Multiple-Symbol Differential Detection with Interference Suppression

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Abstract — **A multiple symbol differential detection detector is presented for communication systems with reception diversity operating over slow fading Rayleigh channel and in the presence of interference. The channel for the signal of interest is assumed unknown. The maximum likelihood decision statistic for the detector is developed. An approximate upper bound for the bit error probability is also developed. The main result in this paper is that with increasing observation interval, the performance of this non-coherent detector approaches that of optimum combining detector with differential encoding at the transmitter.**

I. Introduction

Multiple symbol differential detection (MSDD) was first proposed by Divsalar and Simon [1] for uncoded BPSK and the additive white Gaussian noise (AWGN) channel. They developed a decision statistic assuming a channel with unknown, but fixed phase over multiple symbol intervals. With this detector, the decision is made based on more than two received symbols. They showed that for a long observation interval, the performance (in terms of the required signal-to-noise ratio for a given bit error probability) of noncoherent MSDD approaches that of coherent detection (with differential encoding at the transmitter). With a differential scheme at the receiver, differential encoding is necessary at the transmitter as well. The main advantage of differential MSDD detection is that the knowledge of the channel state information is not required. In a communication scenario when recovering channel phase is very difficult or impossible, MSDD is a good option. In [2], the same authors extended the method to trellis coded M-PSK. In [3], Ho and Fung applied MSDD to a correlated fading channel. Simon and Alouini applied MSDD to channels with reception diversity in [4]. Using similar method as in [1], they showed that with increasing observation interval, the MSDD detector with diversity can approach the performance of a coherent detector with maximal-ratio combining. Since their detector is noncoherent, the estimation of the channel state information (CSI) is not required.

Previously published literature on MSDD assumes operation with additive white Gaussian noise. In this paper, we discuss the MSDD detector with reception diversity in the presence of an interference. The interference is assumed spatially correlated. The channel of the desired signal is assumed to be slow fading Rayleigh channel. The interference is assumed non-fading. The decision statistic of the maximum-likelihood detector is developed. We also derive an analytical expressions for the pairwise error probability (PEP). A closed-form expression for the bit error probability (BEP) is intractable, but we find an approximation of its upper bound. Simulations for differential binary PSK demonstrate the theory developed in the paper. The main result of the paper is, with increasing number of symbols, the performance of the MSDD detector approaches that of optimum combining (with differential encoding). Therefore, MSDD can be used as a robust (with respect to the CSI) detection scheme for wireless communications.

II. System model

Consider a wireless communications system operating over a slow fading channel with one transmit antenna and M receive antennas. At the receiver, the signal of interest is received in the presence of a spatially correlated interference and additive white Gaussian noise. Perfect time synchronization is assumed. The sampled output of the matched filter at antenna m and time k can be expressed as

$$r_{k,m} = \sqrt{P_s} c_m s_k + z_{k,m}, \quad m = 1, \cdots, M, \tag{1}$$

where P_s is the power of the signal of interest, c_m is the complex gain between the desired user and the mth receive antenna, s_k is the transmitted differentially encoded M-PSK symbol. s_k is expressed as $s_k = \exp\left(j2\pi\left(i_k - 1\right)/M_0\right)$, $i_k = 1, 2, \ldots, M_0$, where M_0 is the number of different PSK symbols used by the transmitter. The term $z_{k,m}$ represents the spatial interference plus white Gaussian noise. The signal model in vector notation is

$$\mathbf{r}_k = \sqrt{P_s}\mathbf{c} s_k + \mathbf{z}_k, \tag{2}$$

where $\mathbf{r}_k = \left[r_{k,1}, \ldots, r_{k,M}\right]^T$, $\mathbf{c} = \left[c_1, \cdots, c_M\right]^T$, and $\mathbf{z}_k = \left[z_{k,1}, \ldots, z_{k,M}\right]^T$.

The following assumptions are made with respect to the interference: (1) $z_{k,m}$ is complex-valued, Gaussian, with $E\left[z_{k,m}\right] = 0$ and $E\left[z_{k,m} z^*_{k+j,m}\right] = 0$ for $j \neq 0$ (superscript $*$ denotes complex conjugate), (2) the spatial covariance matrix of $\mathbf{z}_k$ can be expressed as a sum of a rank one interference and additive white noise:

$$\begin{aligned} \mathbf{R}_z &= E\left[\mathbf{z}_k \mathbf{z}_k^H\right] \\ &= P_I \mathbf{c}_I \mathbf{c}_I^H + \mathrm{diag}[N_1, N_2, \cdots, N_M], \end{aligned} \tag{3}$$

where P_I is the interference power, $\mathbf{c}_I$ is a constant vector, $E\left[\mathbf{c}_I^H \mathbf{c}_I\right] = M$, N_m is the power of the white noise component at the mth antenna and the superscript H denotes the

Hermitian transpose. The interference covariance matrix is assumed known at the receiver.

The user's channel coefficients c_m are complex-valued, Gaussian with zero mean. The channel coefficients are mutually independent. It is further assumed that the channel is slow fading, meaning that the channel is static over the observation interval. The channel takes on independent realizations in different observations intervals.

