Parallel Concatenated Turbo Space-Time Coded Modulation

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Abstract— **We present a new channel coding scheme that utilizes concatenated space-time codes with multilevel modulation and multiple transmit/receive antennas. The proposed scheme is analogous to binary and to trellis coded modulation turbo codes and is referred to as *turbo space-time coded modulation* (turbo-STCM). Turbo-STCM is a new bandwidth efficient coded modulation designed to operate over fading channels. It combines the advantages of powerful turbo codes with the transmit/receive diversity of space-time coded modulation. At the receiver, a symbol-by-symbol *maximum a posteriori* decoding algorithm is presented. Simulation results are provided for 4-PSK signal sets with date rate of 2/b/s/Hz, achieving significant coding gain and the diversity advantage.**

I. Introduction

In future wireless communications systems, high data rates will be expected to be reliably transmitted over multipath fading channels. The wireless channel is subject to various impairments such as time-varying multipath fading and interference. Wireless modems that use temporal signal processing alone are limited in their capability to address the problem of multipath fading. In addition to temporal diversity, spatial diversity is an effective mechanism for mitigating fading effects. Receive diversity has been known and investigated for some time, but recently transmit diversity was recognized as alternative and complementary to receive diversity. Recent results show the advantages of joint channel coding and multiple transmit/receive antennas [1].

We present a new class of codes that extends the concept of turbo coding to include space-time encoders as constituent building blocks of turbo codes. The codes are referred to as *turbo space-time coded modulation* (turbo-STCM). The motivation behind the turbo-STCM concept is to combine the important properties of turbo coding and space-time coded modulation (STCM) into a unified design framework. Turbo coding consists of a concatenation of simple codes yielding a single powerful code [2]. Turbo codes allow efficient decoding algorithms and have been shown to operate close to the Shannon limit when moderate to low bit error rates (as low as 10^{-5}) are required. Like turbo codes in the temporal domain, STCM is a newly invented family of codes [3], [4]. It combines the diversity advantage of space-time processing with the bandwidth efficiency and error correction coding of trellis coded modulation (TCM) [5].

Turbo codes were originally proposed for binary modulation. Due to their remarkable performance, turbo codes become an important subject of intensive current research. The link between turbo coding and STCM is very interesting but not obvious. Recently, researchers have taken some steps in that direction. First, schemes were suggested that extend turbo coding to higher order modulations [6]. A series combination between turbo coding and STCM was suggested in [7] with the turbo code serving as an outer code, and STCM serving as the inner code. Alternatively, in [8], turbo coding is concatenated with spatial diversity. In contrast, we take a different approach, where turbo codes and STCM are not concatenated but rather *merged*, with STCM serving as constituent codes.

The paper is organized as follows. In the section II, we introduce a turbo-STCM encoder based on a parallel architecture. In Section III we present an iterative *maximum a posteriori* (MAP) soft-decoder, and section IV contains an example of a 2 b/s/Hz code. Numerical results include a comparison with other 2 b/s/Hz codes such as STCM and turbo-TCM. Finally, section V provides the conclusions.

II. Turbo-STCM Encoder

The goal of turbo-STCM is to apply the basic structure of turbo codes to space-time coded modulation (STCM). The turbo-STCM encoder consists of two STCM modules in systematic form operating in a parallel concatenation structure. The integration of STCM modules in turbo-STCM is similar to the integration of TCM in the turbo-TCM structure of [6]. In particular, there are some important differences between the binary turbo codes in [2] and turbo-STCM: (1) the constituent STCM codes need to be systematic at the symbol level rather than the bit level, (2) the interleaver operates on symbols, (3) the turbo-STCM output consists of multiple streams of symbols that are being transmitted through multiple transmit antennas respectively.

Consider a wireless data system that employs turbo-STCM to transmit data sequences of length L symbols, through N transmit antennas. A schematic of a constituent STCM encoder is shown in Fig. 1. In the specific encoder considered, the input to the encoder consists of pairs of bits and the output is formed by $N = 2$ streams of 4-PSK symbols. Referring to the figure 1, the two labels associated with each trellis branch represent the phases of the 4-PSK signals transmitted by each antenna. The signal mapping is also shown in Fig. 1, which represents a recursive systematic code in the sense that the symbol transmitted by

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the first antenna corresponds to the two input bits, while the other antenna transmits the two parity bits.

