Multiple-Symbol Differential Detection for Space-Time Block Codes

Chunjun Gao and A.M. Haimovich
and Debang Lao[1]
CCSPR, Department of ECE
New Jersey Institute of Technology
Newark, New Jersey 07102, USA
e-mail: cxg0333@oak.njit.edu,
haimovic@njit.edu

Abstract — **The loss of approximately 3 dB signal to noise ratio is always paid with conventional differential detection for space-time block code compared to the related coherent detection. In this paper, a multiple-symbol differential detection (MSDD) technique is proposed for space-time block codes (STBC), which greatly reduces the performance loss by extending the observation interval for decoding. The technique uses maximum likelihood block sequence detection instead of traditional block-by-block detection. The generalized decision metric for an observation interval of N blocks is derived. It is shown that for a moderate number of blocks, MSDD provides 1.5 dB performance improvement over conventional differental detection.**

I. INTRODUCTION

Space-time coding explores the utilization of multiple transmit antennas to improve the spectral efficiency and the performance over fading channels of wireless communications systems. Research on space-time coding has made a significant progress in recent years.

Tarokh [1] first proposed a space-time trellis coding scheme on a Rayleigh fading channel. The scheme was shown to provide a good trade-off between constellation size, data rate, diversity advantage and trellis complexity. A much simpler space-time block code (STBC) scheme, which provides full diversity advantage, but is not optimized for coding gain, was proposed by Alamouti [2]. Alamouti's scheme for two transmit antennas supports a maximum likelihood detection scheme based only on linear processing at the receiver. Tarokh [3] generalized the scheme to multiple transmit antennas (three, four or eight), to obtain full diversity for real-valued constellations. For complex constellations, full diversity can be obtained only at the cost of reduced coding rate. Due to its relative simplicity of implementation, Alamouti's scheme [2] has been adopted by 3G standards, such as W-CDMA and cdma2000.

The design of the space-time codes mentioned above is based on the assumption that perfect estimation of channel state information (CSI) is available at the receiver. This is reasonable when channel changes slowly compared with the symbol rates, since the transmitter can send training symbols, which enable the receiver to estimate the channel accurately. For cases when accurate channel estimation is not possible or the effort associated with channel estimation is to be avoided, it is of interest to develop techniques, which do not require CSI.

Tarokh and Jafarkhani [4] first came up with a differential STBC scheme for a slow Rayleigh fading channel with two transmit antennas. The scheme employs block-by-block detection, in which neither transmitter nor receiver know the CSI. The same authors generalized the differential detection for STBC to more than two transmit antennas [5]. Similar to single antenna channels, a loss of approximately 3 dB is always paid for this differential scheme compared to the related coherent scheme. Hochwald and Seldom [6] proposed a new class of differential modulation schemes for multiple transmit antennas based on unitary space-time modulation. At the same time, a related differential modulation scheme was proposed by Hughes in terms of group codes [7]. These schemes utilize constellations of unitary matrices or group codes to achieve full transmit diversity without knowledge of CSI, but with a loss of about 3 dB in performance. While Tarokh's scheme has properties similar to the 2×2 unitary matrixes in [6] or 2×2 group codes in [7], it outperforms both.

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Multiple-symbol differential detection (MSDD) was first presented for one transmit antenna over the additive white Gaussian noise (AWGN) channel by Divsalar and Simon [8]. By extending the observation interval to more than two symbols, the technique makes use of maximum likelihood sequence detection instead of symbol-by-symbol detection as in conventional differential detection. The performance of MSDD depends on the number of observation symbols. For a moderate number of symbols, MSDD bridges the performance gap between non-coherent and coherent communications. In [9] and [10], MSDD is applied to the flat Rayleigh fading channel.

Motivated by MSDD, Fan [11] extended the observation interval of differential STBC [4] to 3 blocks. As a result, a performance improvement of about 0.5 dB for BPSK message was demonstrated. In this paper, we generalize differential STBC for larger observation intervals. The decision metric for N blocks of observation interval is derived. It is shown that for an observation interval of $N = 8$ blocks there is a gain of 1.5 dB over differential STBC with two blocks and a gain of 1.0 dB over STBC with three blocks.