Consider a symbol sequence of length K running from time $k-(K-1)$ to k. Assume the channels are static within the duration of this sequence. Using vector notation,

$$\underline{\mathbf{r}}_k = \sqrt{P_s}\mathbf{H}\mathbf{s}_k + \underline{\mathbf{z}}_k, \quad (4)$$

where the $\underline{\mathbf{r}}_k = \left[\mathbf{r}_k, \ldots, \mathbf{r}_{k-(K-1)}\right]^T$, $\mathbf{H} = \mathbf{c} \otimes \mathbf{I}_M$, the symbol $\otimes$ denotes the Kronecker product, and $\mathbf{I}_M$ is an identity matrix of rank M. The signal vector is defined as $\mathbf{s}_k = \left[s_k, \ldots, s_{k-(K-1)}\right]^T$. $\underline{\mathbf{z}}_k = \left[\mathbf{z}_k, \ldots, \mathbf{z}_{k-(K-1)}\right]^T$.

III. Decision Statistic

We use MSDD to detect the transmitted symbols $\mathbf{s}_k$. The decision is made after K symbols are received. The maximum likelihood sequence detection rule for the sequence of K symbols $\mathbf{s}_k$ is given by

$$\widehat{\mathbf{s}}_k = \arg\max_{\mathbf{s}_k} p(\underline{\mathbf{r}}_k|\mathbf{s}_k) \quad (5)$$

We express the likelihood conditioned on the channel vector $\mathbf{c}$ as

$$p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{c}) = \pi^{-KM}|\mathbf{R}_z|^{-K} \exp \left\{ -\sum_{i=0}^{K-1} (\mathbf{r}_{k-i} - \sqrt{P_s}\mathbf{c}s_{k-i})^H \mathbf{R}_z^{-1} (\mathbf{r}_{k-i} - \sqrt{P_s}\mathbf{c}s_{k-i}) \right\} \quad (6)$$

Diagonalizing the interference spatial covariance matrix $\mathbf{R}_z$, we obtain $\mathbf{R}_z = \mathbf{U}_z\Lambda_z\mathbf{U}_z^H$, where $\Lambda_z = \text{diag}[\lambda_1, \cdots, \lambda_M]$, $\lambda_1, \cdots, \lambda_M$ are the eigenvalues of $\mathbf{R}_z$, and $\mathbf{U}_z$ is a unitary matrix whose columns are the eigenvectors of $\mathbf{R}_z$.

From $\mathbf{R}_z = \mathbf{U}_z\Lambda_z\mathbf{U}_z^H$, we have $\mathbf{R}_z^{-1} = \mathbf{U}_z\Lambda_z^{-1}\mathbf{U}_z^H$. Substitute that into (6) to obtain

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{c}) &= \pi^{-KM}|\mathbf{R}_z|^{-K} \\ &\exp\left\{ -\sum_{i=0}^{K-1} (\mathbf{U}_z^H\mathbf{r}_{k-i} - \sqrt{P_s}\mathbf{U}_z^H\mathbf{c}s_{k-i})^H \right. \\ &\left. \Lambda_z^{-1}(\mathbf{U}_z^H\mathbf{r}_{k-i} - \sqrt{P_s}\mathbf{U}_z^H\mathbf{c}s_{k-i}) \right\} \end{aligned} \quad (7)$$

Now define the whitened received signal vector $\mathbf{x}_k = [x_{k-i,1}, x_{k-i,2}, \ldots, x_{k-i,M}]^T$, such that

$$\mathbf{x}_k = \mathbf{U}_z^H\mathbf{r}_k \quad (8)$$

Also define the modified channel vector $\mathbf{g} = \mathbf{U}_z^H\mathbf{c}$. Since $\mathbf{c}$ is complex Gaussian and $\mathbf{U}_z$ is unitary, $\mathbf{g}$ has the same probability density function as $\mathbf{c}$. It follows from (7) that

$$p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{g}) = \pi^{-KM}|\mathbf{R}_z|^{-K} \exp\left\{ -\sum_{i=0}^{K-1} (\mathbf{x}_{k-i} - \sqrt{P_s}\mathbf{g}s_{k-i})^H \Lambda_z^{-1} (\mathbf{x}_{k-i} - \sqrt{P_s}\mathbf{g}s_{k-i}) \right\} \quad (9)$$

Let $g_m = \alpha_m e^{j\phi_m}$. After expanding the exponent and taking out the terms that do not depend on $\mathbf{g}$ and s_{k-i}, we get

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{g}) &= \pi^{-KM}|\mathbf{R}_z|^{-K} \exp\{-C_0\} \\ &\prod_{m=1}^{M} \exp\left\{ -KP_s\lambda_m^{-1}\alpha_m^2 + \right. \\ &\left. 2\sqrt{P_s}\lambda_m^{-1}|y_m|\alpha_m \cos(\phi_m - \theta_m) \right\}, \end{aligned} \quad (10)$$

where

$$C_0 = \sum_{i=0}^{K-1}\sum_{m=1}^{M} \lambda_m^{-1}|x_{k-i,m}|^2 \quad (11)$$

and the complex variable $y_m = |y_m|e^{i\theta_m}$ is defined by

$$y_m = \left(\sum_{i=0}^{K-1} x_{k-i,m}s_{k-i}^* \right) \quad (12)$$

Next, average $p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{g})$ over the channels $\mathbf{g}$,

$$p(\underline{\mathbf{r}}_k|\mathbf{s}_k) = \int p(\underline{\mathbf{r}}_k|\mathbf{s}_k, \mathbf{g})d\mathbf{g} \quad (13)$$