The architecture of a turbo-STCM encoder with $N = 2$ transmit antennas is shown in Fig. 2, where the set-partitioning is given in Fig. 1. The operation of the turbo-STCM encoder is best illustrated by an example. For clarity of presentation, we assume a short sequence of six pairs of bits. In practice, sequences will be much longer such that the output of the interleaver is pseudo-random with respect to its input symbol sequences. Assume that the data block to be transmitted consists of the following pairs of bits: $(00, 01, 11, 10, 00, 11)$. For brevity of notation, we use the mapping in Fig. 1 to list the input sequence in terms of four level symbols $(0, 1, 3, 2, 0, 3)$. Following the trellis transmission in Fig. 1, the output of encoder STCM1 consists of the sequence of pairs of 4-PSK symbols $(00, 10, 31, 23, 01, 32)$. Prior to being fed to STCM2, the input sequence is interleaved by a bits pairwise interleaver. Let the interleaver output be $(3, 3, 0, 1, 0, 2)$. When this sequence is fed to encoder STCM2, the output is given by $(30, 33, 02, 13, 00, 22)$. The output of STCM2 is then de-interleaved (IL^{-1}) to ensure matching of the systematic parts of the output of each STCM encoder (i.e., the first systematic term of each pair of symbols). With that in mind, transmission of the systematic components can be accomplished by STCM1, and STCM2 supplies only the parity symbol. Thus, IL^{-1} can be applied only to the parity symbol at STCM2's output. Following the de-interleaver, the parity symbol sequence becomes: $(0, 3, 0, 2, 2, 3)$. Subsequently, antenna 1, which transmits the systematic parts of the codeword, is connected only to STCM1. A selector alternatively selects the output of encoder STCM1 or the de-interleaved output of encoder STCM2. Thus, the symbol sequences transmitted by the two antennas are $(0, 1, 3, 2, 0, 3)$ and $(0, 3, 1, 2, 1, 3)$, respectively. This arrangement ensures that the 4-PSK symbols are transmitted systematically and that the parity symbols are alternately chosen from STCM1 and STCM2. This ensures that each information pair bits decide the 4-PSK symbol vector exactly once and systematically.

Let consider the turbo-STCM receiver utilize M receive antennas. The signals received at the different antennas are assumed to undergo independent, flat Rayleigh fading. Let the signals at the turbo-STCM decoder input be organized in a sequence of $M \times 1$ vectors, $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_L)$, where each component of $\mathbf{y}_k$ represents the signal received at time k, at one of the M receive antennas. Then the received signal at the receiver array at time k is given by

$$\mathbf{y}_k = \mathbf{H}\mathbf{u}_k + \mathbf{z}_k, \tag{1}$$

where $\mathbf{H}$ is the $M \times N$ matrix of channel coefficients assumed fixed during a time interval refered to a frame(quasistatic flat fading), the elements of channel matrix $\mathbf{H}_{ij}$ is the channel path gain from transmit antenna j to receive antenna i, $\mathbf{u}_k$ is the $N \times 1$ vector of transmitted symbol streams, and $\mathbf{z}_k$ is the $M \times 1$ noise vector with independent components. The noise is assumed complex-valued, Gaussian distributed with zero-mean and variance $N_0/2$ for each real and imaginary components.

III. The MAP Decoder

Decoding of turbo-STCM is based on the *maximum a posteriori* (MAP) criterion. The MAP optimal solution for the transmitted signals, $N \times 1$ symbols (one symbol for each transmit antenna) at time k is given by

$$\hat{\mathbf{u}}_k = \arg\max_i P(\mathbf{u}_k = i \mid \mathbf{y}), \qquad k = 1, 2, \ldots L, \tag{2}$$

where $i \in \{1, \ldots, m^N\}$ denotes the symbol vector index and m is the number of symbols in the transmitter output alphabet, as shown in Fig. 1. The optimal MAP decoder is very complex, but a major advantage of turbo codes is the availability of relatively simple sub-optimal algorithms based on decoders associated with the constituent codes. Each STCM APP decoder estimates their own *a posteriori probability* (APP). Data is shared between STCM APP decoders and an iterative process is applied to refine the results.

