Decode-and-Forward Cooperative Diversity with Power Allocation in Wireless Networks

J. Luo, R. S. Blum, L. J. Cimini, L. J. Greenstein, A. M. Haimovich

***Abstract*— We study power allocation for the decode-and-forward cooperative diversity protocol in a wireless network under the assumption that only mean channel gains are available at the transmitters. In a Rayleigh fading channel with uniformly distributed node locations, we aim to find the power allocation that minimizes the outage probability under a short-term power constraint, wherein the total power for all nodes is less than a prescribed value during each two-stage transmission. Due to the computational and implementation complexity of the optimal solution, we derived a simple near-optimal solution. In this near-optimal scheme, a fixed fraction of the total power is allocated to the source node in Stage I. In Stage II, the remaining power is split equally among a set of selected nodes if the selected set is not empty, and otherwise is allocated to the source node. A node is selected if it can decode the message from the source and its mean channel gain to the destination is above a threshold. In this scheme, each node only needs to know its own mean channel gain to the destination and the number of selected nodes. Simulation results show that the proposed scheme achieves an outage probability close to that for the optimal scheme obtained by numerical search, and achieves significant performance gain over other schemes in the literature.**

***Index Terms*— Cooperative diversity, decode-and-forward, power allocation, outage probability, relay networks**

I. Introduction

Cooperative diversity is a set of techniques that exploit the potential of spatially dispersed user antennas to improve communications reliability [1]-[3]. The use of cooperation to achieve diversity in wireless systems was studied in [1] from an information theoretic point of view, where superposition block Markov encoding and backward decoding were employed to achieve the rate region of cooperation. Later, several low-complexity amplify-and-forward and decode-and-forward relay schemes were proposed and studied in [2], [3]. These algorithms were extended to large networks, and both repetition-based and space-time-coded cooperation were considered. In these papers, a common transmission power was assumed at each node.

Recently, various resource allocation problems were studied in [4]-[10] for cooperative diversity systems. For parallel-relay AWGN channels, the optimum power allocations were derived for both amplify-and-forward and decode-and-forward protocols in [4]. Under the assumption of a Rayleigh fading channel where the instantaneous channel state information is available at the transmitters, various allocation problems were examined for a cooperative diversity system with three nodes in [5]-[8]. To model systems with very limited feedback, it is common to assume only mean channel gains are known at the transmitters. However, to the best of our knowledge, only a few published studies have considered allocation based on mean channel gain [9], [10]. Under the assumption of identical mean channel gains for all source-relay and relay-destination links, the power allocation that minimizes the pair-wise error probability was derived for an amplify-and-forward protocol using linear dispersion space-time codes in [9]. For a cooperative diversity system with three nodes, the power allocation that minimizes the outage probability was derived under the assumption that selection diversity is employed at the relay node in [10]. As shown in [10], even for a simple system with three nodes, the optimal allocation is the solution to a set of non-linear equations, which have to be solved using iterative algorithms.

In this paper, we study the power allocation problem for decode-and-forward cooperative diversity in a wireless network with random node locations where only mean channel gain information is available at the transmitters. It is assumed that the instantaneous channel power gains between nodes are exponentially distributed, corresponding to flat Rayleigh fading. The power allocation problem is to find the powers of the source and relay nodes that minimizes the outage probability under a short-term power constraint. As shown in Section III, both the computational complexity and implementation complexity of the optimal solution are very high. Therefore, we focus on deriving a simple near-optimal solution for the allocation problem. The cooperative diversity protocol employed in this paper is similar to that used in [3]. This protocol consists of two transmission stages. In Stage I, the source node transmits and all other nodes listen. In Stage II, the nodes that can decode the message, as well as the source node, re-encode the same message and transmit to the destination.

This paper is organized as follows. In Section II, the cooperative diversity protocol and the channel model are introduced, and the power allocation problem is formulated. In Section III, the challenges for obtaining the optimal solution are discussed. These challenges motivate us to focus on the simple near-optimal solution in this work. A near-optimal power allocation is derived for a cooperative diversity system in Section IV for a network with $N \leq 10$ nodes. Conclusions are drawn in Section V.

