Reduced-Rank Array Processing for Wireless Communications with Applications to IS-54/IS-136

Alexander M. Haimovich, *Senior Member, IEEE*, Amit Shah, *Member, IEEE*, and Xiao Wu, *Member, IEEE*

***Abstract*—We study the application of the eigencanceler, a reduced rank method based on the eigendecomposition of the estimated covariance matrix, to the wireless communication problem. Simple closed-form bounds are obtained for the bit-error rate of binary phase-shift keying modulation in the presence of cochannel interference in systems using sample matrix inversion (SMI) and the eigencanceler. The application of SMI and the eigencanceler to a flat fading time-division multiple-access system is studied in the context of the IS-54/IS-136 standard. It is shown that adaptive antennas in conjunction with reduced-rank processing can be used to increase capacity of such systems by reducing the frequency reuse factor from 7 to 1.**

***Index Terms*—Cochannel interference, fading channels, optimum combining, space diversity.**

I. INTRODUCTION

WE INVESTIGATE the application of a reduced-rank method referred to as *eigencanceler* to implement an optimum combiner in a flat fading Rayleigh channel with unknown cochannel interference (CCI). It is well known in array processing that a loss in the signal-to-noise-and-interference ratio (SNIR) occurs when the array covariance matrix is estimated from a limited size training set. In work motivated by radar applications, it was found that reduced-rank methods can significantly reduce these losses by providing improved statistical stability [1], [2]. In this letter, we are concerned with the effects of training data on the performance of adaptive arrays for wireless communications in the following cases: 1) an optimum combiner of binary phase-shift keying (BPSK) signals in a flat Rayleigh fading channel in the presence of CCI and 2) a system modeled after the IS-54/IS-136 standard utilizing optimum combining with multiple CCI sources. The emphasis is on the development of simple bounds that express the relation between the bit-error rate (BER), number of training samples, and the number of array elements. These simple expressions allow a better insight into the various mechanisms and tradeoffs affecting performance. The specific contributions of the paper are as follows: 1) develop BER bounds for optimum combining with training data; 2) demonstrate the advantages of reduced-rank processing with the eigencanceler in an IS-54/IS-136 system.

Section II provides the signal model. Optimum combining with training data is analyzed in Section III for full-rank and reduced-rank processing. Performance of IS-54/IS-136 system is investigated in Section IV, and Section V contains conclusions.

Paper approved by K.-C. Chen, the Editor for Wireless Data Communications of the IEEE Communications Society. Manuscript received April 22, 1997; revised July 13, 1998 and May 16, 1999. This paper was presented in part at MILCOM'97, Monterey, CA, November 2–5, 1997, and in part at the ASAP'97 Fifth Annual Adaptive Sensor Array Processing Workshop, Lexington, MA, March 12–14, 1997.

A. M. Haimovich is with the Center for Communications and Signal Processing, Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102 USA (e-mail: haimovic@njit.edu).

A. Shah and X. Wu were with the Center for Communications and Signal Processing, Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102 USA. They are now with Lucent Technologies, Bell Laboratories Innovations, Whippany, NJ 07981 USA (e-mail: amitshah@lucent.com; susanwu@lucent.com).

Publisher Item Identifier S 0090-6778(00)04002-X.

II. SIGNAL MODEL

Consider the uplink of a mobile communication system employing a base station with an N element linear antenna array. After coherent demodulation and matched filtering, the array output sampled at $t = kT$ is represented by the N-dimensional vector

$$\mathbf{u}[k] = \sqrt{P_s}\mathbf{c}_s a[k] + \sum_{i=1}^{L} \sqrt{P_i} z_i[k]\mathbf{c}_i + \mathbf{n}[k] \tag{1}$$

where $\sqrt{P_s}$ and $\sqrt{P_i}$ are, respectively, the signal and CCI amplitudes, the data symbols $a[k]$ are mutually independent and assume values $\in\{-1, 1\}$ with equal probabilities, and $\mathbf{n}[k]$ is the vector of ambient noise. The noise is complex-valued, stationary, zero-mean white Gaussian with covariance matrix $E[\mathbf{nn}^H] = N_o\mathbf{I}$, where the superscript H indicates complex conjugate and transposed. The quantity $z_i[k]$ incorporates information on the ith CCI and is given by [3]

$$z_i[k] = \sum_{m=-\infty}^{\infty} b_i[m]g\left(kT - mT - \tau_i\right) \tag{2}$$

