On the Spectral and Power Requirements for Ultra-Wideband Transmission

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***Abstract*— UWB systems based on impulse radio have the potential to provide very high data rates over short distances. In this paper, a new pulse shape is presented that satisfies the FCC spectral mask. Using this pulse, the link budget is calculated to quantify the relationship between data rate and distance. It is shown that UWB can be a good candidate for high rate transmission over short ranges, with the capability for reliably transmitting 100 Mbps over distances at about 10 meters.**

I. Introduction

Ultra-wideband (UWB) technology has been proposed as an alternative air interface for Wireless Personal Area Networks because of its low power spectral density, high data rate, and robustness to multipath fading. The Federal Communications Commission (FCC) has defined an intentional UWB device as one that has a bandwidth equal to or greater than 20% of the center frequency, or that has a bandwidth equal to or greater than 500 MHz. The FCC has also permitted UWB devices to operate using spectrum occupied by existing radio services as long as emission restrictions, in the form of a spectral mask, are met [1].

Impulse radio is one of the popular choices for UWB transmission because of its ability to resolve multipath, as well as the relatively low implementation complexity associated with carrierless (baseband) pulses [2]. Impulse radio does not use a sinusoidal carrier to shift the signal to a higher frequency, but instead communicates with a baseband signal composed of subnanosecond pulses (referred to as *monocycles*) [3]. Because of the short duration of the pulses, the spectrum of the UWB signal can be several gigahertz wide. Impulse radio systems employ a pulse train with pulse amplitude modulation (PAM) or pulse-position modulation (PPM). Previously, UWB systems using PAM [4] and PPM [5] have been analyzed, especially the distance as a function of throughput. However, the standard monocycles used in [4] and [5] do not satisfy the FCC spectral rules. Here, we analyze the transmission range as a function of data rate using a new pulse shape that meets the FCC regulations.

In this paper, we focus on the spectral and power requirements for UWB transmission. In Section II, we compute the power spectral density (PSD) of the Gaussian-based monocycle, which does not satisfy the regulatory rules. A new pulse that meets the FCC emission key is proposed in Section III. This pulse is based on higher-order derivatives of the Gaussian pulse. In Section IV, the transmission range and data rate of a UWB system using the proposed pulse is presented.

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II. Gaussian Pulse and Spectrum

A Gaussian pulse is one candidate for the monocycle in UWB impulse radio systems. If a Gaussian pulse is transmitted, due to the derivative characteristics of the antenna, the output of the transmitter antenna can be modeled by the first derivative of the Gaussian pulse. Therefore, if a general Gaussian pulse is given by

$$x(t) = \frac{A}{\sqrt{2\pi}\sigma}\exp(-\frac{t^2}{2\sigma^2}), \tag{1}$$

then the output of the transmitter antenna will be

$$x^{(1)}(t) = -\frac{At}{\sqrt{2\pi}\sigma^3}\exp(-\frac{t^2}{2\sigma^2}), \tag{2}$$

where the superscript $^{(n)}$ denotes the n-th derivative. The pulse at the output of the receiver antenna is then given by

$$x^{(2)}(t) = A(\frac{t^2}{\sqrt{2\pi}\sigma^5} - \frac{1}{\sqrt{2\pi}\sigma^3})\exp(-\frac{t^2}{2\sigma^2}). \tag{3}$$

A UWB transmitted signal, using PAM, with uniformly spaced pulses in time can be represented as

$$s(t) = \sum_{k=-\infty}^{\infty} a_k x^{(n)}(t-kT), \tag{4}$$

where T is the pulse-spacing interval and the sequence $\{a_k\}$ represents the information symbol. The PSD of the transmitted signal, $P(f)$, is [6, p. 207]

$$P(f) = \frac{\sigma_a^2}{T}|X_n(f)|^2 + \frac{\mu_a^2}{T^2}\sum_{k=-\infty}^{\infty}\left|X_n(\frac{k}{T})\right|^2\delta(f-\frac{k}{T}), \tag{5}$$

where $X_n(f)$ is the Fourier transform of the n-th derivative of a Gaussian pulse, σ_a^2 and μ_a are the variance and mean, respectively, of the symbol sequence $\{a_k\}$, and $\delta(\cdot)$ is the Dirac delta function. The second term in (5), composed of discrete spectral lines, will vanish if the information symbols have zero mean. In what follows, we assume this is true and also assume that $\sigma_a^2 = 1$. Strictly speaking, the duration of the Gaussian pulse and all of its derivatives is infinite. Here, we define the pulse width, T_p, as the interval in which 99.99% of the energy of the pulse is contained. Using this definition, it can be shown that $T_p \approx 7\sigma$ for the first derivative of the Gaussian pulse.