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II. System Model and Allocation Problem

We examine a network with N nodes distributed uniformly in the probabilistic sense, i.e., the probability distribution for each node's location is uniform over the two-dimensional area of the network. For a given arbitrary source-destination pair (s,d), the remaining $N-2$ nodes serve as potential relay nodes. Let $h_{s,d}$, $h_{s,i}$, and $h_{i,d}$ denote the instantaneous channel power gains (the amplitude square of the instantaneous channel gain) between source and destination, between source and node i, and between node i and destination, respectively. It is assumed that they are independent, exponentially distributed random variables with means $m_{s,d}$, $m_{s,i}$, and $m_{i,d}$, respectively. Without loss of generality, the noise variance is normalized to 1. We use a cooperative diversity protocol similar to that in [3], except that the same information is transmitted in two stages (in [3], incremental coding is employed in the second stage):

1) Stage I: Source s transmits with power p_{s1}, and all other nodes including the destination d and $N-2$ potential relay nodes listen. With coherent demodulation, the received signal at the destination due to the transmitted signal $x_{s1}(t)$ and noise $n_d(t)$ can be characterized by

$$y_d(t) = \sqrt{p_{s1}h_{s,d}}x_{s1}(t) + n_d(t)\,. \tag{1}$$

Similarly, the received signal at node i is

$$y_i(t) = \sqrt{p_{s1}h_{s,i}}x_{s1}(t) + n_i(t), \quad \text{for} \quad i = 1, \ldots, N-2\,. \tag{2}$$

In (1) and (2), x_{s1} has normalized unit power, and n_d and n_i are zero-mean Gaussian random variables with unit variance. It is assumed in this paper that if the received signal-to-noise ratio (SNR) exceeds a prescribed decoding threshold η, the received message can be decoded correctly. Let $\mathcal{D}$ denote the decoded set that includes all the nodes whose received SNRs exceed η.

2) Stage II: All the decoded nodes, as well as the source node, re-encode the message and transmit with the assigned power. Let p_j denote the power of node $j \in \mathcal{D}$ and p_{s2} denote the power of the source node in Stage II. Two transmission schemes are studied in this stage [3]:
   - Nodes transmit in independent channels, e.g., in separate time slots or separate frequency bands, using repetition codes.
   - Nodes transmit simultaneously using space-time codes.

For both schemes, applying the optimal signal processing and combining the received signals from the two stages, the received SNR at the destination is

$$\mathrm{SNR}_d = (p_{s1}+p_{s2})h_{s,d} + \sum_{j\in\mathcal{D}} p_j h_{j,d}, \tag{3}$$

and an outage is said to occur if $\mathrm{SNR}_d < \eta$. An illustration of the two-stage transmission protocol is presented in Fig. 1.

In wireless communications, battery energy in terminals is a precious resource. For a given source-destination pair in a cooperative diversity system, an important problem is to minimize the outage probability subject to a power constraint, mathematically stated as

$$\min_{p_{s1}} \mathrm{E}_{\mathcal{D}}\left\{ \min_{p_{s2},p_j,j\in\mathcal{D}} \Pr\left\{ (p_{s1}+p_{s2})h_{s,d} + \sum_{j\in\mathcal{D}} p_j h_{j,d} < \eta \middle| \mathcal{D} \right\}\right\} \tag{4}$$

$$\text{subject to} \quad p_{s1}+p_{s2}+\sum_{j\in\mathcal{D}} p_j \le P \tag{4a}$$

$$p_{s1}h_{s,j} \ge \eta \quad \text{for} \quad j \in \mathcal{D} \tag{4b}$$

$$p_{s1}h_{s,j} < \eta \quad \text{for} \quad j \notin \mathcal{D}\,. \tag{4c}$$

Constraints (4b) and (4c) define the conditions for the decoded set $\mathcal{D}$. The set $\mathcal{D}$ is a random set that varies with the instantaneous channel gains, and $\mathrm{E}_{\mathcal{D}}\{\cdot\}$ denotes the expectation over D. We assume that

(a) Mean channel gains are known at the transmitters.
(b) Instantaneous channel gains are known at the receivers.
(c) The decoded set $\mathcal{D}$ is known in Stage II.

Note that in (4) the minimization over p_{s1} is applied after the averaging implied by $\mathrm{E}_{\mathcal{D}}\{\cdot\}$, since the decoded set $\mathcal{D}$ is unknown to the source node before the transmission in Stage I. Due to this causality constraint on $\mathcal{D}$, the source power p_{s1} and p_{s2} are treated differently in (4).