The rest of this paper is organized as follows: The system model is set up in Section II. In section III, the generalized decision metric is derived for differential STBC with an observation interval of N blocks over the slow Rayleigh fading channel. Special cases for $N = 2$ [4] and $N = 3$ [11] are obtained as special cases of the generalized decision metric. Numerical results are given in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

A. Signal Model

Consider a wireless communication system operating over a slow, flat Rayleigh fading channel in which space-time block coded symbols are sent from two transmit antennas and received by P receive antennas. Transmit diversity is achieved by sending a symbol through a transmit antenna followed by the its complex conjugate/negative being sent by the other antenna at the next time slot. Consequently, we divide the time interval for transmitting the k^{th} symbol into two time slots. The signal received at antenna j, $1 \le j \le P$, at time slot t $(t = 1, 2)$ is given by

$$y_{j,k}^{(t)} = \sqrt{E_s}\sum_{i=1}^{2} h_{i,j} d_{i,k}^{(t)} + \eta_{j,k}^{(t)}, \qquad (1)$$

where $h_{i,j}$ is the Gaussian complex fading path gain from transmit antenna i to receive antenna j. Paths gains are modeled as quasi-static (unchanged over a frame of length L), samples of independent, complex-valued Gaussian random variables with zero-mean and variance $1/2$ per dimension. Path gains vary independently from frame to frame. Noise samples $\eta_{j,k}^{(t)}$ are modeled as independent, zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension; $d_{i,k}$ is the k^{th} transmitted symbol from antenna i; E_s is the total symbol energy on two transmit antennas at time slot t $(t = 1, 2)$.

When the channel state information is known, the optimal receiver computes the decision metric [3]

$$\sum_{t=1}^{2}\sum_{j=1}^{P}\left|y_{j,k}^{(t)} - \sqrt{E_s}\sum_{i=1}^{2} h_{i,j} d_{i,k}^{(t)}\right|^2 \tag{2}$$

over all symbols $d_{i,k}^{(1)}$ $(k = 1, 2, ..., L)$ at time slot 1

$$c_{1,1}, c_{2,1}, c_{1,2}, c_{2,2}, \ldots, c_{1,L}, c_{2,L} \tag{3}$$

and all symbols $d_{i,k}^{(2)}$ $(k = 1, 2, ..., L)$ at time slot 2

$$-c_{2,1}^*, c_{1,1}^*, -c_{2,2}^*, c_{1,2}^*, \ldots, -c_{2,L}^*, c_{1,L}^* \tag{4}$$

to decide in favor of the sequences that minimizes the metric.

In this paper, we are concerned with detectors for situations when the channel is unknown.

B. Differential Encoding of STBC

The differential encoding scheme used in this paper is the one recently proposed by Tarokh and Jafarkhani [4] and it is based on the Alamouti transmit diversity scheme [2]. The scheme is presented here with a modified notation motivated by multiple symbol differential detection.

Define the message matrix $\mathbf{S}_k$,

$$\mathbf{S}_k = \begin{bmatrix} s_{1,k} & s_{2,k} \\ -s_{2,k}^* & s_{1,k}^* \end{bmatrix}, \tag{5}$$

where the superscript "*" denotes complex conjugation and the symbols $s_{1,k}$, $s_{2,k}$ belong to an M-PSK constellation $\mathcal{F}$,

$$\mathcal{F} = \left\{ \frac{e^{j2\pi m/(M-1)}}{\sqrt{2}} \mid m = 1, 2, ..., M \right\}, \tag{6}$$

where $j = \sqrt{-1}$.

Note that the vectors $[s_{1,k}\ ,\ s_{2,k}]$ and $[-s_{2,k}^*\ \ s_{1,k}^*]$ have unit length and are orthogonal to each other. It follows that

$$\mathbf{S}_k \mathbf{S}_k^H = \mathbf{I}_2, \tag{7}$$

where the superscript "H" denotes Hermitian operation (transpose conjugation) and $\mathbf{I}_2$ is the 2×2 identity matrix. Transmission of $\mathbf{S}_k$ already incorporates transmit diversity [2]. We wish to differentially encode the message $\mathbf{S}_k$.