The FCC has issued UWB emission limits in the form of a spectral mask for indoor and outdoor systems [1]. In the band from 3.1 GHz to 10.6 GHz, UWB can use the FCC Part 15 rules with a peak value of -41 dBm/MHz. Outside

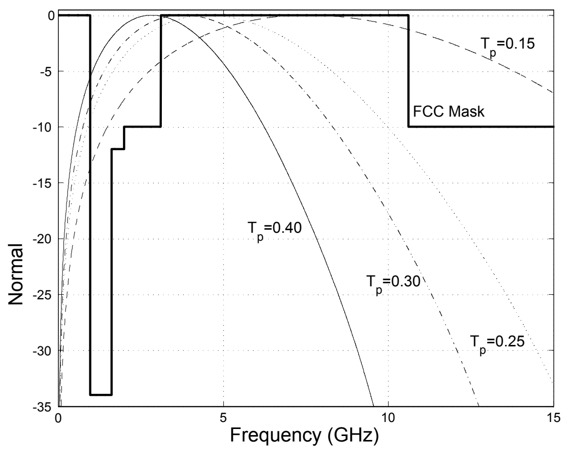

Fig. 1. Power spectral density for the first-derivative Gaussian pulse for various values of the pulse width. The FCC spectral mask for indoor systems is shown for comparison.

of this band, the PSD must be decreased. From 0.96 GHz to 1.61 GHz, the reduction in admissible transmitted power is necessary to protect GPS transmissions. To protect PCS transmission for outdoor systems in the band from 1.99 GHz to 3.1 GHz, the required backoff is 20 dB, rather than the 10 dB for indoor systems.

In Fig. 1, the normalized PSD for the first derivative ($n = 1$) of the Gaussian pulse is plotted for several values of the pulse width, T_p. The normalization factor is the peak value allowed by the FCC, -41 dBm/MHz. It is clear that the PSD of the first derivative pulse does not meet the FCC requirement no matter what value of the pulse width is used. Therefore, a new pulse shape must be found that satisfies the FCC emission requirements. One possibility is to shift the center frequency and adjust the bandwidth so that the requirements are met. This could be done by modulating the monocycle with a sinusoid to shift the center frequency and by varying the values of σ. For example, for a pulse width $T_p = 0.3$ ns, by shifting the center frequency of the monocycles by 3 GHz, the PSD will fall completely within the spectral mask. Impulse UWB, however, is a carrierless system; modulation will increase the cost and complexity. Therefore, alternative approaches are required for obtaining a pulse shape which satisfies the FCC mask.

III. New Pulse Shape Satisfying FCC Mask

In the time domain, the higher-order derivatives of the Gaussian pulse resemble sinusoids modulated by a Gaussian pulse-shaped envelope [7]. As the order of the derivative increases, the number of zero crossings in time also increases; more zero crossings in the same pulse width correspond to a higher "carrier" frequency sinusoid modulated by an equivalent Gaussian envelope. These observations lead to considering higher-order derivatives of the Gaussian pulse as candidates for UWB transmission. Specifically, by choosing the order of the derivative and a suitable pulse width, we can find a pulse that satisfies the FCC's mask. In this section, we derive the spectrum of the higher-order derivatives of the Gaussian pulse and, then choose a pulse shape that meets the emission requirements.

A. Spectrum of Pulses Based on Higher-Order Derivatives

Using the general Gaussian pulse in (1), its n-th derivative can be determined recursively from

$$x^{(n)}(t) = -\frac{n-1}{\sigma^2}x^{(n-2)}(t) - \frac{t}{\sigma^2}x^{(n-1)}(t). \quad (6)$$

The Fourier transform of the n-th order derivative pulse is

$$X_n(f) = A(j2\pi f)^n \exp\{-\frac{(2\pi f\sigma)^2}{2}\}. \quad (7)$$

Consider the amplitude spectrum of the n-th derivative

$$|X_n(f)| = A(2\pi f)^n \exp\{-\frac{(2\pi f\sigma)^2}{2}\}. \quad (8)$$

The frequency at which the maximum value of (8) is attained, the *peak emission frequency,* f_M, can be found by differentiating (8) and setting it equal to zero. Differentiating (8) gives

$$\frac{d\,|X_n(f)|}{df} = A(2\pi f)^{n-1}2\pi \exp\{-\frac{(2\pi f\sigma)^2}{2}\}[n-(2\pi f\sigma)^2]. \quad (9)$$