III. Challenges and Approach

We now present the challenges to solving (4). First, the outage probability is a complicated non-convex function of the power allocation. In the following, we present a closed-form expression for the outage probability $\Pr\left\{\sum_{i=1}^N h_i p_i < \eta\right\}$ with h_i being an exponential random variable with mean m_i. Using characteristic functions and partial fraction techniques, closed-form expressions for the outage probability are presented below:

1) When $m_i p_i \neq m_j p_j$ for any $i \neq j$,

$$\begin{aligned} P_{\text{out}} &= \Pr\left\{\sum_{i=1}^N h_i p_i < \eta\right\} \\ &= \sum_{i=1}^N \left(1-\exp\left(-\frac{\eta}{m_i p_i}\right)\right)\prod_{j\neq i}\frac{1}{1-\frac{m_j p_j}{m_i p_i}} \\ &= 1-\sum_{i=1}^N \exp\left(-\frac{\eta}{m_i p_i}\right)\prod_{j\neq i}\frac{1}{1-\frac{m_j p_j}{m_i p_i}}\,. \end{aligned} \tag{5}$$

2) Though somewhat more complicated, similar results can be obtained when some values of $m_i p_i$ are equal.

The second reason for the difficulty in solving (4) is that, in a cooperative diversity system, the outage probability has to be averaged over all possible decoded sets $\mathcal{D}$, and the number of decoded sets increases exponentially with the number of nodes in the network.

Due to the complexity of the outage probability expression, even for a system with $N = 3$ nodes, no simple closed-form solution can be obtained for the allocation problem (4). Moreover, the numerically obtained optimal solution depends on all mean channel gains and the instantaneous decoded set, and thus it would be difficult to implement in a practical system. Therefore, we focus here on finding a simple near-optimal solution for (4).

IV. Power Allocations For a Cooperative Diversity System

The allocation problem in (4) can be broken down into the following sub-problems:

1) For a given p_{s1} and the decoded set $\mathcal{D}$, find the power allocation p_j and p_{s2} in Stage II that minimizes the conditional outage probability.
2) Find the optimal p_{s1} that minimizes the outage probability averaged over all possible $\mathcal{D}$.

We present the near-optimal solution for each sub-problem in turn.

A. Power Allocation for Stage II

For given p_{s1} and the decoded set $\mathcal{D}$, the allocation problem for Stage II solves

$$\min_{p_{s2},p_j,j\in\mathcal{D}} \Pr\left\{(p_{s1}+p_{s2})h_{s,d}+\sum_{j\in\mathcal{D}}p_jh_{j,d}<\eta\middle|\mathcal{D}\right\} \quad (6)$$
$$\text{subject to} \quad p_{s2}+\sum_{j\in\mathcal{D}}p_j\le P-p_{s1}.$$

Problem (6) with given p_{s1} and $\mathcal{D}$ is in fact a power allocation problem for a transmit diversity system. In [12], a near optimal power allocation scheme, called the Selective Equal-Power scheme, was derived for a transmit diversity system with only mean channel gain information. In this scheme, the power is split equally among a set of selected nodes, where a node is selected if its mean channel gain to the destination is above a selection threshold.

Let $\tilde{\mathbf{p}}_{\rm II}=(\tilde{p}_{s2},\tilde{p}_j,j\in\mathcal{D})$, denote the Selective Equal-Power allocation. Applying the result in [12], the proposed scheme consists of two steps:

1) *Independent node selection within* $\mathcal{D}$. The selected set is
$$S=\left\{j\in\mathcal{D}\middle|m_{j,d}\ge\frac{\eta}{P-p_{s1}}\right\}.$$
Here the selection threshold $\frac{\eta}{P-p_{s1}}$ depends on the decoding threshold and the remaining power $P-p_{s1}$ [12].
2) *Power assignment*.
   - When S is not empty, power is equally split among the selected nodes as
$$\tilde{p}_j=\begin{cases}(P-p_{s1})/|S| & j\in S\\ 0 & \text{otherwise}\,,\end{cases} \quad (7)$$
where $|S|$ is the cardinality of set S. In order to split the power equally among the selected nodes, knowledge of the number of selected nodes is needed. This knowledge can be acquired through one-bit feedback from each node on its selection status.
   - When S is empty, set $\tilde{p}_{s2}=P-p_{s1}$. With this assignment, the total power for each two-stage transmission is equal to the power constraint P.

Numerical results in [12] show that the Selective Equal-Power scheme is near-optimal for a wide range of cases tested.

B. Power Allocation for Stage I

We next examine the power allocation for the source node at Stage I. In this section, it is assumed that $\tilde{\mathbf{p}}_{\rm II}$ proposed in Section IV-A is employed in Stage II. Rather than determine the optimal p_{s1} for the specific mean channel gains, we show that a greatly simplified solution results in very small performance loss for a wide range of conditions.