After some manipulations it can be shown that

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k) &= \pi^{-KM}|\mathbf{R}_z|^{-K} \exp\{-C_0\} \\ &\prod_{m=1}^{M} \int_0^\infty \int_0^{2\pi} \exp\left\{ -KP_s\lambda_m^{-1}\alpha_m^2 + \right. \\ &\left. 2\sqrt{P_s}\lambda_m^{-1}|y_m|\alpha_m \cos(\phi_m - \theta_m) \right\} \\ &p(\alpha_m)p(\phi_m)d\alpha_m d\phi_m \end{aligned} \quad (14)$$

For Rayleigh fading channels,

$$p_{\alpha_m}(\alpha_m) = \frac{2\alpha_m}{\Omega_m} \exp\left(-\frac{\alpha_m^2}{\Omega_m} \right) \qquad p_{\phi_m}(\phi_m) = \frac{1}{2\pi} \quad (15)$$

where $\Omega_m = \overline{\alpha_m^2}$ is the mean square of the amplitude of the mth channel.

Substitute (15) into (14) and separate the integrations,

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k) &= \pi^{-KM}|\mathbf{R}_z|^{-K} \exp\{-C_0\} \prod_{m=1}^{M} \frac{2}{\Omega_m} \\ &\int_0^\infty \exp\left\{ -\left(KP_s\lambda_m^{-1} + \frac{1}{\Omega_m} \right) \alpha_m^2 \right\} \alpha_m d\alpha_m \\ &\frac{1}{2\pi} \int_0^{2\pi} \exp\left\{ 2\sqrt{P_s}\lambda_m^{-1}|y_m|\alpha_m \cos(\phi_m - \theta_m) \right\} d\phi_m \end{aligned} \quad (16)$$

After the second integration is carried out,

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k) &= \pi^{-KM}|\mathbf{R}_z|^{-K} \exp\{-C_0\} \prod_{m=1}^{M} \frac{2}{\Omega_m} \\ &\int_0^\infty \exp\left\{ -\left(KP_s\lambda_m^{-1} + \frac{1}{\Omega_m} \right) \alpha_m^2 \right\} \\ &I_0(2\sqrt{P_s}\lambda_m^{-1}|y_m|\alpha_m)\alpha_m d\alpha_m, \end{aligned} \quad (17)$$

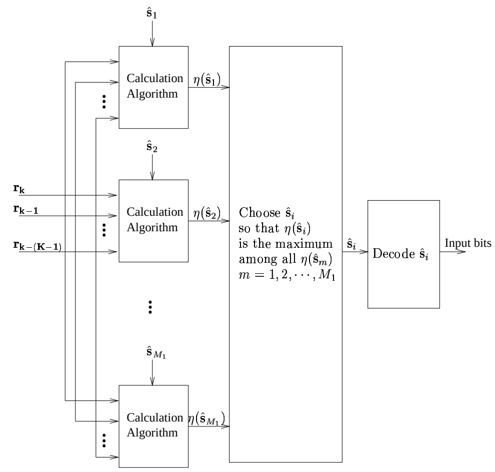

Figure 1: Detector structure

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind. Finally, after computing the integral, we obtain

$$\begin{aligned} p(\underline{\mathbf{r}}_k|\mathbf{s}_k) &= \pi^{-KM}|\mathbf{R}_z|^{-K}\exp\{-C_0\} \\ & \left(\prod_{m=1}^{M}\frac{\lambda_m}{KP_s\Omega_m+\lambda_m}\right) \\ & \exp\left\{P_s\sum_{m=1}^{M}\frac{\Omega_m|y_m|^2}{\lambda_m(KP_s\Omega_m+\lambda_m)}\right\} \end{aligned} \tag{18}$$

In (18), only the argument of the exponential function is dependent on $\mathbf{s}_k$. Due to the monotonicity of the exponential function, maximizing $p(\underline{\mathbf{r}}_k|\mathbf{s}_k)$ with respect to $\mathbf{s}_k$ is equivalent to maximizing $\left\{\sum_{m=1}^{M}\frac{\Omega_m|y_m|^2}{\lambda_m(KP_s\Omega_m+\lambda_m)}\right\}$. To emphasize the dependency of the variables y_m's on the transmitted symbols, we note that from (12), y_m's are a function of the symbol vector $\mathbf{s}_k$.

