plotted the results in Fig. 3. Details on the simulation methodology are given in Section IV. Each point in the figure represents the average from simulations of the incremental number of blocked nodes in the state indicated by the abscissa. Based on these results, we chose to model this relation as linear, i.e., the incremental number of blocked neighbors decreases linearly with the state (the number of current transmissions). A linear regression to the data is also shown in the figure (solid line). By inspecting Fig. 3, we note that serendipitously, the linear fit passes through the data at $s=1$ and $s=L$. This allows further simplification of the linear model as follows

$$\widetilde{\beta}_s = ms + b \text{ where } m = \frac{\beta_L - \beta_1}{L-1} = \frac{-\beta_1}{L-1} \text{ and } b = \frac{L\beta_1}{L-1} \qquad (6)$$

On average, at state $s = L$, all the RF neighbors of the RX/TX pair are blocked by other transmissions and therefore $\beta_L = 0$. Also note that the cumulative number of blocked nodes at each state of the Markov chain can be found by summing up the incremental number of blocked nodes up to and including state s.

$$\begin{aligned}B_s &= \sum_{i=1}^{s}\widetilde{\beta}_i = \sum_{i=1}^{s}\left(\frac{-\beta_1 \cdot i}{L-1} + \frac{L\beta_1}{L-1}\right) = \frac{-\beta_1}{L-1}\frac{s(s+1)}{2} + \frac{sL\beta_1}{L-1} \\ &= \frac{s\beta_1(2L-s-1)}{2(L-1)}\end{aligned} \qquad (7)$$

We will make use of this relation later in the sequel.

B. Evaluation of NBP

NBP was earlier defined as the probability that a node is blocked from transmitting provided that it has a packet to send and that it is not busy transmitting or receiving other packets. This definition is different from the TBP in that it assumes there is always a destination node that is free ($\alpha_s = 0$). The NBP is obtained by multiplying the probability of blocking at each state by the probability of being in that state:

$$P_{NBP} = \sum_{s=1}^{L} P(b \mid s)P(S = s), \qquad (8)$$

where $P(b \mid s)$ is the conditional NBP. Note that state $s=0$ is omitted because the probability of being blocked in that state is 0. In order to find these probabilities, we must first solve the balanced equation of the Markov chain system model. To solve this equation, we must know the number of possible

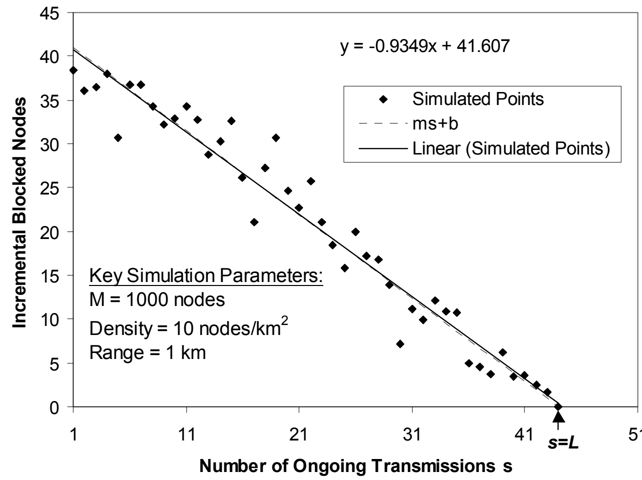

Fig. 3. Incremental Blocked Nodes Experiment

states L. To than end, we use the linear model developed in the previous subsection. The number of states was previously defined as the average number of possible transmissions in the network. It can be found by summing up the number of transmissions $2L$ and the number of blocked nodes B_L (obtained from (7) with $s=L$) to equal the network size M in the L^{th} state of the Markov chain:

$$M = B_L + 2L = \frac{L\beta_1(2L - L - 1)}{2(L-1)} + 2L = L\left(\frac{\beta_1}{2} + 2\right) \qquad (9)$$

The above equation can be solved for L to yield:

$$L = \frac{2M}{\beta_1 + 4} \qquad (10)$$

Note that as we move up from state to state, the number of potential active packet generators decreases (since nodes that are transmitting or receiving or are blocked can not start any new transmissions). Therefore the transmission initiation rate and transmission service rate at each state s can be respectively written as:

$$\lambda_s = F(s)\lambda, \ \ \mu_s = s\mu, \qquad (11)$$

where the transmission rate function (number of nodes that can initiate transmission) is defined by:

$$F(s) \equiv M - 2s - B_s\,. \qquad (12)$$

The probability of being in each state of this Markov chain can now be found by solving the balanced equation of the Markov chain [2]:

$$P(S = s) = G(s)\rho^s P(S = 0)$$

where $\qquad (13)$

$$\mathrm{G(s)} \equiv \frac{1}{\mathrm{s!}}\prod_{i=0}^{s-1} F(i),$$

for $1 \le s \le L$, and by definition $G(0) \equiv 1$. The quantity ρ was previously defined as the node packet arrival rate to packet service rate ratio. By definition, the sum of probabilities of all the states is equal to one, hence $P(S = s)$ can be found as follows:

$$1=\sum_{i=0}^{L}P(S=i)\Rightarrow P(S=s)=\frac{G(s)\rho^s}{\sum_{i=0}^{L}G(i)\rho^i} \tag{14}$$