Let $\rho=p_{s1}/P$ denote the fraction of power allocated to the source node at Stage I. Then the allocation problem is to find the ρ that minimizes the outage probability. Let $P_o(\tilde{\mathbf{p}}_{\rm II}|\rho P;\mathcal{D})$ denote the conditional outage probability achieved by $\tilde{\mathbf{p}}_{\rm II}$ in Stage II for a given $\mathcal{D}$ and $p_{s1}=\rho P$. In a network with $N-2$ relay nodes, there are $2^{(N-2)}$ possible decoded sets. Let $\mathcal{B}(i)$ with $i=0,1,\ldots,2^{(N-2)}-1$ enumerate all possible sets in the space of the decoded sets. Let $\Pr\{\mathcal{D}=\mathcal{B}(i);\rho P\}$ denote the probability that $\mathcal{B}(i)$ is a decoded set. The corresponding outage probability at the destination is

$$\begin{aligned}P_o(\rho)&=\mathrm{E}_{\mathcal{D}}\left\{P_o(\tilde{\mathbf{p}}_{\rm II}|\rho P,\mathcal{D})\right\}\\ &=\sum_{i=0}^{2^{(N-2)}-1}\Pr\{\mathcal{D}=\mathcal{B}(i);\rho P\}\,P_o(\tilde{\mathbf{p}}_{\rm II}|\rho P;\mathcal{B}(i))\,. \quad (8)\end{aligned}$$

The allocation problem will find

$$\rho^*=\arg\min_{0\le\rho\le1}P_o(\rho)\,. \quad (9)$$

For the assumed (exponential) channel power gain distribution, $\Pr\{\mathcal{D}=\mathcal{B}(i);\rho P\}$ is given by

$$\begin{aligned}&\Pr\{\mathcal{D}=\mathcal{B}(i);\rho P\}\\ &=\prod_{j\in\mathcal{B}(i)}\Pr\{h_{s,j}\rho P\ge\eta\}\prod_{j\notin\mathcal{B}(i)}\Pr\{h_{s,j}\rho P<\eta\}\\ &=\prod_{j\in\mathcal{B}(i)}\exp\left(-\frac{\eta}{\rho Pm_{s,j}}\right)\prod_{j\notin\mathcal{B}(i)}\left(1-\exp\left(-\frac{\eta}{\rho Pm_{s,j}}\right)\right). \quad (10)\end{aligned}$$

The conditional outage probability $P_o(\tilde{\mathbf{p}}_{\rm II}|\rho P,\mathcal{D})$ can be evaluated using (5). When N is large, computing the outage probability by evaluating all $2^{(N-2)}$ terms in (8) becomes computationally intensive. Thus, for $N\ge15$, we use Monte Carlo simulation to compute the outage probability. That is, for a given node configuration, the instantaneous channel gains are generated based on an exponential distribution with given mean channel gains for 10^4 runs. For each run, the decoded set is determined, and the conditional outage probabilities $P_o(\tilde{\mathbf{p}}_{\rm II}|\rho P,\mathcal{D})$ is calculated using (5) and is averaged over 10^4 runs to obtain the outage probability $P_o(\rho)$.

The optimum power ratio ρ^* can be obtained numerically through exhaustive search. Though the optimal ρ^* can be any value between 0 and 1 depending on the topology of the network, we observe from the simulation results that a fixed $\rho\in[0.5,0.6]$ is near-optimal for a wide-range of settings. In

Fig. 2, the outage probability versus P/η performance with the optimal ρ^*, $\rho = 0.5$, and $\rho = 0.6$ is presented for a randomly generated network configuration with $N = 10$ nodes. The mean channel power gains are assumed to follow the path loss model $m_{ij} = c d_{ij}^{-\beta}$, where d_{ij} is the distance between nodes i and j; β is the path loss exponent (we use $\beta = 4$); and c is a propagation parameter whose value is immaterial to this study and is set so that $m_{s,d} = 1$. Here the corresponding mean channel gains are given in the caption of Fig. 2. As we can see, in this example, a fixed power ratio $\rho = 0.5$ or $\rho = 0.6$ achieves outage probability close to the minimum outage probability.

