ADAPTIVE ANTENNA ARRAYS USING EIGENVECTOR METHODS

Alexander Haimovich (*)
Yeheskel Bar-Ness
Center for Communications
and
Signal Processing Research
Electrical Engineering Department
New Jersey Institute of Technology
Newark, NJ 07102

This paper considers the application of eigenvector techniques to the solution of interference cancelation problems. The conventional Minimum Variance and Maximum Signal-to-Noise Ratio criteria are extended to a case where the eigen-structure of the array correlation matrix is used to cancel directional interference sources. It is shown that while the method has good interference cancelation properties it results in higher pattern distortion relative to conventional arrays.

Some of the more common criteria that have been put forth to determine the weighing of the array sensors are the Minimum Noise Variance and the Maximum Signal to Noise Ratio [1]. It has been shown that those techniques have the property that cancelation of directional interferences is in direct proportion with the source strength relative to the background noise [2]. In contrast, techniques based on separate modeling of the background noise and directional sources, derived from the eigen-structure of the array correlation matrix, have been shown to be asymptotically invariant to the background noise and have been applied extensively to spectral estimation and direction finding problems [3,4,5].In interference cancelation applications it is necessary to separate between the desired signal and the unwanted sources. Duvall [6] has used a prefiltered output of the array to obtain the optimal signal to noise beamformer weights and then applied those weights directly at the array outputs for interference cancelation. Eigenvector techniques can be used to solve interference cancelation problems. Citron and Kailath [7] used the prefiltered of the array in conjunction with an eigenvector method to synthesize the weights. In this paper we intend to extend the conventional Minimum Variance and Maximum Signal to Noise Ratio criteria and apply eigenvector methods to find optimal weights for each canceler.

Our model comprises of an N+1 element array with even spacings of $\frac{1}{2}\lambda$ and L interference sources. Signals are assumed narrowband and are represented by their complex envelopes. The background noise is assumed to be independent from sensor to sensor and Gaussian with zero mean and σ_ν^2 variance. Our approach for deriving the weight vector is based on optimizing criteria similar to those in conventional arrays, but restricts the solution to the subspace orthogonal to the interference sources. To this end, first, the observed data vector,x from an N + 1 element array is spatially filtered to remove the desired signal (with assumed known direction of arrival), to preclude its cancelation with the interferences. Refer to Fig.1 for a diagram of the system. Let the filter be represented by the matrix F, with $Fd_{,,} = 0$ where $d_{,,}$ is the direction vector of the desired signal. Then, the correlation matrix $R = E(\tilde{x}\tilde{x}^H)$ is estimated from the transformed data $\tilde{x} = Fx$. $[F]$ has full rank,and nullity equal to unity hence x is N-dimensional. The correlation matrix is given by

$$R = FDPD^H F^H + \sigma_\nu^2 G \tag{1}$$

where $G = FF^H$, D is a matrix whose columns , $\bar{d}_i = \frac{1}{\sqrt{N}}(1,\ldots,e^{j\psi_i},\ldots,e^{jN\psi_i})^T$ are the interference sources (normalized) position vectors, $\psi_i = \pi \sin\theta_i$ and P is the interferences correlation matrix and with those assumed uncorrelated it is diagonal. We

CH2563-5/88/0000-0980$1.00 © 1988 IEEE

define the interference subspace as the span of the columns of D and the noise subspace as the orthogonal complement. Without loss of generality the desired signal is assumed at broadside and F is defined to subtract adjacent elements, then a vector orthogonal to a filtered interference position vector Fd_i, will be also orthogonal to the original position vector $d_i = \frac{1}{\sqrt{N}}(1, \ldots, e^{j\psi_i}, \ldots, e^{j(N-1)\psi_i})$ defined in the N element array, because $Fd_i = (1 - e^{j\psi_i})d_i$. Since $FDPD^H F^H$ has rank L and P is strictly positive definite there are $N - L$ eigenvectors of the pencil matrix $[R; G]$ for which $D^H F^H e_j = 0$ or eqivalently $Fd_i \perp e^j, i = 1, \ldots, L$ and $j = L + 1, \ldots, N$. Hence a weight vector w defined in the noise subspace will completely null the interference sources given infinite observation time. Also note for all e_j in the noise subspace and all e_i in the complementary subspace $e_i^H G e_j = 0$ [8], namely, the noise subspace is G-orthogonal to the L principal eigenvectors of the pencil.