Similar to the initial Bahl's algorithm for decoding convolutional codes [9], and given received signals, it is possible to obtain the APP for each possible symbol vector $\mathbf{u_k}$ at each STCM APP decoder. The turbo-STCM decoder is shown schematically in Fig. 3. It features an extension of the decoder for turbo-TCM suggested in [6]. Unlike classical binary turbo codes, turbo-STCM transmits the systematic and parity symbols through different transmit antennas. The systematic and parity components can not be separated as they combine at the receiver antenna. That complicates the design of the turbo-STCM. The decoder features two parallel MAP decoders denotes APP1 and APP2 in Fig. 3. The output of each APP decoder is split into two components: (1) a-priori and (2) systematic and extrinsic. Each APP decoder will pass only the systematic and extrinsic component to the other APP decoder. That ensures that the systematic information is not be used more than once in each corresponding STCM APP decoder.

Let the symbol $i \in \{1, \cdots, m^N\}$ be the input associated with the trellis branch transition from time $k-1$ to time k. In the following derivation, the trellis state at $k-1$ is indexed by s',the trellis state at k by s and the trellis state at $k+1$ by s''. The received signal consists of a sequence of $M \times 1$ symbol vectors $\mathbf{y} = \mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_L$. The MAP decoding algorithm computes and maximizes the APP at time k:

$$\max_i P(\mathbf{u}_k = i/\mathbf{y}) = \max_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s/\mathbf{y}) \tag{3}$$

where $P(\mathbf{u}_k = i/\mathbf{y})$ is the *a posteriori* probability of the signal $\mathbf{u}_k$. The sums are taken over all trellis branch transitions from state s' to state s when $\mathbf{u}_k = i$.

Bayes rule can be applied to obtain

$$P(\mathbf{u}_k = i, s', s/\mathbf{y}) = \frac{P(\mathbf{u}_k = i, s', s, \mathbf{y})}{\sum_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s, \mathbf{y})} \tag{4}$$

Note that $P(\mathbf{y}) = \sum_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s, \mathbf{y})$. Next, we concentrate on computing $P(\mathbf{u}_k = i, s', s, \mathbf{y})$. Utilizing a procedure similar to Bahl's algorithm for decoding convolutional codes [9],

$$P(\mathbf{u}_k = i, s', s, \mathbf{y}) = \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s), \quad (5)$$

where the recursive expression for the forward probability from state s' to state s is

$$\begin{aligned} \alpha_k(s) &= P(s, \mathbf{y}_{t<k}, \mathbf{y}_k) \\ &= \sum_{s'} \alpha_{k-1}(s')\gamma(s', s, \mathbf{y}_k) \quad (6) \end{aligned}$$

The backward probability $\beta_k(s)$ in recursive form in terms of the reverse transitions from state s'' at time $k+1$ to state s at time k is:

$$\begin{aligned} \beta_k(s) &= P(\mathbf{y}_{t>k}/s) \\ &= \sum_{s''} \beta_{k+1}(s'')\gamma(s, s'', \mathbf{y}_{k+1}) \quad (7) \end{aligned}$$

The branch probability $\gamma(s', s, \mathbf{y}_k)$ is given by

$$\begin{aligned} \gamma(s', s, \mathbf{y}_k) &= P(s, \mathbf{y}_k/s') \\ &= \sum_i P(\mathbf{y}_k/\mathbf{u}_k = i, s, s')P(\mathbf{u}_k = i/s, s')P(s/s') \\ &= \sum_i \gamma_i(s', s, \mathbf{y}_k), \quad (8) \end{aligned}$$

By combining (6),(7) and (8), the APP can be expressed

$$\begin{aligned} P(\mathbf{u}_k = i/\mathbf{y}) &= \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s/\mathbf{y}) \\ &= \frac{\sum_s \sum_{s'} \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s)}{\sum_s \sum_{s'} \sum_j \alpha_{k-1}(s')\gamma_j(s', s, \mathbf{y}_k)\beta_k(s)} \\ &= \frac{\sum_s \sum_{s'} \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s)}{P(\mathbf{y})}, \quad (9) \end{aligned}$$

where the above equation comprises a-priori, systematic and extrinsic recursive terms. The probability $P(\mathbf{y})$ is not necessary to compute since at each iteration calculation of the APP(9) will be normalized by all i.