The message $\mathbf{S}_k$ can be differentially encoded in a way similar to standard single-antenna DPSK [12]. To initialize transmission, the transmitter sends a message matrix $\mathbf{C}_0$, which does not carry any message. Let

$$\mathbf{C}_0 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{8}$$

Now, define the code matrix $\mathbf{C}_k$,

$$\mathbf{C}_k = \begin{bmatrix} c_{1,k} & c_{2,k} \\ -c_{2,k}^* & c_{1,k}^* \end{bmatrix} \tag{9}$$

The differential STBC scheme consists of sending the code matrix $\mathbf{C}_k$ rather than transmitting the message matrix $\mathbf{S}_k$ directly. The differentially encoded message $\mathbf{C}_k$ at time k is obtained by applying the previous code matrix $\mathbf{C}_{k-1}$ to the current message $\mathbf{S}_k$:

$$\mathbf{C}_k = \mathbf{S}_k \mathbf{C}_{k-1}. \tag{10}$$

This process is initialized with $\mathbf{C}_1 = \mathbf{S}_1 \mathbf{C}_0$. These relations are similar to single-antenna DPSK. The only difference is, that the variables here are matrices rather than scalars.

The relation in (10) is consistent with the encoding algorithm in [4]:

$$\begin{bmatrix} c_{1,k} \\ c_{2,k} \end{bmatrix} = s_{1,k}\begin{bmatrix} c_{1,k-1} \\ c_{2,k-1} \end{bmatrix} + s_{2,k}\begin{bmatrix} -c_{2,k-1}^* \\ c_{1,k-1}^* \end{bmatrix} \tag{11}$$

and

$$\begin{bmatrix} -c_{2,k}^* \\ c_{1,k}^* \end{bmatrix} = -s_{2,k}^*\begin{bmatrix} c_{1,k-1} \\ c_{2,k-1} \end{bmatrix} + s_{1,k}^*\begin{bmatrix} -c_{2,k-1}^* \\ c_{1,k-1}^* \end{bmatrix} \tag{12}$$

Differential space-time block encoding entails an expansion of the constellation of the symbols that are differentially encoded. For example, when binary phase shift keying (BPSK) messages $\mathbf{S}_k$ are encoded by (10), the elements of $\mathbf{C}_k$ will form a certain 5-PAM constellation which contains the symbols $\{-1, -1/\sqrt{2}, 0, 1/\sqrt{2}, 1\}$. However, it can be verified that at any time, the total energy transmitted is E_s. Similarly, when 4-PSK messages $\mathbf{S}_k$ are encoded, the elements of $\mathbf{C}_k$ will be 9-QAM (quadrature amplitude modulation).

Note that the code matrix $\mathbf{C}_k$ has the same unitary property as the message matrix $\mathbf{S}_k$. Indeed, from the definition (9) it is easily verified that

$$\mathbf{C}_k \mathbf{C}_k^H = \mathbf{I}_2 \tag{13}$$

By multiplying (10) on the left with $\mathbf{C}_{k-1}^H$, based on (13), we get

$$\mathbf{C}_k \mathbf{C}_{k-1}^H = \mathbf{S}_k \mathbf{C}_{k-1} \mathbf{C}_{k-1}^H = \mathbf{S}_k \tag{14}$$

It follows that if the code matrices $\mathbf{C}_k$ are observable at the receiver, the messages $\mathbf{S}_k$ can be decoded from

$$\mathbf{S}_k = \mathbf{C}_k \mathbf{C}_{k-1}^H \tag{15}$$

From the previous relation and from (13), we have the following relation for $k > j$,

$$\mathbf{C}_k \mathbf{C}_j^H = \mathbf{S}_k \mathbf{S}_{k-1} \ldots \mathbf{S}_{k-j+1} \tag{16}$$

This relation will be useful later in the paper.