The peak emission frequency then must satisfy $2\pi f_M\sigma = \sqrt{n}$, and the maximum value of the amplitude spectrum is

$$|X_n(f_M)| = A(\frac{\sqrt{n}}{\sigma})^n \exp(-\frac{n}{2}). \quad (10)$$

Define the normalized PSD, $|P_n(f)|$, as

$$|P_n(f)| \triangleq \frac{|X_n(f)|^2}{|X_n(f_M)|^2} = \frac{(2\pi f\sigma)^{2n}\exp\{-(2\pi f\sigma)^2\}}{n^n\exp(-n)}, \quad (11)$$

which has a peak value of 1 (0 dB). If we consider the n-th derivative of the Gaussian pulse as the UWB transmitted pulse, then the PSD of the transmitted signal is given by

$$|P_t(f)| \triangleq A_{\max}|P_n(f)| = \frac{A_{\max}(2\pi f\sigma)^{2n}\exp\{-(2\pi f\sigma)^2\}}{n^n\exp(-n)}, \quad (12)$$

where $A_{\max}$ is the peak PSD that the FCC will permit. The parameters n and σ can now be chosen to satisfy the FCC mask. In the next subsection, an algorithm will be presented which provides the appropriate pulse shape.

B. New Pulse Shape Parameters

The objective here is to obtain a pulse that matches the FCC's PSD mask as closely as possible, but which also maximizes the bandwidth. We choose values of (n, σ) in (11) to achieve this goal. We first note that the FCC spectral mask has several corner points and that any pulse spectrum must pass through or be below the mask at the corner frequencies (0.96, 1.61, 1.99, 3.1, and 10.6 GHz). Our approach to finding (n, σ) is to first fix f equal to the frequency of one of the corner points. This enables us to rewrite (11) in the following form

$$20n\log_{10}(2\pi f\sigma) - \frac{10(2\pi f\sigma)^2}{\ln 10} - 10n\log_{10} n + \frac{10n}{\ln 10} - R_{dB} = 0, \quad (13)$$

where $R_{dB} \triangleq 10\log_{10}|P_n(f)|$ is the backoff value of the FCC mask at the chosen corner point (for example, if $f = 10.6$

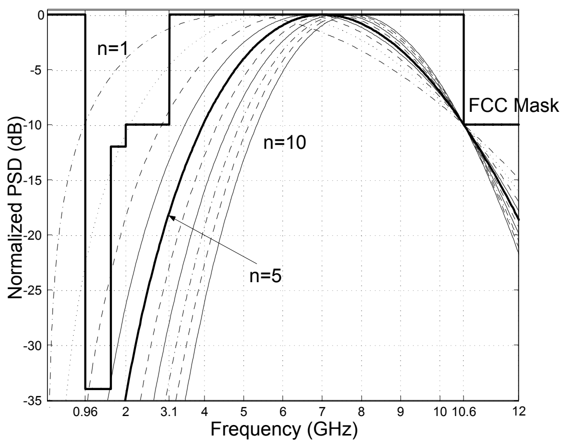

Fig. 2. PSD of the higher-order derivatives of the Gaussian pulse for UWB indoor systems.

GHz, $R_{dB} = -10$ dB for indoor systems). We then start with $n = 1$ and solve (13) for σ. Notice that (13) has two roots for $f\sigma$ with respect to a fixed R_{dB} and n. Taking the peak emission frequency as one end point, the bisection method, or some other root-finding algorithm, can be used to find the roots of (13) numerically. Once σ is found, we simply check if (11), with the value of (n, σ) determined above, meets the FCC mask at the other corner points. If not, we increase n and find a new σ from (13). In Figs. 2 and 3, the normalized PSD for the first-order through tenth-order derivatives of the Gaussian pulse are shown for indoor and outdoor systems, respectively. Note that there are different optimum values of σ for indoor and outdoor systems. For indoor systems, at least the fifth-order derivative should be used, while, for outdoor systems, the seventh or higher should be used. To maintain the bandwidth as wide as possible, the fifth-order derivative should be chosen for indoor systems and the seventh order for outdoor systems.