where the CCI symbols $b_i[m]$ assume values $\in\{-1, 1\}$ with equal probability, are mutually independent, and are independent of $a[k]$. The equivalent impulse response of the transmitter, channel, and receiver $g(t)$ has a raised-cosine pulse shape with excess bandwidth β. The random variable τ_i represents the timing phase of the ith CCI and is assumed uniformly distributed over the interval $[0, T]$, where T is the symbol interval. CCI samples $z_i = z_i[k]$ have the following properties: 1) $E[z_i] = 0$ (since $E[b_i[k]] = 0$); 2) due to the independence between interference sources, the random variables z_i are mutually independent (hence, $E[z_i z_j] = 0$ for $i \neq j$); and 3) $E[z_i^2] = 1 - \beta/4$ (see [3]). It is assumed that transmission of the signal takes place in a flat fading Rayleigh channel in the presence of additive white Gaussian noise and CCI. The vectors $\mathbf{c}_s$ and $\mathbf{c}_i$, respectively, represent the channel complex gains for the signal and CCI. The channel vectors are mutually independent, but have identical distributions, which are complex-valued, zero-mean, multivariate Gaussian with independent terms and unity variance, i.e., for example

0090–6778/00$10.00 © 2000 IEEE

$E[\mathbf{c}_s\mathbf{c}_s^H] = \mathbf{I}$. The fading is assumed quasi-static, i.e., channel vectors are fixed over some time interval of interest referred to as frame, but vary independently from frame to frame. It is further assumed that the interference-plus-noise vector $\mathbf{x}[k] = \sum_{i=1}^{L}\sqrt{P_i}z_i[k]\mathbf{c}_i + \mathbf{n}[k]$ has a multivariate Gaussian distribution with zero mean and covariance $E[\mathbf{x}[k]\mathbf{x}[k]^H] = \mathbf{R}$, where $\mathbf{R} = \sum_{i=1}^{L} P_i\mathbf{c}_i\mathbf{c}_i^H + N_o\mathbf{I}$ is the (colored) noise covariance matrix.

III. BER Bound with Training Data

The optimal combiner output is given by $y = \mathbf{w}^H\mathbf{u}$, where the optimal weight vector is $\mathbf{w} = \mathbf{R}^{-1}\mathbf{c}_s$. Scaling of the weight vector has no effect on the output, hence the weight vector can also be expressed $\mathbf{w} = \mathbf{R}^{-1}\mathbf{r}$, where $\mathbf{r} = E[a[k]\mathbf{u}[k]\,|\,\mathbf{c}_s] = \sqrt{P_s}\mathbf{c}_s$ is the cross-correlation vector between the desired signal symbol and the received vector. The output SNR conditioned on the channel $\mathbf{c}_s$ is then given by

$$\mu = ||\mathbf{c}_s||^2\frac{P_s}{N_o} = ||\mathbf{c}_s||^2 h \tag{3}$$

where $||\cdot||^2$ denotes the Euclidean norm. Subsequent to the Gaussian assumption on the CCI, the conditional BER is given by $P(e\,|\,\mu) = Q(\sqrt{2\mu})$, where Q is the Gaussian tail function.

Let $\lambda_i,\ i = 1,\cdots,N$, denote the eigenvalues of the noise covariance matrix $\mathbf{R}$ in descending order, $\lambda_1 \geq \lambda_2 \geq \cdots$. Assume that the L interference sources result in a noise covariance matrix with $r < N$ principal eigenvalues, such that $\lambda_r \gg \lambda_{r+1} = \cdots = \lambda_N = N_o$. It can be shown that the upper bound on the average BER is given by [4]