Therefore we can determine that the decision statistic at time k is

$$\eta(\mathbf{s}_k)=\sum_{m=1}^{M}\frac{\Omega_m|y_m(\mathbf{s}_k)|^2}{\lambda_m(KP_s\Omega_m+\lambda_m)} \tag{19}$$

A diagram of the MSDD detector is shown in Fig. 1. In the figure, each $\widehat{\mathbf{s}}_i$ is a sequence of K M-PSK symbols. From (19) we can see different $\mathbf{s}_k$ could have the same $\eta(\mathbf{s}_k)$. That is why differential encoded is needed at the transmitter. In Fig. 1 M_1 is the number of all possible sequences that has different $\eta(\mathbf{s}_k)$. For example, for M_0-PSK, $M_1=M_0^{K-1}$. The calculation algorithm is given by (19).

Let's consider a special case when there is no interference, i.e., $P_I=0$. In this case, $\mathbf{R}_z=\text{diag}[N_1,N_2,\cdots,N_M]$. We get $\lambda_i=N_i$, $i=1,2,\ldots,M$ and $\mathbf{U}_z=\mathbf{I}_M$. Using (8) and (12), for this case we have

$$\mathbf{x}_{k-i}=\mathbf{r}_{k-i} \tag{20}$$

and

$$y_m(\mathbf{s})=\sum_{i=0}^{K-1}r_{k-i,m}s_{k-i}^{*} \tag{21}$$

The decision statistic in (19) becomes

$$\eta(\mathbf{s}_k)=\sum_{m=1}^{M}\frac{\Omega_m|y_m(\mathbf{s}_k)|^2}{N_m(KP_s\Omega_m+N_m)} \tag{22}$$

When the channels have the same power profile $\Omega_m=\Omega_0$, and the noise power is uniform, $N_m=N_0$, for $m=1,2,\cdots,M$. (22) becomes

$$\eta(\mathbf{s}_k)=\frac{\Omega_0}{N_0(KP_s\Omega_0+N_0)}\sum_{m=1}^{M}|y_m(\mathbf{s}_k)|^2, \tag{23}$$

which is equivalent to the decision statistic

$$\begin{aligned} \eta(\mathbf{s}_k) &= \sum_{m=1}^{M}|y_m(\mathbf{s}_k)|^2 \\ &= \sum_{m=1}^{M}\left|\sum_{i=0}^{K-1}r_{k-i,m}s_{k-i}^{*}\right|^2. \end{aligned} \tag{24}$$

As expected, this decision statistic is the same as (9) in [4]. That means (9) in [4] is only a special case for the more general case of (19).

IV. Error Probability Analysis

Due to the complexity of the problem, it is very difficult to find the bit error probability (BEP) for MSDD analytically, except for the case of two symbols $K=2$ and binary PSK. In stead as in [1], we first derive the pairwise error probability (PEP). Subsequently we derive the union bound of the BEP for general cases.

To simplify the problem, we assume that the channels are flat, i.e., $\Omega_m=1$ and $N_m=N_0$ for $m=1,2,\cdots,M$. In this case, the interference spatial covariance matrix is given by

$$\mathbf{R}_z=P_I\mathbf{c}_I\mathbf{c}_I^H+N_0\mathbf{I}_M \tag{25}$$

and the MSDD decision statistic becomes

$$\eta(\mathbf{s})=\sum_{m=1}^{M}\frac{|y_m(\mathbf{s})|^2}{\lambda_m(KP_s+\lambda_m)} \tag{26}$$

The eigenvalues of $\mathbf{R}_z$ can be shown to be [5, p. 441],

$$\lambda_1 = P_I\sum_{m=1}^{M}|c_{I,m}|^2+N_0 \tag{27}$$

$$\lambda_m = N_0 \quad m=2,3,\cdots,M \tag{28}$$

Let $\mathbf{s}_k$ and $\mathbf{s}_k'$ denote two sequences of K differentially encoded M-PSK symbols. The pairwise error probability (PEP) that when $\mathbf{s}_k$ is transmitted but $\mathbf{s}_k'$ is detected ($\mathbf{s}_k\neq\mathbf{s}_k'$) is denoted as $P(\mathbf{s}_k\to\mathbf{s}_k'|\mathbf{s}_k)$.

Define the variable

$$D=\eta(\mathbf{s}_k)-\eta(\mathbf{s}_k') \tag{29}$$

Then the required PEP is given by

$$P(\mathbf{s}_k\to\mathbf{s}_k'|\mathbf{s}_k)=P(D<0|\mathbf{s}_k) \tag{30}$$

Let $\Phi_{D|\mathbf{s}_k}(j\omega)$ be the characteristic function and $p(D|\mathbf{s}_k)$ be the probability density function of D, then

$$\begin{aligned} P(D<0|\mathbf{s}_k) &= \int_{-\infty}^{0}p(D|\mathbf{s}_k)dD \\ &= \int_{-\infty}^{0}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_{D|\mathbf{s}_k}(j\omega)e^{-j\omega D}d\omega\right]dD \end{aligned} \tag{31}$$

Exchange the order of integration in the previous expression,

$$\begin{aligned} P(D<0|\mathbf{s}_k) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_{D|\mathbf{s}_k}(j\omega)d\omega\int_{-\infty}^{0}e^{-j\omega D}dD \\ &= -\frac{1}{2\pi j}\int_{-\infty+j\varepsilon}^{\infty+j\varepsilon}\frac{\Phi_{D|\mathbf{s}_k}(j\omega)}{\omega}d\omega \\ &= -\sum_{\text{u.p.}}\text{residue}\left[\frac{\Phi_{D|\mathbf{s}_k}(j\omega)}{\omega}\right]. \end{aligned} \quad (32)$$

where ε is a small positive number and the summation is taken over the poles in the upper half of the complex plane. It follows that the PEP can be computed from the characteristic function $\Phi_{D|\mathbf{s}_k}(j\omega)$.