To complete the evaluation of (8), now that we have found $P(S=s)$, we must find the conditional NBP $P(b\,|\,s)$. The conditional NBP, can be expressed as the cumulative number of blocked nodes at that state divided by the total number of nodes M minus nodes that are already transmitting:

$$P(b\,|\,s)=\frac{B_s}{M-2s} \tag{15}$$

Plugging these results into (8), yields the expression for NBP:

$$P_{NBP}=\sum_{s=1}^{L}\frac{s\beta_1(2L-s-1)}{2(L-1)(M-2s)}\frac{G(s)\rho^s}{\sum_{i=0}^{L}G(i)\rho^i} \tag{16}$$

C. *Evaluation of TBP*

To derive the TBP, we must recognize that a destination node is not always available to receive a packet. As a result, the balanced equation for the Markov chain must be modified to include this factor. Before proceeding we note that the probability of a blocked neighbor at state s is:

$$P(bn\,|\,s)=P(b\,|\,s\,|\,\text{TX node not blocked})=\frac{B_s}{M-2s-1} \tag{17}$$

The negative one in the denominator of the above equation is do to the condition that the source node is not blocked and therefore does not contribute to the set of nodes that can be blocked. The transmission initiation rate can now be rewritten as:

$$\lambda_s=[1-P(bn\,|\,s)]F(s)\lambda, \tag{18}$$

the destination blocking rate

$$\alpha_s=P(bn\,|\,s)F(s)\lambda, \tag{19}$$

and the transmission service rate

$$\mu_s=s\mu. \tag{20}$$

Hence, the balanced equation can, again, be solved to determine $P(S=s)$:

$$P(S=s)=G'(s)\rho^s P(S=0)$$

where (21)

$$\mathrm{G}'(\mathrm{s})\equiv\frac{1}{\mathrm{s}!}\prod_{i=0}^{s-1}\left[1-P(bn\,|\,s)\right]F(i)$$

for $1\le s\le L$, and by definition $G'(0)\equiv 1$. We have,

$$1=\sum_{i=0}^{L}P(S=i)\Rightarrow P(S=s)=\frac{G'(s)\rho^s}{\sum_{i=0}^{L}G'(i)\rho^i} \tag{22}$$

Note that the probability of successful transmission is the probability that a source is free and that a neighbor is free, i.e.,

$$P(ST\,|\,s)=[1-P(b\,|\,s)][1-P(bn\,|\,s)]] \tag{23}$$

The TBP is the probability that either a source or a neighbor is blocked,

$$P(TBP\,|\,s)=1-P(ST\,|\,s)=1-[1-P(b\,|\,s)][1-P(bn\,|\,s)] \tag{24}$$

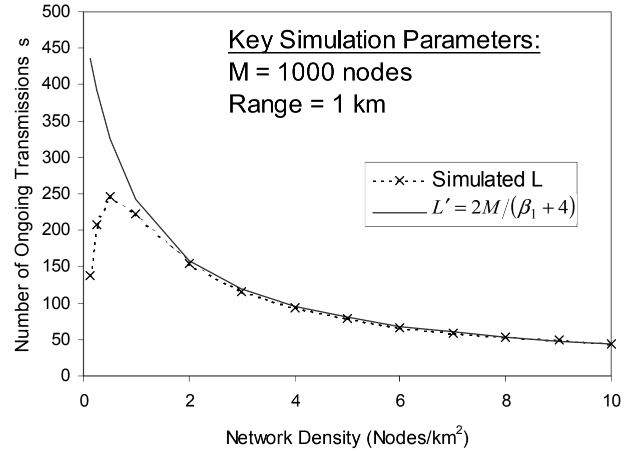

Fig. 4. Number of Possible Transmissions as a Function of Node Density

Consequently, the TBP can be written as:

$$\begin{aligned}P_{TBP}&=\sum_{s=1}^{L}P(TBP\,|\,s)P(S=s)\\&=\sum_{s=1}^{L}\{1-[1-P(b\,|\,s)][1-P(bn\,|\,s)]\}\frac{G'(s)\rho^s}{\sum_{i=0}^{L}G'(i)\rho^i}\end{aligned} \tag{25}$$

IV. Computer Simulations

Two computer simulations were used to verify the model and the equations that were derived in previous sections. The first simulation was used to confirm the assumption that β_s is a linear function. The second simulation was used to verify the TBP equation (25).