As before, the time duration of these pulses is infinite. Using the definition of pulse width previously given, for $n = 5$, $T_p \approx 8.5\sigma$. We choose $T_p = 10\sigma$ for convenience in this paper. The maximum PSD can be controlled by changing the value of the amplitude A of the pulse. In Table I, we summarize the parameter σ, the frequencies where the spectrum is 3 dB down from the peak, the peak emission frequency, and the 3-dB bandwidth for indoor systems, where the frequency is in GHz. These results show that the pulse width will be less than 1 nanosecond for all cases, and the 3-dB bandwidth is 2.5 GHz or greater.

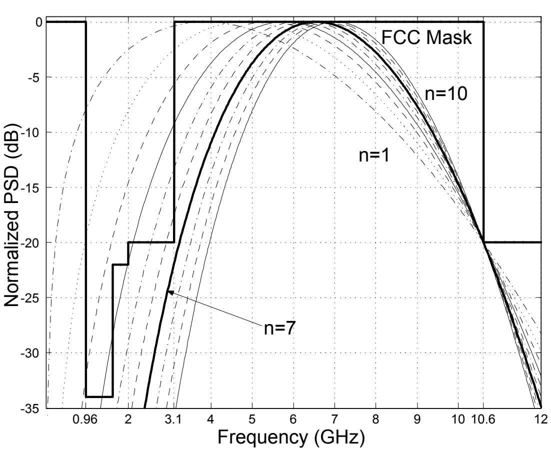

Fig. 3. PSD of the higher-order derivatives of the Gaussian pulse for UWB outdoor systems.

TABLE I

SUMMARY OF THE PARAMETERS FOR UWB INDOOR SYSTEMS

n	σ(ps)	f_L	f_H	f_M	B_{3dB}(GHz)
1	33	2.31	7.84	4.79	5.53
2	39	3.57	8.33	5.78	4.76
3	44	4.33	8.60	6.34	4.28
4	47	4.85	8.79	6.72	3.93
5	51	5.25	8.92	7.01	3.67
6	53	5.57	9.03	7.23	3.46
7	57	5.83	9.12	7.42	3.29
8	60	6.05	9.19	7.57	3.14
9	62	6.24	9.26	7.70	3.01
10	64	6.41	9.30	7.81	2.90

Note that the derivative operation could be implemented as highpass filtering; to transmit the fifth derivative over the air, the Gaussian pulse must be filtered to the fourth-order derivative.

IV. LINK BUDGET AND DATA RATE

The bandwidth of the UWB signal is very large and should be able to transmit a very high data rate; in particular, 100 Mbps should be possible over short distances. In this section, we will determine the permitted distances and data rates using a simple link budget analysis. In [4], the distance as a function of throughput was analyzed for PAM UWB systems. However, the analysis was not performed using the more realistic monocycles presented here, and the path loss was considered constant over the bandwidth of the signal. In this paper, we will quantify the range and data rate relation for an M-PAM using the fifth derivative of the Gaussian pulse as the transmitted monocycle, and a more realistic path loss model.

In the assumed indoor system, the transmitted monocycle, $y(t)$, is the fifth-derivative of the Gaussian pulse,

$$y(t) = A\left(-\frac{t^5}{\sqrt{2\pi}\sigma^{11}} + \frac{10t^3}{\sqrt{2\pi}\sigma^9} - \frac{15t}{\sqrt{2\pi}\sigma^7}\right)\exp\left(-\frac{t^2}{2\sigma^2}\right), \quad (14)$$

and its amplitude spectrum is

$$|Y(f)| = A(2\pi f)^5 \exp\left\{-\frac{(2\pi f\sigma)^2}{2}\right\},$$

where A is a constant chosen to meet the limitations set by the FCC. The normalized pulse waveform is shown in Fig. 4. In the following analysis, additive, white Gaussian noise is assumed at the receiver with a free-space propagation channel model.

For a narrowband signal, the received power at distance d is given by [8]

$$P_r(d) = \frac{P_t}{PL(d)}, \quad (15)$$

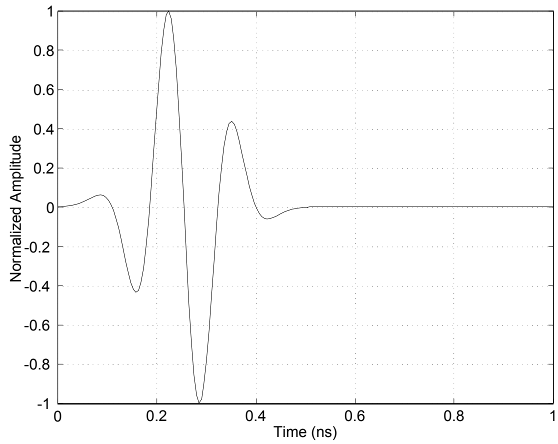

Fig. 4. The pulse shape for the fifth derivative of the Gaussian pulse.

and the free-space path loss, $PL(d)$, is given by

$$PL(d) = \frac{(4\pi)^2 d^2 f_c^2}{G_t G_r c^2}, \tag{16}$$

where P_t is the transmitted power, G_t and G_r are the transmitted and received antenna gains, respectively, f_c is the carrier frequency, and c is the speed of light.