For the Minimum Noise Variance approach the problem is formulated as the minimization of the array output noise power $w^H w$ subject to the mainbeam constraint $w^H d_o = 1$ and the orthogonality constraints set $w^H GE = 0$, where d_o is the desired signal position vector in the N element array and E is a matrix whose columns are the L principal eigenvectors of the pencil $[R; G]$. It is possible to use Lagrange multipliers to obtain the solution but we choose to apply the singular value decomposition.

$$svd(GE) = QSZ^H = [Q_1|Q_2][\frac{S}{o}]Z^H \qquad (2)$$

where $Q = [Q_1|Q_2]$ is $N * N$ unitary, Q_1 and Q_2 have orthogonal L and $(N - L)$ columns respectively, Z is $L * L$ unitary and S is diagonal of rank L [9].Define

$$u = Q^H w = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{(N-L)*1}^{L*1} \qquad (3)$$

Then using (3) in (2) the orthogonality constraint

$$w^H GE = [u_1|u_2]Q_H Q[\frac{S}{o}]Z^H = 0 \qquad (4)$$

is equivalent to $u_1^H S = 0$ or $u_1 = 0$. Formulating now the optimization problem in terms of the vector u with $u_1 = 0$ we have

$$\min_{u_2} u_2^H Q_2^H Q_2^H u_2 \ \ subject\ to\ u_2^H Q_2^H d_o = 1 \qquad (5)$$

The solution is

$$u_2 = \frac{Q_2^H d_o}{\|Q_2^H d_o\|^2}$$

and the weight vector is

$$w_{EV} = [Q_1|Q2] \begin{bmatrix} 0 \\ u_2 \end{bmatrix} = \frac{Q_2 Q_2^H d_o}{\|Q_2^H d_o\|^2} \qquad (6)$$

namely a scaled projection of the desired signal vector onto the noise subspace. Application of the Maximum Signal-to-Ratio criterion leads to the following optmization problem

$$\max_{w} \frac{w^H d_o d_o^H w}{w^H nn^H w} \ subject\ to\ w^H GE = 0 \qquad (7)$$

where n is a combined vector of interferences and noise. However, the constraint implies that the weight vector will be orthogonal to the interference sources hence, we can simplify the formulation to include only background noise namely

$$\max \frac{w^H d_o d_o^H w}{w^H w} \text{ subject to}$$

$$w^H G H = 0 \tag{8}$$

Using again a technique based on the singular value decomposition of GE it can be shown that the optimal weight vector is

$$w_{ES} = Q_2 Q_2^H d_o \tag{9}$$

Thus the weight vectors yielded by the two methods differ only by a constant factor. In essence, the beamformer uses a number of degrees of freedom equal to the number of directional sources, to train nulls on those sources, while the remaining degrees of freedom are employed to beamform in the direction of the desired signal and reduce the background noise.

The performance of the eigen-beamformer is illustrated in Fig.2 that shows the sensitivity pattern of an array with $N + 1 = 11$ elements, in a simulated environment of three interference sources at 15, 18 and 25 degrees and of strength 30dB above the background noise. The desired signal is 10dB above the noise level. The array correlation matrix was estimated as the average of 30 sample correlations $x_k x_k^H$. For comparison the pattern of a conventional Minimum Variance array is presented in the same figure. It is evident that eigen-beamformer does a better job in canceling the interferences.