A. *Simulation 1 - Average Number of Transmissions*

We used a Monte Carlo simulation to confirm the assumption that β_s is a linear function of the state (i.e. relation (6)). This simulation uses as inputs the number of nodes, the network area, and the RF range of the radio. It places the nodes randomly on a X-Y plane and finds the incremental number of blocked nodes β_s at each state. In addition, it also finds the maximum number of transmissions L that can take place simultaneously in a network. All of the collected data is then averaged over multiple random seeds.

The corresponding results for the incremental number of blocked nodes and for the maximum number of transmissions are presented in Fig. 3 and in Fig. 4, respectively. As discussed earlier, Fig. 3 shows the linear regression from which the linear model was deduced. Fig. 4 demonstrates that the average number of possible transmissions L computed using (10) agrees with the simulated results as long as node density is not too small. When node density becomes very small, the probability that there are no RF neighbors becomes large and the equation no longer holds true. It can be shown that the probability of having one or more nodes with no RF neighbors in a 1000 node network approaches 0 for network density of 4 nodes/km^2 or more. For smaller network densities, this probability becomes significantly larger (for 3

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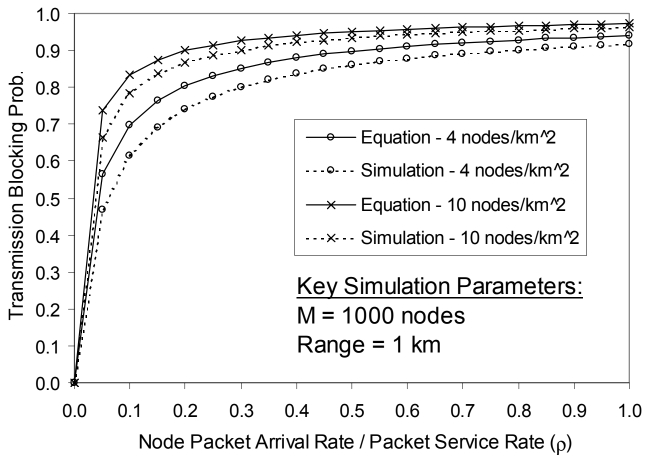

Fig. 5. TBP Simulation Results

nodes/km^2 it is about 0.1). Since (10) was derived using the linear β_1 model, this experiment verifies that the linear model accurately represents the system as long as node density is adequately large.

B. Simulation 2 - TBP

The second simulation experiment aims to verify the TBP equation. This simulation was developed using the OPNET modeling tool and uses all of the inputs of the previous simulation. In addition, it uses the node packet arrival rate to packet service rate ratio ρ, which is varied from 0 to 1. The simulation and analytical results are plotted in Fig. 5 as a function of ρ for network densities of 4 and 10 nodes/km^2. This figure shows that the analytical model represented by (25) has a slight bias and is consistently pessimistic as compared to simulation results. This bias was introduced by the linear model of β_s and carries over into the TBP equation. Also note that this bias is slightly worse for the network density of 4 nodes/km^2. This is because the linear mode is more accurate for higher network densities as was demonstrated by the first simulation and shown in Fig. 4.

It can be seen from the plot in Fig. 5, that TBP becomes large even before ρ reaches 0.1. At first glance this might seem like unrealistically poor performance. However, one must consider that ρ as defined here, refers to each node of the network (M=1000) and that on average, only L=41 transmissions are possible at one time (for network density of 10 nodes/km^2). Therefore, even at $\rho = 0.1$ and at packet service rate of $\mu = 1$ packets/sec, the offered network load would be 100 packets/sec. Another way of interpreting TBP is as the probability of transmitting with zero network access delay or not transmitting at all (since queue size is one). Most practical systems allow for packet queuing and for some channel access delay. If we allowed for a small access delay in our model, the network would probably perform considerably better. Alternatively, we should seek ways to further reduce the number of blocked neighbor nodes β_1 so that more TX/RX pairs could communicate. This in turn would increase L - the average number of possible transmissions. This can be achieved by the use of directional antennas. In general performance benefits of using directional antennas in ad-hoc networks have been demonstrated in [8] and [12].

V. Conclusion

In this paper an analytical model to study, evaluate and quantify the QoS in ad-hoc networks was introduced. This model uses the TBP as a figure of merit of QoS. First we derived an expression for the average number of blocked nodes as a function of network density. Then, we used this expression in our Markov chain model of the multiple user channel access protocol. This model was then used to derive the TBP equation (25).

The TBP equation and the analysis presented in this paper provide an analytical tool to study how QoS can be improved in ad-hoc networks. The use of directional antennas is, clearly, only one of the ways leading to future improvements. Furthermore in our current research we study the expansion of the presented results by refining the TBP equation for different assumptions about each node's queue (e.g. infinite queue, finite queue) and developing a rigorous proof of the linear model.

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