After some algebra, and defining the matrix

$$\mathbf{G} = \begin{pmatrix} \mathbf{I}_M & 0 \\ 0 & -\mathbf{I}_M \end{pmatrix}, \quad (33)$$

it can be shown that the variable D can be expressed as

$$D = \mathbf{f}^H\mathbf{G}\mathbf{f}, \quad (34)$$

where $\mathbf{f} = \left[\mathbf{e}^T(\mathbf{s}_k), \mathbf{e}^T(\mathbf{s}'_k)\right]^T$, and the mth element of the M dimensional vector $\mathbf{e}(\mathbf{s}_k)$ is

$$e_m(\mathbf{s}_k) = b_m y_m(\mathbf{s}_k), \quad (35)$$

where

$$b_m = \sqrt{\frac{1}{\lambda_m(KP_s+\lambda_m)}} \quad m = 1,2,\cdots,M \quad (36)$$

Using the result in appendix B in [6], the characteristic function of D is

$$\Phi_{D|\mathbf{s}_k}(j\omega) = \prod_{k=1}^{2M}\frac{1}{1-2j\mu_k\omega}, \quad (37)$$

where μ_k is the kth eigenvalue of the matrix $\mathbf{M} = \mathbf{R}_f\mathbf{G}$, and $\mathbf{R}_f = E\left[\mathbf{f}\mathbf{f}^H\right]$ is the covariance matrix of $\mathbf{f}$.

Next, we want to get the closed-form expressions for the eigenvalues μ_k. To that end, we need to find a more convenient expression for the vector $\mathbf{f}$. Define the $M\times 1$ vector $\mathbf{w}(\mathbf{s}_k)$, with elements

$$w_m(\mathbf{s}_k) = \sum_{i=0}^{K-1} r_{k-i,m}s^*_{k-i} \quad (38)$$

From (8) and (12), we have

$$\begin{aligned} \mathbf{y} &= [y_1, y_2, \ldots, y_M]^T \\ &= \sum_{i=0}^{K-1}\mathbf{x}_{k-i}s^*_{k-i} \\ &= \sum_{i=0}^{K-1}\mathbf{U}_z^H\mathbf{r}_{k-i}s^*_{k-i} \\ &= \mathbf{U_z^H w} \end{aligned} \quad (39)$$

It follows that

$$\mathbf{e}(\mathbf{s}_k) = \mathbf{B}\mathbf{y}(\mathbf{s}_k) = \mathbf{T}\mathbf{w}(\mathbf{s}_k), \quad (40)$$

where $\mathbf{B} = \text{diag}[b_1,\cdots,b_M]$ and $\mathbf{T} = \mathbf{B}\mathbf{U}_z^H$.

The covariance matrix $\mathbf{R}_f$ can then be expressed as

$$\begin{aligned} &\mathbf{R}_f \\ &= E\begin{bmatrix} \mathbf{e}(\mathbf{s}_k)\mathbf{e}(\mathbf{s}_k)^H & \mathbf{e}(\mathbf{s}_k)\mathbf{e}(\mathbf{s}'_k)^H \\ \mathbf{e}(\mathbf{s}'_k)\mathbf{e}(\mathbf{s}_k)^H & \mathbf{e}(\mathbf{s}'_k)\mathbf{e}(\mathbf{s}'_k)^H \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{T}E\left[\mathbf{w}(\mathbf{s}_k)\mathbf{w}(\mathbf{s}_k)^H\right]\mathbf{T}^H & \mathbf{T}E\left[\mathbf{w}(\mathbf{s}_k)\mathbf{w}(\mathbf{s}'_k)^H\right]\mathbf{T}^H \\ \mathbf{T}E\left[\mathbf{w}(\mathbf{s}'_k)\mathbf{w}(\mathbf{s}_k)^H\right]\mathbf{T}^H & \mathbf{T}E\left[\mathbf{w}(\mathbf{s}'_k)\mathbf{w}(\mathbf{s}'_k)^H\right]\mathbf{T}^H \end{bmatrix} \end{aligned} \quad (41)$$

After some calculations, we can get

$$E\left[\mathbf{w}(\mathbf{s}_k)\mathbf{w}^H(\mathbf{s}_k)\right] = K^2P_s\mathbf{I}_M + K\mathbf{R}_z \quad (42)$$

$$E\left[\mathbf{w}(\mathbf{s}'_k)\mathbf{w}(\mathbf{s}'_k)^H\right] = \left|v(\mathbf{s}'_k)\right|^2 P_s\mathbf{I}_M + K\mathbf{R}_z \quad (43)$$

$$E\left[\mathbf{w}(\mathbf{s}_k)\mathbf{w}(\mathbf{s}'_k)^H\right] = KP_s v^*(\mathbf{s}'_k)\mathbf{I}_M + v^*(\mathbf{s}'_k)\mathbf{R}_z \quad (44)$$

$$E\left[\mathbf{w}(\mathbf{s}'_k)\mathbf{w}(\mathbf{s}_k)^H\right] = KP_s v(\mathbf{s}'_k)\mathbf{I}_M + v(\mathbf{s}'_k)\mathbf{R}_z, \quad (45)$$

where $v(\mathbf{s}'_k) = \mathbf{s}'^H_k\mathbf{s}_k$ is the correlation coefficient between the transmitted sequence $\mathbf{s}_k$ and the detected sequence $\mathbf{s}'_k$.