$$P_e < 0.5(1+h)^{-(N-r)}. \tag{4}$$

In practice, the true noise covariance matrix $\mathbf{R}$ as well as the cross-correlation vector $\mathbf{r}$ are not available and need to be estimated from the data. Assuming the availability of a training sequence of length K, the cross-correlation vector is estimated from the expression $\hat{\mathbf{r}} = 1/K\sum_{k=1}^{K} a[k]\mathbf{u}[k]$. The estimated cross correlation can then be used to estimate the interference vector $\hat{\mathbf{x}}[k] = \mathbf{u}[k] - a[k]\hat{\mathbf{r}}$. The estimated interference and noise covariance matrix is given by $\hat{\mathbf{R}} = 1/K\ \sum_{k=1}^{K}\hat{\mathbf{x}}[k]\hat{\mathbf{x}}[k]^H$. We assume that $\mathbf{r} = \hat{\mathbf{r}}$ and focus on the effect of estimating the covariance matrix. The goal is to investigate the effect of $\hat{\mathbf{R}}$ on expression (4). In the following, it is assumed that the covariance matrix is estimated from a training set $\hat{\mathbf{x}}(k), k = 1,\cdots,K$. The training set is assumed to have the same statistics as the interference-plus-noise during normal operation, i.e., $\hat{\mathbf{x}}(k)$ has a multivariate Gaussian distribution with zero mean and covariance $\mathbf{R}$. We define the conditioned signal-to-noise and interference ratio (CSNR) ρ as the ratio of the SNIR when a specified weight vector $\mathbf{w}$ is used, to the optimal SNIR $\mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}$. The CSNR is then given by

$$\rho = \frac{|\mathbf{w}^H\mathbf{r}|^2}{\mathbf{w}^H\mathbf{R}\mathbf{w}}\frac{1}{\mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}}. \tag{5}$$

The quantity ρ is a random variable that takes on values $0 \leq \rho \leq 1$ ($\rho = 1$ when $\mathbf{w} = \mathbf{R}^{-1}\mathbf{r}$) and with density $f_\rho(\rho)$. The BER conditioned on both the channel and CSNR is $P(e\,|\,\rho,\mu) = Q(\sqrt{2\rho\mu})$. The performance penalty is made evident by the inequality $Q(\sqrt{2\rho\mu}) \geq Q(\sqrt{2\mu})$. The mean BER can be found by averaging over both the CSNR and fading

$$P_e = \int_0^\infty\int_0^1 P(e\,|\,\rho,\mu)f_\rho(\rho)f_\mu(\mu)\,d\mu\,d\rho \tag{6}$$

where $f_\mu(\mu)$ is the density of μ [4]. We proceed to analyze the BER for two different methods of deriving the weight vector $\mathbf{w}$. The *sample matrix inversion* (SMI) weight vector is given by $\mathbf{w} = \hat{\mathbf{R}}^{-1}\mathbf{r}$. The CSNR becomes

$$\rho = \frac{\left(\mathbf{r}^H\hat{\mathbf{R}}^{-1}\mathbf{r}\right)^2}{\mathbf{r}^H\hat{\mathbf{R}}^{-1}\mathbf{R}\hat{\mathbf{R}}^{-1}\mathbf{r}}\frac{1}{\mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}}. \tag{7}$$

The density of ρ for the SMI has been found in the classical paper [5] and is given by the beta function

$$f_\rho(\rho) = \frac{\Gamma(K+1)}{\Gamma(N-1)\Gamma(K+2-N)}(1-\rho)^{N-2}\rho^{K+1-N}. \tag{8}$$

We first evaluate $P(e\,|\,\mu) = \int_0^1 P(e\,|\,\rho,\mu)f_\rho(\rho)d\rho$. Using $Q(\sqrt{2\rho\mu}) < 0.5e^{-\rho\mu}$ and [6, eq. (13.2.1)], it can be shown that

$$P(e\,|\,\mu) < 0.5{}_1F_1(K+2-N, K+1, -\mu) \tag{9}$$

where ${}_1F_1$ is the confluent hypergeometric function. This expression is dependent on the number of samples K used to estimate the covariance matrix. Using (9) and the Kummer transformation ${}_1F_1(a,b,-\mu) = e_1^{-\mu}F_1(b-a,b,\mu)$ [9, eq. (13.2.1)], we obtain the expression

$$P_e < 0.5\Gamma^{-1}(N-r)h^{-(N-r)} \times \int_0^\infty e^{-\mu(1+1/h)}\mu^{-(N-r-1)}{}_1F_1(N-1,K+1;\mu)\,d\mu \tag{10}$$

which is recognized as a Laplace transform. Applying relation [7, p. 510], and after some algebra, this transform is evaluated as

$$P_e < 0.5(1+h)^{-(N-r)} \times {}_2F_1(N-1, N-r, K+1; h/(1+h)) \tag{11}$$

where ${}_2F_1$ is the Gauss hypergeometric function. This result is consistent with (4), since as the number of training samples $K \to \infty$, $\lim_{K\to\infty}\ {}_2F_1\left(N-1, N-r, K+1; h/(1+h)\right) = 1$ [8, p. 3], and (11) reverts to (4). The effect of finite K can be assessed by evaluating the function ${}_2F_1$. This function is available in software packages such as Mathematica, or it can be approximated by a series. Better insight into the effect of training is obtained by applying the asymptotic expansion of ${}_2F_1$, [9, p. 238]