C. Receiver Model

To simplify notation and with no loss of generality, assume that there is only a single receive antenna. Now, we seek to observe the received signals when the differential code matrix $\mathbf{C}_k$ is transmitted. Since only one receive antenna is assumed, from (1), the corresponding received signals can be written

$$\begin{bmatrix} y_{1,k}^{(1)} \\ y_{1,k}^{(2)} \end{bmatrix} = \sqrt{E_s}\begin{bmatrix} c_{1,k} & c_{2,k} \\ -c_{2,k}^* & c_{1,k}^* \end{bmatrix}\begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} + \begin{bmatrix} \eta_{1,k}^{(1)} \\ \eta_{1,k}^{(2)} \end{bmatrix}, \tag{17}$$

where $y_{1,k}^{(1)}$ and $y_{1,k}^{(2)}$ represent two received signals at time slots 1 and 2, respectively.

Reconstructing (17) we get

$$\begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix}^T = \sqrt{E_s}\begin{bmatrix} c_{1,k} \\ c_{2,k} \end{bmatrix}^T \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \end{bmatrix}^T, \tag{18}$$

where $r_{1,k} = y_{1,k}^{(1)}$, $r_{2,k} = -y_{1,k}^{(2)*}$, $n_{1,k} = \eta_{1,k}^{(1)}$, and $n_{2,k} = -\eta_{1,k}^{*(2)}$ and "T" denotes the transpose operation. Meanwhile, from (18), we have

$$\begin{bmatrix} -r_{2,k}^* \\ r_{1,k}^* \end{bmatrix}^T = \sqrt{E_s}\begin{bmatrix} -c_{2,k}^* \\ c_{1,k}^* \end{bmatrix}^T \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} -n_{2,k}^* \\ n_{1,k}^* \end{bmatrix}^T \tag{19}$$

Putting (18) and (19) together, we get

$$\begin{bmatrix} r_{1,k} & r_{2,k} \\ -r_{2,k}^* & r_{1,k}^* \end{bmatrix} = \sqrt{E_s}\begin{bmatrix} c_{1,k} & c_{2,k} \\ -c_{2,k}^* & c_{1,k}^* \end{bmatrix}\begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix} + \begin{bmatrix} n_{1,k} & n_{2,k} \\ -n_{2,k}^* & n_{1,k}^* \end{bmatrix} \quad (20)$$

Utilizing compact matrix notation for (18), we obtain

$$\mathbf{R}_k = \sqrt{E_s}\mathbf{C}_k\mathbf{H}_k + \Psi_k \quad (21)$$

where

$$\mathbf{R}_k = \begin{bmatrix} r_{1,k} & r_{2,k} \\ -r_{2,k}^* & r_{1,k}^* \end{bmatrix},\ \mathbf{C}_k = \begin{bmatrix} c_{1,k} & c_{2,k} \\ -c_{2,k}^* & c_{1,k}^* \end{bmatrix}$$
$$\mathbf{H}_k = \begin{bmatrix} h_{1,1} & -h_{2,1}^* \\ h_{2,1} & h_{1,1}^* \end{bmatrix},\ \Psi_k = \begin{bmatrix} n_{1,k} & n_{2,k} \\ -n_{2,k}^* & n_{1,k}^* \end{bmatrix}$$

For the MSDD signal model, consider an observation interval consisting of N blocks of symbols, where, consistent with differential decoding, each block is defined as two symbols at two time slots. A frame consists of L symbol blocks. The channel is assumed constant during a frame, which implies that the channel is fixed during the observation interval. Starting from the kth block $\mathbf{R}_k$, the received sequence can be expressed as

$$\mathbf{R} = \sqrt{E_s}\mathbf{C}\mathbf{H} + \Psi, \quad (22)$$

where

$$\begin{aligned} \mathbf{R} &= [\mathbf{R}_k\ \mathbf{R}_{k-1}\ ...\ \mathbf{R}_{k-N+1}]^T \\ \mathbf{C} &= diag\{\mathbf{C}_k, \mathbf{C}_{k-1}, ...\mathbf{C}_{k-N+1}\} \\ \mathbf{H} &= [\mathbf{H}_k\ \mathbf{H}_{k-1}\ ...\ \mathbf{H}_{k-N+1}]^T \\ \Psi &= [\Psi_k\ \Psi_{k-1}\ ...\ \Psi_{k-N+1}]^T. \end{aligned}$$

The matrices $\mathbf{R}$, $\mathbf{H}$, and Ψ are $2N \times 2$ and $\mathbf{C}$ is $2N \times 2N$. For convenience, we also define the $2N \times 2N$ matrix $\mathbf{S} = diag\{\mathbf{S}_k, \mathbf{S}_{k-1}, ...\mathbf{S}_{k-N+1}\}$.