Noting that

$$\mathbf{T}\mathbf{T}^H = \mathbf{B}\mathbf{U}_z^H\mathbf{U}_z\mathbf{B}^H = \mathbf{B}\mathbf{B}^H \quad (46)$$

$$\mathbf{T}\mathbf{R}_z^H\mathbf{T}^H = \mathbf{B}\Lambda_z\mathbf{B}^H \quad (47)$$

and substituting (42) - (47) into (41), we finally have $\mathbf{R}_f$ as

$$\begin{pmatrix} K^2P_s\mathbf{B}\mathbf{B}^H + K\mathbf{B}\Lambda_z\mathbf{B}^H & KP_s v^*(\mathbf{s}'_k)\mathbf{B}\mathbf{B}^H + v^*(\mathbf{s}'_k)\mathbf{B}\Lambda_z\mathbf{B}^H \\ KP_s v(\mathbf{s}'_k)\mathbf{B}\mathbf{B}^H + v(\mathbf{s}'_k)\mathbf{B}\Lambda_z\mathbf{B}^H & \left|v(\mathbf{s}'_k)\right|^2 P_s\mathbf{B}\mathbf{B}^H + K\mathbf{B}\Lambda_z\mathbf{B}^H \end{pmatrix} \quad (48)$$

Using (33), (48) and $\mathbf{M} = \mathbf{R}_f\mathbf{G}$ we can get $\mathbf{M}$. When $P_I \neq 0$, the eigenvalues of $\mathbf{M}$ can be found as:

$$\mu_1 = \frac{K_v b_1^2 P_s + \sqrt{K_v^2 b_1^4 P_s^2 + 4\left[KP_s+\lambda_1\right]K_v\lambda_1 b_1^4}}{2} \quad (49)$$

$$\mu_2 = \frac{K_v b_1^2 P_s - \sqrt{K_v^2 b_1^4 P_s^2 + 4\left[KP_s+\lambda_1\right]K_v\lambda_1 b_1^4}}{2} \quad (50)$$

$$\mu_m = \frac{K_v b_2^2 P_s + \sqrt{K_v^2 b_2^4 P_s^2 + 4\left[KP_s+\lambda_2\right]K_v\lambda_2 b_2^4}}{2} \quad (51)$$

for $m = 3,5,\cdots,2M-1$ and

$$\mu_m = \frac{K_v b_2^2 P_s - \sqrt{K_v^2 b_2^4 P_s^2 + 4\left[KP_s+\lambda_2\right]K_v\lambda_2 b_2^4}}{2} \quad (52)$$

for $m = 4,6,\cdots,2M$, where $K_v = K^2 - \left|v(\mathbf{s}'_k)\right|^2$.

It can be seen that $\mu_m > 0$ when $m = 1,3,\cdots,2M-1$ and $\mu_m < 0$ when $m = 2,4,\cdots,2M$.

Using (37) and (49) to (52) into (32), we get the following expressions for the PEP

$$\begin{aligned} & P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) = P(D<0|\mathbf{s}_k) \\ = & -\sum_{\text{Pole } \omega=-j\frac{1}{2\mu_2}, -j\frac{1}{2\mu_4}} \text{Residue}\left[\frac{1}{\omega}\frac{1}{(1-2j\mu_1\omega)}\frac{1}{(1-2j\mu_2\omega)}\right. \\ & \left.\frac{1}{(1-2j\mu_3\omega)^{M-1}}\frac{1}{(1-2j\mu_4\omega)^{M-1}}\right] \\ = & -\frac{(\mu_2)^{2M-1}}{(\mu_1-\mu_2)(\mu_3-\mu_2)^{M-1}(\mu_4-\mu_2)^{M-1}} \\ & -\frac{(-1)^M \mu_4^{M-1}}{\mu_3^{M-1}}\frac{1}{(M-2)!}\sum_{k=0}^{M-2}\frac{(M-2+k)!}{k!} \\ & \left[1-\frac{\mu_1}{(\mu_1-\mu_2)}\frac{\mu_1^{M-1-k}}{(\mu_1-\mu_4)^{M-1-k}}+\right. \\ & \left.\frac{\mu_2}{(\mu_1-\mu_2)}\frac{\mu_2^{M-1-k}}{(\mu_2-\mu_4)^{M-1-k}}\right]\frac{\mu_3^{M-1+k}}{(\mu_3-\mu_4)^{M-1+k}} \end{aligned} \quad (53)$$

From the previous relation, it follows that the PEP depends only on the correlation between $\mathbf{s}_k$ and $\mathbf{s}'_k$ instead of on individual $\mathbf{s}_k$ and $\mathbf{s}'_k$.