$${}_2F_1(a,b,c+m;z) = 1 + \frac{abz}{m} + o\left(m^{-2}\right) \tag{12}$$

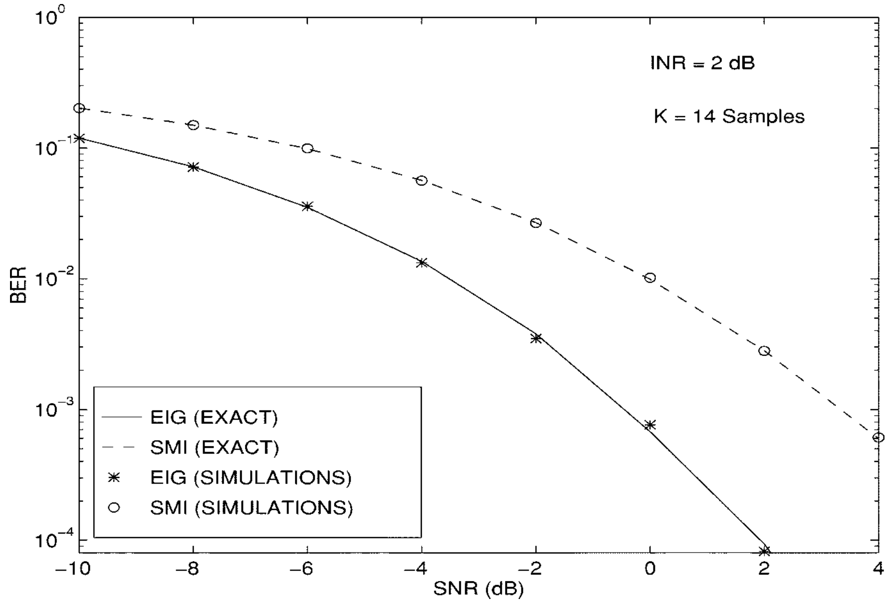

Fig. 1. Comparison of theoretical expressions and simulation results.

where $m > 0$. For the problem at hand, we have

$$ {}_2F_1(N-1, N-r, K+1; h/(1+h)) = 1 + \frac{(N-1)(N-r)}{K}\frac{h}{1+h} + o\left(K^{-2}\right). \quad (13) $$

Substitution of (13) in (11) yields

$$ P_e < 0.5(1+h)^{-(N-r)} \times \left(1 + \frac{(N-1)(N-r)}{K}\frac{h}{1+h} + o\left(K^{-2}\right)\right). \quad (14) $$

The significance of this relation is that for a given interference rank r, the BER increases *quadratically* with the number of degrees of freedom.

The SMI performance is compared with that of the eigencanceler. The eigencanceler is derived from the eigendecomposition of $\hat{\mathbf{R}}$, and its weight vector when the interference rank is r, is given by $\mathbf{w} = (\mathbf{I} - \hat{\mathbf{Q}}_1\hat{\mathbf{Q}}_1^H)\mathbf{r}$ [10], where the columns of $\hat{\mathbf{Q}}_1$ are the r eigenvectors associated with the principal eigenvalues of the sample covariance matrix $\hat{\mathbf{R}}$. It can be shown that for large interference-to-noise ratio (INR), the density function of the CSNR ρ for the eigencanceler is given by [11]

$$ f_\rho(\rho) = \Gamma^{-1}(r)K^r e^{-K(1-\rho)}(1-\rho)^{r-1}. \quad (15) $$

Using (15) in (6), we evaluate the conditional BER bound $P(e\,|\,\mu) < 0.5\int_0^1 e^{-\mu} f_\rho(\rho)d\rho$. We make the assumption that the values of the output SNR μ for which the density function is nonvanishing are such that $\mu < K$. With this assumption, the previous integral can be evaluated as

$$ P(e\,|\,\mu_0) < 0.5e^{-\mu}\left(1 - \frac{\mu}{K}\right)^{-r}\left[1 - \frac{\Gamma(r, K-\mu)}{\Gamma(r)}\right] \quad (16) $$

where $\Gamma(a, x) = \int_x^\infty e^{-t}t^{a-1}\,dt$. The series expansion of the terms in the parenthesis yields

$$ P(e\,|\,\mu_0) < 0.5e^{-\mu}\left[1 + \frac{\mu r}{K} + o\left(\frac{\mu}{K}\right)^2\right] \times \left[1 - \frac{e^{-(K-\mu)}(K-\mu)^{r-1}}{\Gamma(r)}\left(1 + o\left(K-\mu\right)^{-1}\right)\right]. \quad (17) $$