An analytical analysis of the performance of the proposed beamformer is further developed for the case of one interference. In this context,the beamformer interference cancelation capabilities and the effect of adaptation on the sidelobes level and their relation to various array and environment parameters are studied. A comparison with the conventional Minimum Variance array reveals that the improved interference cancelation capabilities are countered by higher sidelobes. Specifically, while the eigen-beamformer has theoretically, zero response in the direction of an interference source, it can shown that the sensitivity for the conventional array is finite and equal to $g_I = \varphi\rho(1 - \gamma)$, where φ is a scaling factor, $\rho = d_o^H d_i$ depends on the relative separation between the desired signal and the interference, and γ is a function of the interference to background noise ratio, $\gamma \rightarrow 1$ when that ratio tends to infinity. The amount of distortion or change in sidelobes , may be indicated by $w^H w$, the normalised output noise power. For any beamformer in the abscence of interference $w^H w = 1$. The noise power for the array under consideration can be calculated using (6)

$$w_{EV}^H w_{EV} = \frac{1}{\| Q_2^H d_o \|^2} \tag{10}$$

Using some of the quantities defined in (2) we show that $w_{EV}^H w_{EV} \geq 1$,indeed $1 = d_o^H d_o = d_o^H Q Q^H d_o \geq d_o^H Q_2 Q_2^H d_o = \frac{1}{w_{EV}^H w_{EV}}$. If a single interference is present it can be shown that (10) transforms into

$$w_{EV}^H w_{EV} = 1 + \frac{\|\rho\|^2}{(1 - \|\rho\|)^2} \tag{11}$$

and that the minimum variance array yields

$$w_{MV}^H w_{MV} = 1 + \frac{\gamma 2\|\rho\|^2(1 - \|\rho\|^2)}{(1 - \gamma^2\|\rho\|^2)^2} \tag{12}$$

Since $\gamma \leq 1$, $w_{EV}^H w_{EV} \geq w_{MV}^H w_{MV} \geq 1$ namely the presence of interference adaptation causes an increase in the background noise contribution that being larger for the eigenvector array. Also notice that the noise power depends on the separation parameter ρ. Fig. 3 shows the noise power as a function of the interference angle for an 11 element eigen-beamformer. The bearing of the desired signal is assumed at broadside. As the interference source closes on the desired signal, pattern distortions are accentuated, weights have to be set larger to maintain signal level and more background noise is taken into the system.

References

[1] R. A. Monsingo and T. W. Miller, *Introduction to Adaptive Arrays*, J. Wiley 1980.

[2] J. E. Hudson, *Adaptive Array Principles*, Peter Peregrinus1981.

[3] V. F. Pisarenko, "The Retrival of Harmonics from a Covariance Function", Geophy. J. R. Astr. Soc. , No. 33, 1973.

[4] R. O. Schmidt,*A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation*, PhD Thesis, Stanford University, 1981.

[5] D. H. Johnson, "The application of spectral estimation methods to bearing estimation problems", Proc. IEEE, vol.70,pp. 1018-1028, Sept. 1982.

[6] K. M. Duvall, *Signal Cancelation in Adaptive Arrays: The Phenomenon and a Remedy*, PhD Thesis, Stanford University,1983.

[7] T. K. Citron ans T. Kailath, "Eigenvector Methods and Beamforming - A First Approach", 17-th Asilomar Conference on Circuit Circuit Systems, 1983

[8] B. N. Parlett, *The Symmetric Eigenvalue Problem*, Prentice Hall 1980

[9] G. H. Golub and C. F. Van Loan, *Matrix Computations*, The Johns Hopkin University Press, 1985

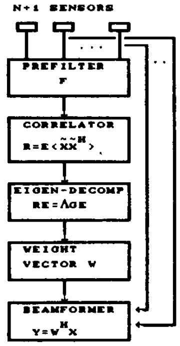

FIG. 1 - SYSTEM DIAGRAM

Fig.2 - Sensitivity patterns

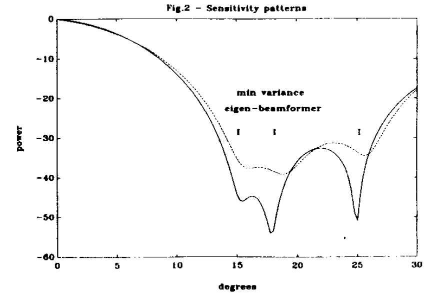

Fig.3 - Eigen-beamformer noise power