Next, we find the union bound of the BEP.

Let $\mathbf{u}_k$ be the sequence of $(K-1)\log_2 M_0$ information bits that produces $\mathbf{s}_k$ and $\mathbf{u}'_k$ be the sequence of $(K-1)\log_2 M_0$ bits that results from the detection of $\mathbf{s}'_k$. When $\mathbf{s}_k$ is transmitted but $\mathbf{s}'_k$ is detected ($\mathbf{s}_k \neq \mathbf{s}'_k$), the pairwise bit error probability is

$$P_b\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) = \frac{1}{(K-1)\log_2 M_0} h(\mathbf{u}_k, \mathbf{u}'_k) P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) \quad (54)$$

where $h(\mathbf{u}_k, \mathbf{u}'_k)$ denotes the Hamming distance between $\mathbf{u}_k$ and $\mathbf{u}'_k$.

Let $P_b(\mathbf{s}_k)$ be the bit error probability that $\mathbf{s}_k$ is transmitted but an error sequence (any error sequence) is detected. $P_b(\mathbf{s}_k)$ is upper bounded by the union of all pairwise bit error events,

$$\begin{aligned} P_b(\mathbf{s}_k) & \leq \sum_{\mathbf{u}'_k \neq \mathbf{u}_k} P(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k) \\ & = \frac{1}{(K-1)\log_2 M_0} \\ & \quad \sum_{\mathbf{u}'_k \neq \mathbf{u}_k} h(\mathbf{u}_k, \mathbf{u}'_k) P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) \end{aligned} \quad (55)$$

Since differential M-PSK is symmetric, $P_b(\mathbf{s}_k)$ does not change with $\mathbf{s}_k$ and is actually equal to the BEP for no matter what sequence is transmitted. Denote the BEP with P_b, then

$$\begin{aligned} P_b & \leq \frac{1}{(K-1)\log_2 M_0} \\ & \quad \sum_{\mathbf{u}'_k \neq \mathbf{u}_k} h(\mathbf{u}_k, \mathbf{u}'_k) P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) \end{aligned} \quad (56)$$

It can be shown that sequences that have smaller $|v(\mathbf{s}'_k)|$ have smaller PEP. When a sequence has a very small $|v(\mathbf{s}'_k)|$, its PEP is so small that its contribution to the union bound can be neglected. In [1] and [4], as an approximation when calculating the upper bound, the authors took into account only the error sequences that have the largest $|v(\mathbf{s}'_k)|$. Follow the same reasoning, the approximate upper bound is given by

$$\begin{aligned} B & \approx \frac{1}{(K-1)\log_2 M_0} \\ & \quad \sum_{\substack{\mathbf{u}'_k \neq \mathbf{u}_k \\ |v(\mathbf{s}'_k)| = |v(\mathbf{s}'_k)|_{\max}}} h(\mathbf{u}_k, \mathbf{u}'_k) P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right) \\ & = \frac{1}{(K-1)\log_2 M_0}\left[\sum_{\substack{\mathbf{u}'_k \neq \mathbf{u}_k \\ |v(\mathbf{s}'_k)| = |v(\mathbf{s}'_k)|_{\max}}} h(\mathbf{u}_k, \mathbf{u}'_k)\right] \\ & \quad P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right)\Big|_{|v(\mathbf{s}'_k)| = |v(\mathbf{s}'_k)|_{\max}}, \end{aligned} \quad (57)$$

where $|v(\mathbf{s}'_k)|_{\max}$ is the maximum of all $|v(\mathbf{s}'_k)|$.

In the following, we consider the special case of binary differential PSK where $M_0 = 2$. For convenience, we choose $\mathbf{u}_k = [0, 0, \cdots, 0]$. Assume the information bit 0 is encoded as phase shift 0, bit 1 is encoded as phase shift π, and the first transmitted symbol is 1. Then corresponding $\mathbf{s}_k$ would be $[1, 1, \cdots, 1]$.

When $K = 2$, $M_0 = 2$, there is only one error sequence, which is $\mathbf{u}'_k = [1]$, the corresponding $\mathbf{s}'_k$ would be $\mathbf{s}'_k = [1, -1]$, therefore $h(\mathbf{u}, \mathbf{u}'_k) = 1$, and $|v(\mathbf{s}'_k)| = 0$. The PEP is exactly the BEP,

$$P_b = P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right)\Big|_{|v(\mathbf{s}'_k)|=0} \quad (58)$$

When $K > 2$, it is shown in [1] that

$$\left|v\left(\mathbf{s}'_k\right)\right|_{\max} = K-2 \quad (59)$$

and

$$\sum_{\substack{\mathbf{u}'_k \neq \mathbf{u}_k \\ |v(\mathbf{s}'_k)| = |v(\mathbf{s}'_k)|_{\max}}} h(\mathbf{u}_k, \mathbf{u}'_k) = 2(K-1) \quad (60)$$

Substitute (59) and (60) into (57), then we get for $K > 2$ and $M_0 = 2$,

$$B \approx 2\, P\left(\mathbf{s}_k \to \mathbf{s}'_k|\mathbf{s}_k\right)\Big|_{|v(\mathbf{s}'_k)|=K-2} \quad (61)$$

B could be calculated with any sequences $\mathbf{s}_k$ and $\mathbf{s}'_k$ such that $|v(\mathbf{s}'_k)| = K-2$.