In order to show that a fixed $\rho \in [0.5, 0.6]$ is near-optimal for a wide range of configurations, we randomly generate 100 configurations with N nodes. For each configuration, the minimum outage probability $P_o(\rho^*)$ with the optimal ρ^* (determined through numerical search) and the outage probability $P_o(\rho)$ with a fixed ρ are calculated. Then the cumulative distribution function (CDF) of the ratio $x = \frac{P_o(\rho)}{P_o(\rho^*)}$ are computed over 100 random configurations. This experiment is done for $\rho = 0.5$ and $\rho = 0.6$ with $N = 3$ nodes, as well as $\rho = 0.5$, $\rho = 0.6$, $\rho = 0.7$ with $N = 30$ nodes, respectively, and the results are presented in Fig. 3. Since for a given P/η the outage probability varies in a large range for different number of nodes as well as different configurations, to have a fair comparison, here for each configuration, P/η is chosen such that $P_o(\rho = 0.5)$ is between 0.001 and 0.01. As shown in Fig. 3, for over 95% of all configurations, $P_o(\rho)$ at fixed $\rho = 0.5, 0.6$ is within two times the minimum outage probability. Similar results are observed for the case with the path loss exponent $\beta = 2$.

From above numerical results, we demonstrate that a fixed ρ is near-optimal for a wide range of conditions. It is difficult, however, to prove the near-optimality of the fixed ρ in general. In the following, we present analysis suggesting that $\rho = 0.6$ is near-optimal for $N \le 6$ at asymptotically high power with *any* mean gains. Providing a more general proof is a topic for further investigation. At asymptotically high power, as shown in Appendix A, the outage probability can be approximated as

$$P_o(\rho) \quad \approx \quad \hat{P}_o(\rho) = \left(\frac{\eta}{P}\right)^{(N-1)} \sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K(\rho)\,, \tag{11}$$

where

$$f_K(\rho) = \begin{cases} \frac{1}{\rho^{(N-2)}}, & K = 0 \\ \frac{1}{\rho^{(N-1-K)}(1-\rho)^K}, & 1 \le K \le N-2 \end{cases}, \tag{12}$$

and $a_K(\mathbf{m}), 0 \le K \le N$ are described in Appendix A as a function of of the mean gains $\mathbf{m}$.

Lemma 1 *Let $\hat{P}_{\mathrm{o,min}}$ denote the minimum outage probability. The outage probability approximation $\hat{P}_o(\rho)$ at asymptotically high power is bounded as*

$$\hat{P}_{\mathrm{o,min}} \le \hat{P}_o(\rho) \le b(\rho)\hat{P}_{\mathrm{o,min}} \tag{13}$$

where

$$b(\rho) = \max_{0 \le K \le N-2} \frac{f_K(\rho)}{f_K^*}, \tag{14}$$

$$f_K^* = \begin{cases} 1 & K = 0 \\ \frac{1}{\left(1-\frac{K}{N-1}\right)^{(N-1-K)}\left(\frac{K}{N-1}\right)^K} & K \ge 1 \end{cases}. \tag{15}$$

The proof is in Appendix B. We list the values of $b(\rho = 0.6)$ for $3 \le N \le 6$ in Table I. For $N = 3$ nodes with any mean gains, the scheme with a fixed $\rho = 0.6$ achieves a performance within 1.667 times the minimum outage probability; and for $N = 6$ nodes with any mean gains, it achieves a performance within 7.71 times the minimum outage probability. As N increases beyond 6, the upper bound in (13) becomes increasingly loose, and poorly reflects the actual relationship between $\hat{P}_o(\rho)$ and the minimum outage probability.

C. Discussions and Numerical Results

In summary, the proposed scheme $\tilde{\mathbf{p}}$ for the cooperative diversity system, called *Fixed-Ratio Selective Equal-Power*, is as follows:

1) During Stage I, the source node transmits a fraction $\rho = 0.6$ of the total allowed power.
2) During Stage II, a selected set S is first determined within the decoded set $\mathcal{D}$. If S is empty, the source node continues to transmit with the remaining power $(1-\rho)P$. Otherwise, the remaining power $(1-\rho)P$ is equally split among the selected relay nodes.

The main advantages of the proposed scheme are its near-optimal performance and its simplicity. In this scheme, the relay selection procedure is done in a distributed fashion and independently by each node. This scheme only requires that:

1) Each node knows its own mean channel gain to the destination. No global information on the mean channel gains is required.
2) The number of the selected relaying nodes is known at Stage II.