Keeping only the first order terms of K, and assuming $K \gg \mu$, the conditional BER is bound by

$$ P(e\,|\,\mu) < 0.5e^{-\mu}\left(1 + \frac{\mu r}{K}\right). \quad (18) $$

Substitution of (18) in (6) yields

$$ P_e < 0.5(1+h)^{-(N-r)}\left(1 + \frac{r(N-r)}{K}\frac{h}{1+h} + o(K^{-2})\right). \quad (19) $$

For large K, this expression can be approximated by the first two terms in the parenthesis. The eigencanceler's advantage over SMI is evident in the *linear* rather than *quadratic* increase in BER as a function of the degrees of freedom N.

A. Numerical Results

The theory developed above is illustrated through numerical results obtained from simulations. The simulation model consisted of two sources: a desired signal with specified SNR and an interference with INR $= 2$ dB. The modulation was BPSK, the number of antenna elements $N = 9$, and the number of training samples used for estimating the covariance matrix $K = 14$ (this conforms with the training specified by the IS-54/IS-136 time-division multiple-access (TDMA) standards). The fit between simulation data and theory is illustrated in Fig. 1. The simulation results represent the error count of 280 000 Monte Carlo runs. The channel was assumed fixed over the processing interval, but was varied randomly from run to run. Theoretical

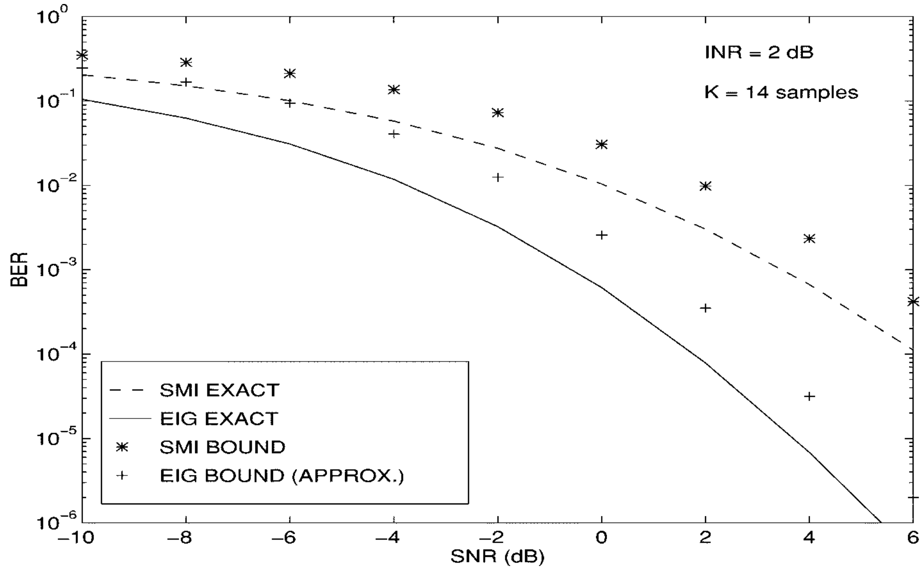

Fig. 2. Bound and exact BER for SMI and eigencanceler with $N = 9$ elements and $K = 14$ training samples.