For the indoor UWB transmission system using the pulse given in (14), the PSD is very wideband and, therefore, (16) must be modified to account for variations across the bandwidth of the signal. In particular, the transmitted and received powers should be calculated using the integral of the PSD within a frequency region, and the total transmitted power should be based on the FCC restrictions. Therefore, the transmitted power is

$$P_t = \int_{-\infty}^{+\infty} |P_t(f)|\, df = \int_{-\infty}^{+\infty} A_{\max} |P_n(f)|\, df, \tag{17}$$

where $|P_t(f)|$ and $|P_n(f)|$ are defined in (12) and (11), respectively, and $A_{\max} = -41$ dBm/MHz is the maximum PSD permitted. Based on these parameters and $\sigma = 51$ ps and $n = 5$, the total transmitted power is $P_t = -5.095$ dBm. If we assume that the received signal occupies a band from f_L to f_H, the received power at distance d becomes

$$P_r(d) = \int_{f_L}^{f_H} \frac{A_{\max} |P_n(f)|}{PL(d,f)} df, \tag{18}$$

where $PL(d,f)$ is the wideband path loss for the free-space propagation model, with the center frequency in (16) replaced by the variable f. With this frequency-dependent path loss, the received power becomes

$$P_r(d) = \frac{A_{\max} G_t G_r c^2}{(4\pi)^2 d^2} \int_{f_L}^{f_H} \frac{|P_n(f)|}{f^2} df. \tag{19}$$

In the calculations that follow, the noise spectral density is $N_0 = kT_0F \cdot LM = -102.83$ dBm/MHz, where k is Boltzmann's constant 1.38×10^{-23} Joules/K, T_0 is room temperature (300 K), the noise figure is $F = 6$ dB, and a link margin, LM, of 5 dB is assumed. Normalized by the maximum signal PSD, we have $N_0/A_{\max} \approx -62$ dB. Then, if the receiver bandwidth is chosen corresponding to a 62-dB signal bandwidth, the maximum signal-to-noise ratio (SNR) can be achieved.

The received power (19) necessary at distance d to achieve a given average SNR can be computed from the relation

$$P_r(d) = SNR + P_N + LM, \tag{20}$$

where P_N is the received noise power and is equal to N_0B, and where B is the noise equivalent bandwidth of the receiver. If the symbol rate is assumed equal to the pulse repetition rate, a single UWB pulse is transmitted for each data symbol, and the energy per information symbol equals the energy per pulse. Then, the average output SNR is given by

$$SNR = \frac{E_s/T_s}{N_0B} = \frac{E_sR_s}{N_0B} = \frac{E_b}{N_0} \cdot \frac{R_b}{B}, \tag{21}$$

where E_s is the received symbol energy, T_s is the symbol duration, and R_s is the symbol rate. For uncoded systems, $E_s = E_b \log_2 M$ and $R_s = R_b/\log_2 M$, where E_b is the bit energy and R_b is the bit rate. The ratio $\beta \triangleq B/R_b$ is related to the duty cycle of the pulse and can be thought of as the pulse processing gain. By increasing the occupied bandwidth of the pulse or reducing the pulse repetition frequency, the overall data rate and the distance can be increased. This factor is what allows UWB systems to operate at a very low average transmit PSD, while achieving useful data rates and transmission ranges [4]. For a target bit error rate (BER), the required E_b/N_0 for PPM and PAM can be obtained in [6, p. 262-268].