III. Decision Metric for MSDD

Assume that the observation interval consists of N blocks.If the CSI is known, from expression (22), conditioned on the transmitted symbols $\mathbf{C}$ and the channel $\mathbf{H}$, the matrix $\mathbf{R}$ is a complex-valued, zero-mean Gaussian random matrix. Its probability density function (PDF) is given by [7]

$$p(\mathbf{R}|\mathbf{H},\mathbf{C}) = \frac{1}{(\pi)^{4N}}\exp\left\{-tr\left[\left(\mathbf{R}-\sqrt{E_s}\mathbf{C}\mathbf{H}\right)^H \left(\mathbf{R}-\sqrt{E_s}\mathbf{C}\mathbf{H}\right)\right]\right\}, \quad (23)$$

where "tr" denotes the trace function. If the code matrices are equally likely, the optimal receiver is the maximum-likelihood detector [13], so we can detect $\mathbf{C}$ by

$$\begin{aligned} \widehat{\mathbf{C}} &= \arg\max_{\tilde{c}_{1,k-i},\,\tilde{c}_{2,k-i}\in F} p(\mathbf{R}|\mathbf{H},\widetilde{\mathbf{C}}) \\ &= \arg\min_{\tilde{c}_{1,k-i},\,\tilde{c}_{2,k-i}\in F} tr\left[\left(\mathbf{R}-\sqrt{E_s}\widetilde{\mathbf{C}}\mathbf{H}\right)^H \left(\mathbf{R}-\sqrt{E_s}\widetilde{\mathbf{C}}\mathbf{H}\right)\right] \end{aligned} \quad (24)$$

No surprise, if (24) is spread out, we got the same form as (2). Expression (24) is the decision metric for the case that channel is known, but we are concerned with decision metric for situations when the channel is unknown.

Differential encoding can be applied when the channel $\mathbf{H}$ is unknown, but fixed over some time interval. In this case, the transmitter sends the code matrix $\mathbf{C}_k$ instead of sending messages $\mathbf{S}_k$ directly. For a block of N observations, the received matrix $\mathbf{R}$ given that message matrix $\mathbf{S}$ is transmitted (through code matrix $\mathbf{C}$) has a multivariate Gaussian conditional PDF

$$p(\mathbf{R}|\mathbf{S}) = \frac{1}{(\pi)^{4N}\det\Lambda}\exp\{-tr\left(\mathbf{R}^H\Lambda^{-1}\mathbf{R}\right)\}, \quad (25)$$

where Λ is the covariance matrix of R, $\Lambda = E\{\mathbf{R}\mathbf{R}^H|\mathbf{S}\}$. Since path gains are assumed constant during a frame, $\mathbf{H}_i = \mathbf{H}_j,\ i \neq j$. We have,

$$\begin{aligned} \Lambda &= E\left[\left(\sqrt{E_s}\mathbf{C}\mathbf{H}+\Psi\right)\left(\sqrt{E_s}\mathbf{C}\mathbf{H}+\Psi\right)^H\right] \\ &= E\left[E_s\mathbf{C}\mathbf{H}\mathbf{H}^H\mathbf{C}^H + \Psi\Psi^H\right] \\ &= E_s\mathbf{C}\left(\mathbf{I}_2 \otimes \mathbf{1}_N\right)\mathbf{C}^H + N_0\mathbf{I}_{2N}, \end{aligned} \quad (26)$$

where "⊗" denotes the Kronecker product, and $\mathbf{1}_N$ represents an $N \times N$ matrix with all elements equal 1.

Using the unitary property of the matrix $\mathbf{C}$, it can be shown that $\det\Lambda$ is independent of the messages $\mathbf{S}_k, \mathbf{S}_{k-1}, ...\mathbf{S}_{k-N+1}$.