V. Numerical Results

Numerical results presented in this section include computer simulation results and the theoretical results obtained using equations in the earlier sections. The simulation uses binary differential PSK modulation with uniform channel and noise profiles ($\Omega_m = 1$ and $N_m = N_0$ for $m = 1, 2, \cdots, M$). The signal to interference ratio β is defined as $\beta = P_s/P_I$. The signal to noise ratio SNR is defined as SNR $= P_s/N_0$. We assume $\mathbf{c}_I = [c_{1,I}, \cdots, c_{M,I}]^T$,where $c_{i,I} = e^{j\theta_i}, \theta_i$ is a random variable uniformly distributed between 0 and 2π. For comparison purposes, Figures 2 and 3 also contain performance curves for optimum combining. The BEP for optimum combining for BPSK (denoted as P_{oc}) is given by (10.25) in [5]. Using (4.201) in [7], the BEP for optimum combining for binary differential PSK is

$$P_{\text{d\_oc}} = 2P_{\text{oc}}(1-P_{\text{oc}})$$

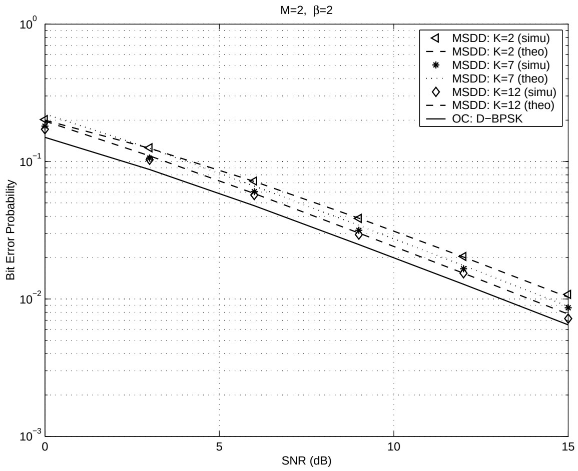

Figure 2: Bit error probability of MSDD and optimum combining, $M = 2$, $\beta = 2$

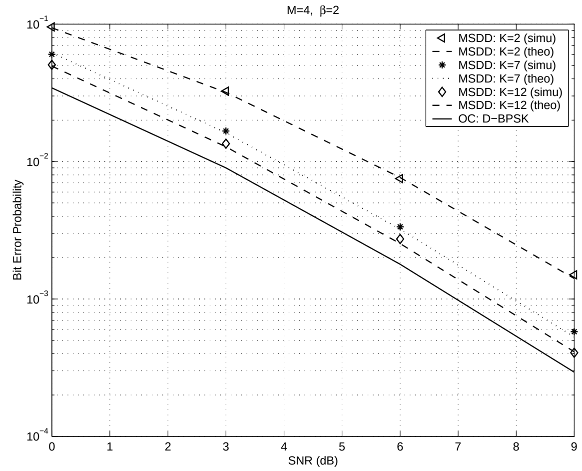

Figure 3: Bit error probability of MSDD and optimum combining, $M = 4$, $\beta = 2$

Figures 2 and 3 show the BEP versus SNR for $M = 2$ and 4 antennas at the receiver, respectively. In both figures, the signal to interference ratio was kept constant at $\beta = 2$. Curves labeled 'simu' represent simulation results, while curves labeled 'theo' show theoretical results. Theoretical results were computed using (53) and (58) (for $K = 2$) or (61 (for $K > 2$). We can see that the theoretical results are very close to the simulation results. Thus they are useful for evaluating the performance.

From the figures, it is clearly observed that as the MSDD frame length K increases for 2 to 12, the performance of MSDD approaches that of optimum combining. For example, for $M = 2$, at BEP $= 2 \times 10^{-2}$, when $K = 2$, the SNR difference between MSDD and optimum combining is about 2.1 dB. When $K = 12$, the difference is only 0.8 dB. The improvement for $M = 4$ is more significant. At BEP $= 2 \times 10^{-3}$, when $K = 2$, the SNR difference between MSDD and optimum combining is about 2.6 dB. When $K = 12$, the difference is only about 0.6 dB. That means there is 2 dB improvement going from $K = 2$ to $K = 12$.

VI. Conclusion

In this paper, we have developed a MSDD detector to suppress interference. The interference was assumed non-fading, temporally white and with known spatial correlation matrix. A decision statistic was derived based on the maximum-likelihood criterion. Closed-form expressions were obtained for the pairwise error probability and the union bound of the bit error probability for this detector. This noncoherent MSDD detector can achieve performance close to that of optimum combining as the observation interval increases. The advantage of MSDD over optimum combining is that it does not require knowledge of the channel of the desired user, which is necessary for optimum combining. Although nonfading of the interference is assumed in the paper, the method developed can be applied to fading interference without much change.

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