From (19), (20), and (21), the relation between the distance d and the data rate R_b can be obtained as

$$d = \frac{c}{4\pi}\sqrt{\frac{A_{\max}G_tG_r}{E_b/N_0 \cdot R_b \cdot kT_0F \cdot LM} \int_{f_L}^{f_H} \frac{|P_n(f)|}{f^2} df}. \tag{22}$$

From (22), we observe that the transmission range can be increased for a given data rate by increasing the receiver antenna gain, using channel coding, and/or reducing the noise figure. In Fig. 5, the achievable distance in a free-space environment is presented for M-PAM at a 100-Mbps bit rate. Results are shown for two values of the target BER with G_t and G_r set to 0 dBi. For BER=10^{-6}, binary PAM can reach about 7 meters for a receiver with a bandwidth equal to the 3-dB bandwidth of the pulse. As indicated in Fig. 5, the range can also be extended if the receiver bandwidth is increased; however, the improvement is not that significant.

In Fig. 6, the achievable distance is plotted as a function of bit rate for various values of M. For these results, the target BER is set at 10^{-6}, and the receiver bandwidth is set equal to the 62-dB bandwidth of the pulse; this achieves the maximum SNR. As expected, higher data rates can be achieved at shorter distances. For binary PAM, 100 Mbps can be reliably transmitted at a distance of approximately 8 meters; while, 20 Mbps can be transmitted more than 18 meters. For 4-PAM, 100 Mbps can be reliably transmitted over a distance of only 5 meters. This occurs because, although PAM is a spectrally efficient modulation technique, it is not power efficient. For

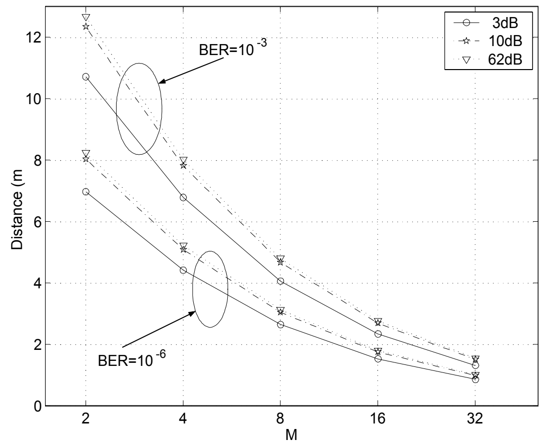

Fig. 5. Transmission range for M-PAM for 100 Mbps data with different receiver bandwidth.

an AWGN channel, the lower order PAM signal would result in the best performance for this power-limited system. In Fig. 6, the allowed distance is also plotted as a function of bit rate for various values of M for BER=10^{-3}. This is the condition specified in [4]. From Fig. 6, binary PAM can be reliably transmitted at 100 Mbps over about 13 meters. In [4], the permitted distance is about 20 meters. However, the results in [4] are optimistic because the actual pulse spectrum and frequency variation of the path loss were not considered.

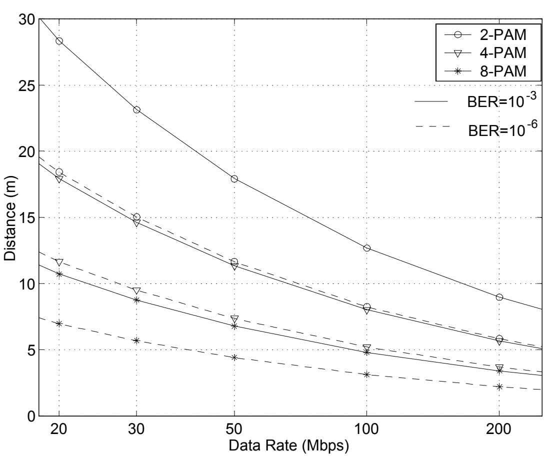

Fig. 6. The achievable range for various data rates for M-PAM UWB systems with receiver bandwidth set to achieve the maximum SNR.

V. Conclusion

In this paper, a new monocycle, based on the higher-order derivatives of the Gaussian pulse, has been proposed. This new pulse satisfies the FCC emission limits for UWB systems. In particular, we demonstrated that the fifth-derivative of the Gaussian pulse meets the regulatory requirements. Based on this pulse and a more accurate frequency-dependent path loss calculation, the link budget was computed for an indoor UWB system, quantifying the relationship between data rate and distance. It was shown that UWB can be a good candidate for high-rate transmission over short ranges with the capability for reliably transmitting 100 Mbps within large rooms.

If there is sufficiently large bandwidth, PPM can provide significant performance improvements over PAM for power-limited applications. However, the PSD of a PPM signal is not linearly related to the PSD of the individual pulse [9], and the spectral lines are not easily removed [10, p. 235]. Further investigation is necessary to assess the performance of PPM systems. In addition, the performance of both PAM and PPM in multipath environments [11] should be investigated.

Acknowledgment

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