The proposed scheme $\tilde{\mathbf{p}}$ was derived by finding near-optimal solutions for two transmission stages separately. Now, we compare its performance with the best numerical solution $\mathbf{p}^*$ obtained from the cyclic coordination search algorithm [11]. Due to the high computational complexity, we can only obtain the $\mathbf{p}^*$ with $N \le 10$. The same experiment employed in Section IV-B is performed over 100 random configurations. As shown in Fig. 4, in over 95% of the cases, the proposed scheme achieves an outage probability within two times the minimum outage probability.

We now show that the proposed scheme significantly improves system performance compared to the following power allocation schemes.

- *Best-Select One*. At Stage II, if the decoded set is not empty, only the decoded node with the largest mean gain transmits; otherwise, the source node continues to transmit with the remaining power.
- *Best-Select Two*. At Stage II, if there are more than two decoded nodes, the two decoded nodes with the largest

mean channel gain transmit with equal power; otherwise, Best-Select One is applied.

- *Equal-Power*. All decoded nodes and the source node transmit with the same constant power p. This is the power allocation scheme used in [3].

In previous numerical results, the outage probability metric was used to characterize the performance of a given source-destination pair. Now, in order to demonstrate the performance of the network (considering all possible source-destination pairs) the metric *link-failure probability*, i.e., the percentage of links whose outage probability is below a target value, is used. Here, each node-node connection represents one of the $N(N-1)/2$ source-destination pairs for a network with N nodes. In Fig. 5, the link-failure probabilities of various schemes are plotted for a network with 10 uniformly distributed nodes in a square area. Here, the link-failure probability is evaluated over 40 randomly generated network configurations, and the target link-outage probability is set to 0.01. The abscissa represents the average total power consumed for each two-stage transmission over all possible source-destination pairs and configurations[1]. As we can see, the proposed scheme achieves significant power gains over both the Equal-Power scheme and the Best-Select One scheme. In this example, with the target link-outage probability being 0.01, the Best-Select Two scheme achieves a performance close to that of the proposed scheme. If the target link-outage probability is lower, the gain of the proposed scheme over the Best-Select Two scheme will be more significant.

V. Conclusions

We have investigated cooperative relaying in an N-node distributed network. The aim has been to find the transmit power allocation among the source and relay nodes that minimize the outage probability for a given source-destination pair subject to a total power constraint. Due to high computational complexity of the optimal solution, a simple near-optimal scheme is derived by solving two sub-problems separately: (1) choosing the ratio, ρ, of the total power, P, that should be allocated to the source node in Stage I; and (2) choosing the distribution of the remaining power, $(1-\rho)P$, among the selected relay nodes in Stage II. For the second stage, we specify a Selective-Equal-Power allocation, which was shown previously to be near-optimal [12]. For the first stage, we have shown here that a fixed $\rho = 0.6$ is near-optimal under a wide range of conditions. We have shown that for $N \le 10$ the proposed scheme achieves an outage probability within two times the minimum outage probability in over 95% of 100 randomly generated configurations. We have also compared our scheme with other practical power allocations described in the literature and found it to be superior to all of them. Assessing the proximity of the new scheme to optimal under a more comprehensive range of conditions is a topic for further research.

[1]For the Equal-Power scheme, the total transmission power varies with the number of decoded nodes, while for all other schemes, the total transmission power for each transmission is a fixed value P. Therefore, to have a fair comparison, the average power over all transmissions is used.

Appendix

A. Outage Probability Approximation for Asymptotically Large Power

Under the assumption that $\tilde{\mathbf{p}}_{II}$ is employed in Stage II, the outage probability at the destination is

$$P_o(\rho) = \sum_{i=0}^{2^{(N-2)}-1} \Pr\left\{\mathcal{D} = \mathcal{B}(i); \rho P\right\} P_o(\tilde{\mathbf{p}}_{\mathrm{II}}|\rho P; \mathcal{B}(i)) \,. \tag{16}$$

For very large power, we have the following approximations:

$$\exp\left(-\tfrac{\eta}{\rho P m_{s,j}}\right) \approx 1 - \tfrac{\eta}{\rho P m_{s,j}} \approx 1, \tag{17}$$

$$1 - \exp\left(-\tfrac{\eta}{\rho P m_{s,j}}\right) \approx \tfrac{\eta}{\rho P m_{s,j}} \,. \tag{18}$$