Define the matrices $\mathbf{A}$, $\mathbf{F}$, $\mathbf{B}$, and $\mathbf{D}$:

$$\begin{aligned} \mathbf{A} &= N_0\mathbf{I}_{2N}, & (27) \\ \mathbf{F} &= E_s\mathbf{I}_{2N}, & (28) \\ \mathbf{B} &= \mathbf{C}\left(\mathbf{I}_2 \otimes \mathbf{1}_{N\times 1}\right), & (29) \\ \mathbf{D} &= \left(\mathbf{I}_2 \otimes \mathbf{1}_{N\times 1}\right)^H\mathbf{C}^H, & (30) \end{aligned}$$

where $\mathbf{1}_{N\times 1}$ is a vector of ones. Then (26) can be expressed as

$$\Lambda = \mathbf{B}\mathbf{F}\mathbf{D} + \mathbf{A} \quad (31)$$

Using the matrix inversion lemma

$$(\mathbf{A}+\mathbf{B}\mathbf{F}\mathbf{D})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{F}^{-1}+\mathbf{D}\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\mathbf{D}\mathbf{A}^{-1}$$

and (26), we have

$$\Lambda^{-1} = \frac{1}{N_0}\mathbf{I}_{2N} - \frac{E_s}{N_0\left(E_s+N_0\right)}\mathbf{C}\left(\mathbf{I}_2 \otimes \mathbf{1}_N\right)\mathbf{C}^H \quad (32)$$

Since the natural logarithm is a monotonically increasing function of its argument, maximizing $p(\mathbf{R}|\mathbf{S})$ over $\mathbf{S}$ in (25) is equivalent to maximizing $\ln\left(p(\mathbf{R}|\mathbf{S})\right)$ over $\mathbf{S}$. Choose the sequence $\mathbf{S}$ to maximize logarithm of (25), which results in the decision metric

$$\begin{aligned} \widehat{\eta} &= tr\left[-\ln(\det\Lambda) - \left(\mathbf{R}^H\Lambda^{-1}\mathbf{R}\right)\right] & (33) \\ &= tr\left[-\ln(\det\Lambda) - \frac{1}{N_0}\mathbf{R}^H\mathbf{R} + \frac{E_s}{N_0\left(E_s+N_0\right)}\mathbf{R}^H\left(\mathbf{C}\left(\mathbf{I}_2 \otimes \mathbf{1}_N\right)\mathbf{C}^H\right)\mathbf{R}\right] & (34) \end{aligned}$$

As $\det\Lambda$, $\mathbf{R}^H\mathbf{R}$, N_0, and E_s are independent of transmitted messages, they can be ignored. Then the decision metric becomes:

$$\widetilde{\eta} = tr\left[\mathbf{R}^H\mathbf{C}\left(\mathbf{I}_2 \otimes \mathbf{1}_N\right)\mathbf{C}^H\mathbf{R}\right] \quad (35)$$

Expanding (35), the metric can be expressed

$$\begin{aligned}\widetilde{\eta} &= tr\left[\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\mathbf{R}_{k-j}\right]\\ &= tr\left[\sum_{i=0}^{N-1}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-i}^{H}\mathbf{R}_{k-i}\right]\\ &\quad +tr\left[\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\mathbf{R}_{k-j}\right]_{i\neq j}\\ &= T+2Retr\left[\sum_{i=1}^{N-1}\sum_{j=0}^{i-1}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\mathbf{R}_{k-j}\right], \end{aligned} \quad (36)$$

where

$$T=tr\left[\sum_{i=0}^{N}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-i}^{H}\mathbf{R}_{k-i}\right] \quad (37)$$

and "Re" denotes real part. Due to the unitary property of $\mathbf{C}_{k-i}$, T is independent of the transmitted symbol sequence. Thus the decision metric becomes

$$\eta = Retr\left[\sum_{i=1}^{N-1}\sum_{j=0}^{i-1}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\mathbf{R}_{k-j}\right] \quad (38)$$

Using the identity for the trace function,

$$Retr\left[\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\mathbf{R}_{k-j}\right] = Retr\left[\mathbf{R}_{k-j}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\right] \quad (39)$$