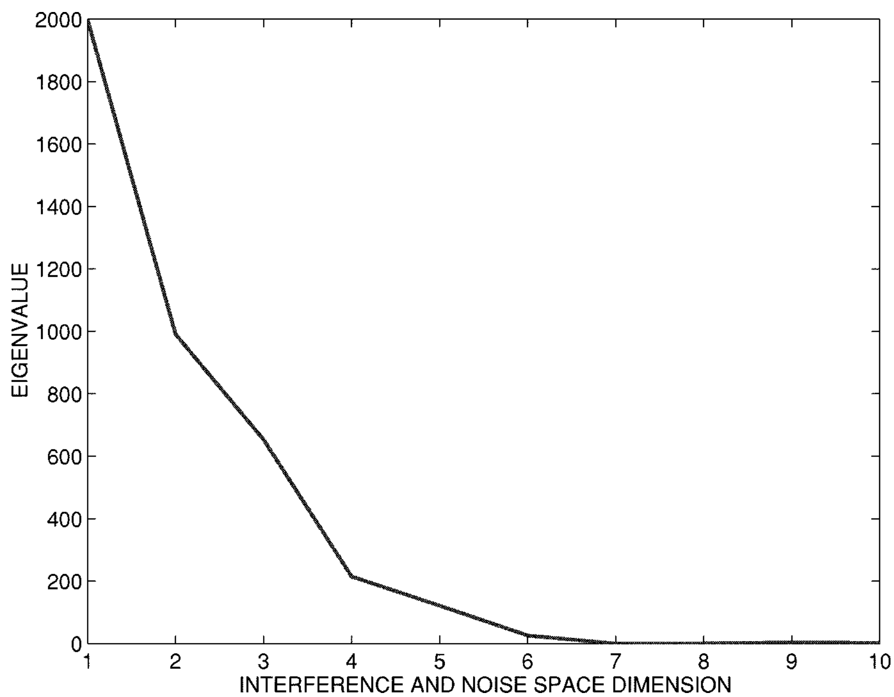

Fig. 3. Eigenvalues of interference plus noise covariance matrix.

curves were generated by evaluating the integral in (6) numerically, using the SMI and eigencanceler probability density expressions in (8) and (15), respectively. A good fit is observed between theory and simulations.

The error bounds and the exact BER expressions are compared in Fig. 2 as a function of the SNR for the case of $K = 14$ training symbols. The approximations are based on (14) and (19), respectively. An error of about 1 dB is observed between the bound and the exact values. The bound error has the following three sources: 1) error due to the approximation of the Gaussian tail function with the quantity $0.5e^{-\mu}$; (2) error due to the assumption that INR is large; and (3) error due to the assumption that K is large. The Gaussian tail approximation is the main source of the bound error. The INR provides negligible error. This is explained by the well-known fact that the output of an adaptive array with sufficient degrees of freedom, and optimum combining is not very sensitive to the interference power.

IV. Application to IS-54/IS-136

The IS-54/IS-136 TDMA system uses $\pi/4$-shifted differential quadrature phase-shift keying (DQPSK) modulation. The probability of symbol error for $\pi/4$-DQPSK is given by [12]

$$P_e(\mu) = \frac{1}{4\pi\sqrt{2}} \int_0^{2\pi} \frac{1}{1 - \dfrac{\cos t}{\sqrt{2}}} \exp\left[-\mu\left(1 - \frac{\cos t}{\sqrt{2}}\right)\right] dt. \tag{20}$$

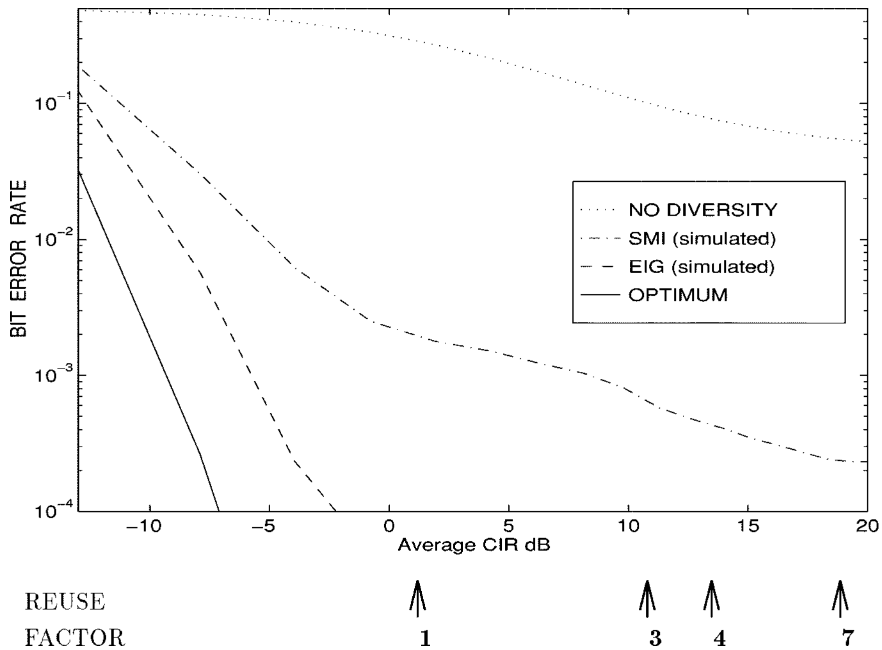

Fig. 4. Average BER versus CIR. Arrows indicate the corresponding reuse factor.