Using (18), we have

$$\begin{aligned}
&\Pr\left\{\mathcal{B}(i); \rho P\right\} \\
&= \prod_{j\in\mathcal{B}(i)} \Pr\left\{h_{s,j}\rho P \ge \eta\right\} \prod_{j\notin\mathcal{B}(i)} \Pr\left\{h_{s,j}\rho P < \eta\right\} \\
&= \prod_{j\in\mathcal{B}(i)} \exp\left(-\frac{\eta}{\rho P m_{s,j}}\right) \prod_{j\notin\mathcal{B}(i)} \left(1 - \exp\left(-\frac{\eta}{\rho P m_{s,j}}\right)\right) \\
&\approx \left(\frac{\eta}{\rho P}\right)^{(N-K)} \frac{1}{\prod_{j\notin\mathcal{B}(i)} m_{s,j}} \,.
\end{aligned} \tag{19}$$

For very large power, the selected set S is equal to the decoded set $\mathcal{D} = \mathcal{B}(i)$. Then the power allocation $\tilde{\mathbf{p}}_{\mathrm{II}}$ with $S = \mathcal{B}(i)$ of cardinality K is given by

$$\tilde{p}_{s2} = \begin{cases} (1-\rho)P & K = 0 \\ 0 & 1 \le K \le N-2 \end{cases}, \tag{20}$$

$$\tilde{p}_j = \begin{cases} (1-\rho)P/K & j \in \mathcal{B}(i) \\ 0 & \text{otherwise} \end{cases}. \tag{21}$$

For very large power, the outage probability is approximated as [3]

$$\Pr\left\{\sum_{i=1}^{N} p_i h_i \le \eta\right\} \approx \frac{\eta^N}{\prod_{i=1}^{N} p_i m_i} \,. \tag{22}$$

Applying (21) and (22), it follows that

$$\begin{aligned}
&P_o(\tilde{\mathbf{p}}_{\mathrm{II}}|\rho P; \mathcal{B}(i)) \\
&= \Pr\left\{(p_{s1} + p_{s2})h_{s,d} + \sum_{j\in\mathcal{B}(i)} p_j h_{j,d} \le \eta\right\} \\
&\approx \begin{cases} \frac{\eta}{P m_{s,d}}, & K = 0 \\ \frac{(\eta/P)^{K+1}}{(K+1)!\rho\left(\frac{1-\rho}{K}\right)^K m_{s,d} \prod_{j\in\mathcal{B}(i)} m_{j,d}} & 1 \le K \le N-2 \end{cases}.
\end{aligned} \tag{23}$$

Applying (19) and (23) and combining the terms in (16) with $B(i)$ of the same cardinality, the outage probability can be approximated as

$$P_o(\rho) \approx \left(\frac{\eta}{P}\right)^{(N-1)} \sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K(\rho) \triangleq \hat{P}_o(\rho) \,, \tag{24}$$

where

$$a_K(\mathbf{m}) = \begin{cases} \frac{1}{m_{s,d}\prod_{j=1}^{N-2} m_{s,j}}, & K=0 \\ \frac{K^K}{m_{s,d}(K+1)!}\sum_{|\mathcal{B}(i)|=K}\frac{1}{\prod_{j\in\mathcal{B}(i)} m_{j,d}\prod_{j\notin\mathcal{B}(i)} m_{s,i}}, & \\ \quad 1 \le K \le N-2 & \end{cases} \quad (25)$$

$$f_K(\rho) = \begin{cases} \frac{1}{\rho^{(N-2)}}, & K=0 \\ \frac{1}{\rho^{(N-1-K)}(1-\rho)^K}, & 1 \le K \le N-2 \end{cases}, \quad (26)$$

and $\mathbf{m}$ denote the vector of all mean channel gains.

B. Proof of Lemma 1

We first show that $f_K^* = \min_\rho f_K(\rho)$. It can be verified that the ρ that minimizes $f_K(\rho)$ is equal to $1-\frac{K}{N-1}$ when $K \ge 1$, and is equal to 1 otherwise. By inserting this ρ into $f_K(\rho)$ in (12), we obtain the expression for f_K^* as presented in (15). Then we have

$$\begin{aligned} \hat{P}_o(\rho) &= \left(\frac{\eta}{P}\right)^{(N-1)}\sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K(\rho) \\ &= \left(\frac{\eta}{P}\right)^{(N-1)}\sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K^* \frac{f_K(\rho)}{f_K^*} \\ &\overset{(a)}{\le} b(\rho)\left(\frac{\eta}{P}\right)^{(N-1)}\sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K^* \\ &\overset{(b)}{\le} b(\rho)\left(\frac{\eta}{P}\right)^{(N-1)}\sum_{K=0}^{N-2} a_K(\mathbf{m}) f_K(\hat{\rho}) \\ &= b(\rho)\hat{P}_o(\hat{\rho}) \,. \end{aligned} \quad (27)$$

Inequality (a) holds since $\frac{f_K(\rho)}{f_K^*} \le b(\rho)$ for all K. Inequality (b) holds since $f_K^* \le f_K(\rho)$ for any ρ.