From (38), (39), and (16), we have

$$\begin{aligned}\eta &= Retr\left[\sum_{i=1}^{N-1}\sum_{j=0}^{i-1}\mathbf{R}_{k-j}\mathbf{R}_{k-i}^{H}\mathbf{C}_{k-i}\mathbf{C}_{k-j}^{H}\right] && (40)\\ &= Retr\left[\sum_{i=1}^{N-1}\sum_{j=0}^{i-1}\mathbf{R}_{k-j}\mathbf{R}_{k-i}^{H}\left(\mathbf{S}_{k-j}\ldots\mathbf{S}_{k-i+1}\right)^{H}\right] && (41)\end{aligned}$$

The differentially encoded message $\widehat{\mathbf{S}}$ can then be detected from

$$\begin{aligned}\widehat{\mathbf{S}} &= \arg\max_{\widetilde{s}_{1,k-i},\,\widetilde{s}_{2,k-i}\in F} Retr\left[\sum_{i=1}^{N-1}\right.\\ &\left.\sum_{j=0}^{i-1}\mathbf{R}_{k-j}\mathbf{R}_{k-i}^{H}\left(\widetilde{\mathbf{S}}_{k-j}\widetilde{\mathbf{S}}_{k-j-1}\ldots\widetilde{\mathbf{S}}_{k-i+1}\right)^{H}\right] \end{aligned} \quad (42)$$

This is the MSDD decision metric for an observation interval of N blocks. Notice that no channel information is required for the signal detection. For an observation interval of N blocks, there are $N-1$ message blocks (the first block $\mathbf{C}_0$ does not contain information). Each block contains two unknown symbols. Hence, for M-PSK symbols, there are $M^{2(N-1)}$ possible message block sequences $\widetilde{\mathbf{S}}_k, \widetilde{\mathbf{S}}_{k-1}, \ldots\widetilde{\mathbf{S}}_{k-N+2}$. As in single antenna MSDD, the complexity of the receiver increases exponentially with the length of the observation interval.

Next, we discuss the special cases of $N=2$ and $N=3$.

A. Two Blocks Observation Interval

When $N=2$, (42) becomes

$$\widehat{\mathbf{S}}_k = \arg\max_{\widetilde{s}_{1,k},\,\widetilde{s}_{2,k}\in F} Retr\left\{\mathbf{R}_k\mathbf{R}_{k-1}^{H}\widetilde{\mathbf{S}}_k^{H}\right\} \quad (43)$$

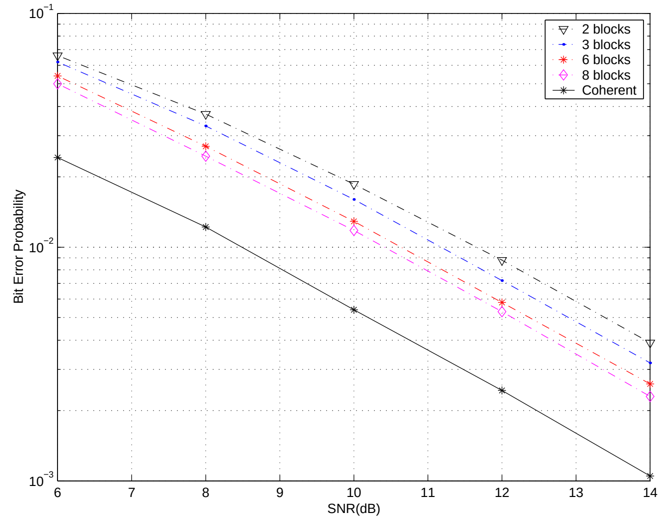

Fig. 1. Multiple symbol differential detection for space-time block code with BPSK signal. Bit Error Probability versus SNR for different length of observation interval, 2 transmit antennas, 1 receive antenna.

Let $\mathbf{S}_k$ be the message matrix that is differentially encoded and transmitted. Then

$$\begin{aligned}\mathbf{R}_k\mathbf{R}_{k-1}^{H} &= \left(|h_{1,1}|^2+|h_{2,1}|^2\right)E_s\mathbf{S}_k+\sqrt{E_s}\mathbf{C}_k\mathbf{H}_k\Psi_{k-1}^{H}\\ &\quad +\sqrt{E_s}\Psi_k\mathbf{H}_{k-1}^{H}\mathbf{C}_{k-1}^{H}+\Psi_k\Psi_{k-1}^{H} \end{aligned} \quad (44)$$

The last expression is similar to (26) in [4]. MSDD is a generalization of the differential space-time codes in [4]. Our notation not only enables to express the MSDD decision statistic, but also provides a simpler way to express known results for two blocks observation interval.