Sub-optimal MAP algorithm

Expression (9) provides the APP for each of the possible transmitted symbol vectors. As a fact, there are two practical problems that affect this computation: (1) it is computationally intensive requiring a large number of multiplications per symbol, (2) it is sensitive to numerical round-off errors. To overcome these problems, a sub-optimal logarithmic form of the algorithm is adopted.

Let the STCM APP1 output at time k be

$$M_1(\mathbf{u}_k) = \log P(\mathbf{u}_k = i \mid \mathbf{y}_k^{(1)}), \quad (10)$$

where $\mathbf{y}^{(1)}$ is the input received data to APP1. Ideally, $\mathbf{y}^{(1)}$ would consists of the noisy and faded signal transmitted by STCM1. However, due to the Turbo-encoder architecture in Fig. 2, the input to STCM APP1 decoder consists of: (1) correct systematic data, (2) parity data that is alternately provided by STCM1 or STCM2. Thus, the input to APP1 consists of a sequence that contains correct STCM1 symbols punctured by STCM2 parity symbols. Subsequently, APP1 is computed from that metric value and the *a priori* information. A distinguishing feature of turbo-STCM iterative decoding is that the *a priori* input to APP1 is provided by the output of APP2. There is a caveat, however. Only new information generated by APP2 can be fed to APP1. The APP2 output $M_2(\mathbf{u_k})$ can be written

$$M_2(\mathbf{u}_k) = \Psi_2\left(\mathbf{y}_k^{(2)} \mid \mathbf{u}_k\right) + L_2(\mathbf{u}_k), \quad (11)$$

where $\Psi_2\left(\mathbf{y}_k^{(2)} \mid \mathbf{u}_k\right)$ is the *extrinsic* information derived by APP2 at the current stage of decoding which will be used as the a-priori information of the APP1 decoder in the next iteration, and $L_2(\mathbf{u_k}) = \log P(\mathbf{u}_k = i)$ is the *a-priori* information into APP2. To preclude multiple uses of the same information, only $\Psi_2\left(\mathbf{y}_k^{(2)} \mid \mathbf{u}_k\right)$ is fed to APP1 as *a priori* information. Similarly, the APP1 output is separated into *extrinsic* and *a priori* parts:

$$\Psi_1\left(\mathbf{y}_k^{(1)} \mid \mathbf{u}_k\right) = M_1(\mathbf{u}_k) - L_1(\mathbf{u}_k). \quad (12)$$

Then $\Psi_1\left(\mathbf{y}_k^{(1)} \mid \mathbf{u}_k\right)$ is fed to APP2 as *a priori* information.

IV. Simulations

A numerical simulation was performed for frames of 1024 information bits and 4-PSK eight-state modulation transmitted over a quasi-static, flat Rayleigh fading channel. For illustration purposes, performance is also provided for a 4-PSK, eight-state STCM scheme with $N = 2$ transmit and $M = 2$ two receive antennas as in [5], and for a 8-PSK, eight-state turbo-TCM scheme [6]. The interleaver/de-interleaver are chosen to be pseudo-random. Note that all schemes correspond to a throughput of 2 b/s/Hz per transmit antenna.

Fig. 4 shows the frame error rate (FER) performance with two transmit and two receive antennas for various number of iterations. FER are plotted versus the ratio E_b/N_0 dB per receive antenna. At FER $= 10^{-2}$, the gain achieved by additional iterations is respectively, 2.8 dB, 1.5 dB and 0.5 dB. Thus, little is gained by performing more than eight iterations. Fig. 5 provides a comparison of several schemes: (1) turbo-STCM 4PSK, 8-state, 2 transmit antennas and 2 receive antennas(2T2R), (2) turbo-STCM with 2 transmit antennas and a single receive antenna (2T1R), (3) turbo-TCM (1T1R), and (4) conventional STCM (2T2R). In this comparison, all iterative decoding algorithms (turbo-STCM and turbo-TCM) perform eight iterations. It can be observed from the figure, that at FER $= 10^{-2}$, 2T2R turbo-STCM provides a 3.3 dB advantage over conventional 2T2R STCM, 9.5 dB over 2T1R turbo-STCM, and 20 dB over single antenna turbo-TCM.