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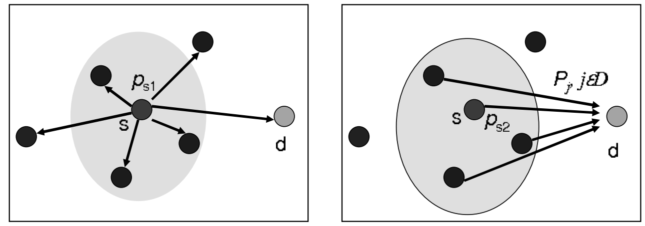

Fig. 1. Decode-and-forward cooperative diversity protocol: two-stage transmission

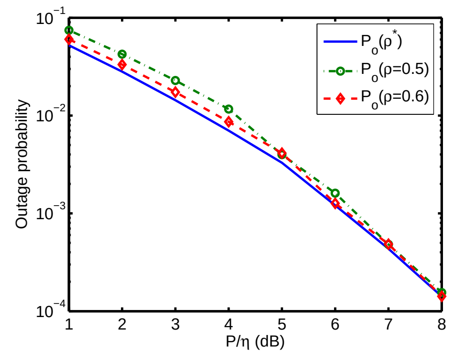

Fig. 2. The outage probability versus P/η performance of the source-destination pair in the relay network with $N = 10$ nodes. The source-destination mean gain is normalized as $m_{s,d} = 1$. The source-relay mean gains are $m_{s,i} = [5.4, 7.8, 0.87, 47.9, 1.4, 1.3, 9.1, 2.9]$. The relay-destination mean gains are $m_{i,d} = [0.1, 0.8, 6.1, 0.5, 0.09, 123.6, 20.5, 0.1]$.

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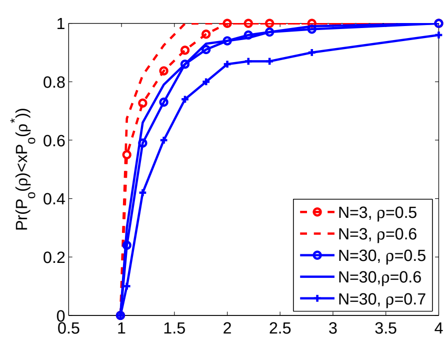

Fig. 3. The probability $\Pr\{P_o(\rho) < xP_o(\rho^*)\}$ in a random network with uniformly distributed nodes. Here N is the number of nodes in the network.

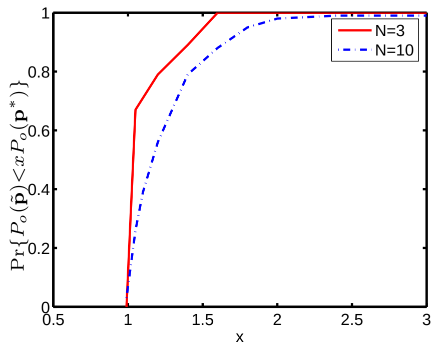

Fig. 4. The probability $\Pr\{P_o(\tilde{\mathbf{p}}) < xP_o(\mathbf{p}^*)\}$ using a cooperative diversity protocol in a network with N uniformly distributed nodes. $\tilde{\mathbf{p}}$: Fixed-Ratio Equal-P with Selection, $\mathbf{p}^*$: optimal numerical solution.

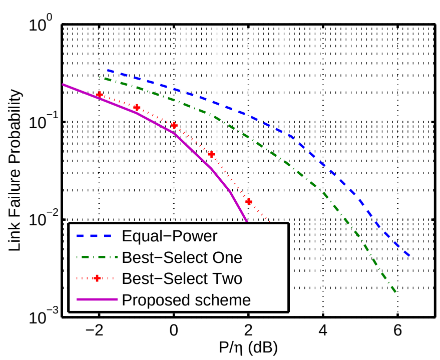

Fig. 5. The link-failure probability of various schemes in a random network with 10 uniformly distributed nodes with target outage probability equal to 0.01.

TABLE I
VALUES OF $b(\rho = 0.6)$ FOR VARIOUS N.

N	3	4	5	6
$b(\rho = 0.6)$	1.667	2.778	4.63	7.71