B. Three Blocks Observation Interval

Another special case of interest is an observation interval of $N=3$. A receiver scheme with significant notational complexity was suggested in [11]. Once again, our notation provides for a simple decision statistic expressed as a special case of (42). We can detect $\mathbf{S}_k, \mathbf{S}_{k-1}$ by

$$\begin{aligned}\left[\widehat{\mathbf{S}}_k\ \widehat{\mathbf{S}}_{k-1}\right] &= \arg\max_{\widetilde{s}_{1,k},\,\widetilde{s}_{2,k}\in F} Retr\left\{\mathbf{R}_k\mathbf{R}_{k-1}^{H}\widetilde{\mathbf{S}}_k^{H}+\right.\\ &\left.\mathbf{R}_{k-1}\mathbf{R}_{k-2}^{H}\widetilde{\mathbf{S}}_{k-1}^{H}+\mathbf{R}_k\mathbf{R}_{k-2}^{H}\left(\widetilde{\mathbf{S}}_k\widetilde{\mathbf{S}}_{k-1}\right)^{H}\right\} \end{aligned} \quad (45)$$

IV. Numerical Results

Using decision metric in (42), we can detect blocks of differentially encoded signals by observing intervals of different lengths. In Fig. 1 and Fig. 2, consist of curves for various observation intervals and for BPSK and 4-PSK modulations, respectively. The curves for $N=2$ correspond to the scheme suggested in [4]. Indeed these curves match those in the reference. The curve for $N=3$ in Fig. 1 matches well the results in [11]. Note that there is an almost 0.5 dB improvement by increasing the observation interval from $N=2$ to $N=3$.

Since the computation complexity of the decision statistic in (42) increases exponentially with N, we present results for obsevation intervals of only 8 blocks (16 symbols). For BPSK, there is about 1.5 dB performance improvement compared to the conventional differential detection. This implies a 1.5 dB and 1 dB gain, respectively, over previously published results for 2 and 3 block observation intervals.

V. Conclusions

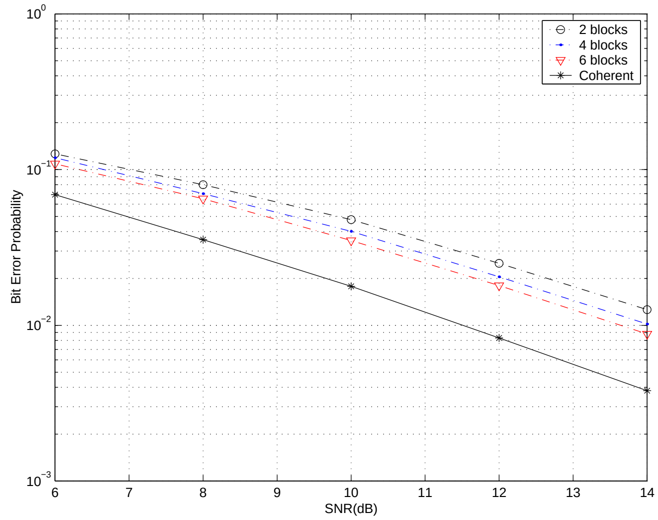

Fig. 2. Multiple symbol differential detection for space-time block code with QPSK signal. Bit Error Probability versus SNR for different length of observation interval, 2 transmit antennas, 1 receive antenna.

A multiple-symbol differential detector is proposed for space-time block codes, in which neither transmitter nor receiver know the CSI. The generalized decision metric for an observation interval of N blocks is derived. It is shown that previously published differential STBC schemes can obtained as special cases of MSDD. Simulation results demonstrate that MSDD can greatly improved the performance of differential STBC. Previously proposed schemes utilizing a two and three block observation interval, incur an SNR performance loss of 3 and 2.5 dB, respectively compared to related coherent detection for BPSK or 4-PSK modulations.

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