V. Conclusions

In this paper we introduced a new class of turbo-codes with space-time codes as the constituent codes. The new turbo-STCM codes combine the high performance of turbo-coding with the diversity advantage of space-time processing and the bandwidth efficiency of coded modulation under a *single* framework. We describe a parallel turbo-STCM encoder scheme with two constituent systematic space-time codes. A MAP iterative decoder is also presented. Performance is studied for a 4-PSK eight-state turbo-code with 2 transmitter antennas and 2 receiver antennas. It is shown that in operation over a fading channel, a significant performance gain is obtained over conventional STCM (about 4 dB). Turbo-STCM codes offer a new way to transmit high-rate data over fading channels in future wireless networks.

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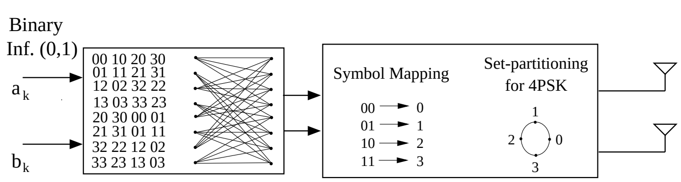

Fig. 1. Recursive systematic STCM encoder for 4-PSK, eight-state, 2 transmit antennas, 2b/s/Hz.

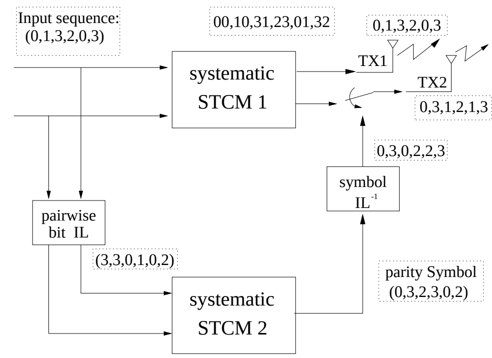

Fig. 2. Turbo-STCM encoder, 2 transmit antennas, 2 b/s/Hz.

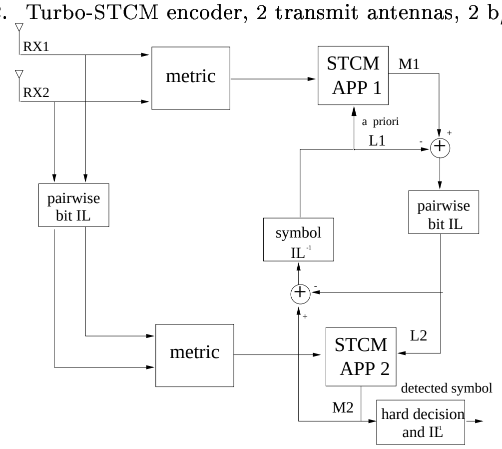

Fig. 3. Turbo-STCM decoder.

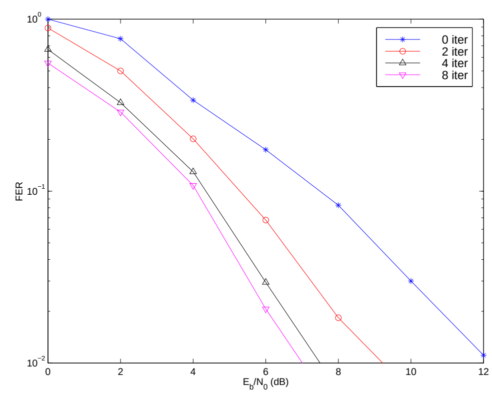

Fig. 4. Iterations of turbo-STCM decoder,2 transmit/2 receive antennas.

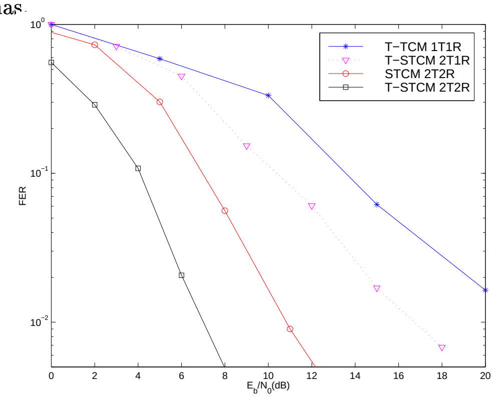

Fig. 5. FER for flat Rayleigh fading, 2 b/s/Hz, input block length $L = 1024$ bits.