A typical IS-54 TDMA frame contains $K = 14$ synchronization symbols that can also be used for array training. Performance of a TDMA system with an antenna array controlled by the SMI or eigencanceler methods was evaluated by simulation. The signal environment was modeled by three cell layers. With all channels fully occupied, interference was provided by 6 CCI sources from the first layer, 12 CCI sources from the second layer, and 18 CCI sources from the third layer, while ignoring interferences from other outlying layers. CCI sources were assumed to be the base stations of the surrounding cells. The normalized eigenvalue distribution of a sample interference and noise covariance matrix for an $N = 9$ element antenna is shown in Fig. 3. Note that most of the interference power is concentrated in the six largest eigenvalues, suggesting the use of a reduced-rank method such as the eigencanceler. Fig. 4 shows the average BER versus the carrier-to-interference ratio (CIR) for an adaptive array with $E_b/N_o = 10$ dB. The capacity of a TDMA system is expressed in terms of its reuse factor. Current 2G standards stipulate a frequency reuse factor of 7. The curves shown in the figure are averages of 1000 runs. Each run consisted of estimation of the array covariance matrix $\hat{\mathbf{R}}$ using $K = 14$ training samples, and a randomly chosen channel vector $\mathbf{c}_s$. Reuse factors are marked by arrows. For BER $= 10^{-3}$, SMI can be applied with reuse factor 3, while the eigencanceler provides higher capacity corresponding to a reuse factor of 1.

V. Conclusion

This letter considered reduced-rank antenna arrays for wireless communications. The paper focused on the eigencanceler, but other reduced-rank methods can be applied. Simple analytical expressions were obtained for the BER bound for the case of BPSK modulation and the presence of colored Gaussian CCI. The performance of a TDMA system as specified by the IS-54/IS-136 standards was studied by simulation. It was shown that reduced-rank processing at the base station can increase the capacity of TDMA systems by reducing the frequency reuse factor from 7 to 1.

References

[1] I. P. Kirsteins and D. W. Tufts, “Adaptive detection using low rank approximation to data matrix,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 30, pp. 55–67, Jan. 1994.

[2] A. Haimovich, “The eigencanceler: Adaptive radar by eigenanalysis methods,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, pp. 532–542, Apr. 1996.

[3] N. C. Beaulieu and A. A. Abu-Dayya, “Bandwidth efficient QPSK in cochannel interference and fading,” *IEEE Trans. Commun.*, vol. 43, pp. 2464–2474, Sept. 1995.

[4] A. Haimovich and A. Shah, “The performance of space-time processing for suppressing narrowband interference in CDMA communications,” *Wireless Pers. Commun.*, vol. 8, no. 7, pp. 233–255, 1998.

[5] I. S. Reed *et al.*, “Rapid convergence rates in adaptive arrays,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-10, pp. 853–863, Nov. 1974.

[6] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. Washington, DC: National Bureau of Standards, 1972. 10th printing.

[7] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*. Philadelphia, PA: Gordon & Breach Science, 1992.

[8] W. Bailey, *Generalized Hypergeometric Series*. New York: Stechert-Hafner, 1964.

[9] Y. Luke, *The Special Functions and Their Approximations*. New York: Academic, 1969, vol. 1.

[10] X. Wu and A. Haimovich, “Adaptive arrays for increased performance in mobile communications,” in *Proc. 6th Int. Symp. Personal, Indoor and Mobile Radio Communications (PIMRC’95)*, Toronto, Canada, Sept. 1995, pp. 653–657.

[11] A. Haimovich, “Asymptotic distribution of the conditioned signal-to-noise ratio in an eigenanalysis-based adaptive array,” *IEEE Trans. Aerosp. Electron. Syst.*, pp. 988–996, July 1997.

[12] F. Adachi, “Postdetection optimal diversity combiner for DPSK differential detection,” *IEEE Trans. Veh. Technol.*, vol. 42, pp. 326–337, Aug. 1993.