# Error Performance Analysis of Turbo Space-Time Coded Modulation over Fading Channels

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*Abstract*— **In this paper, we derive performance bounds for a turbo space-time coded modulation (turbo-STCM) scheme over Rayleigh block-fading channels with multiple transmit and multiple receive antennas. The performance bounds are based on the union bound. It is shown that a subset of dominant error paths provide a good approximation of the bound thereby alleviating the computational effort involved in computing the full union bound. The bound is expressed in terms of parameters of the individual space-time codes that make up the turbo-STCM code, interleaver length, number of transmit/receive antennas, the squared Euclidean distance spectra, and the statistical properties of the channel. The theoretical expressions are demonstrated by a close match with simulation results. This bound provides researchers with a useful analytical tool for the design and analysis of turbo space-time codes.**

## I. Introduction

Turbo-codes [1] when combined with iterative decoding [2] codes have been shown to provide excellent coding gains. Such coding and decoding architectures outperform conventional concatenation of block and/or convolutional codes. Concomitant with the invention of turbo codes, new results in information theory on the capacity of multiantenna systems over multipath fading channels showed large capacities waiting to be harvested [3]. This discovery spurred the invention of new techniques that attempt to capitalize on the richness of the space-time multipath fading channel. Notable among those are space-time codes [4] and BLAST [5].

It is natural to expect that merging turbo-coding and space-time coding could lead to the design of better codes for the fading channel. Recently, several schemes that combine turbo and space-time codes were proposed. Serially concatenated space-time codes and turbo-codes were proposed in [6]. A serial concatenation of space-time codes and recursive convolutional codes was presented in [7]. A different serial concatenation as well as a parallel concatenation scheme are shown in [8]. In [9], the outputs of binary turbo-codes were mapped to QPSK symbols and transmitted through multiple antennas. A scheme that maps the outputs of recursive convolutional encoders to different antennas was proposed in [10]. Another parallel concatenation with the outputs of recursive encoders mapped to different antennas was presented in [11]. Some of these schemes provide full space diversity and full coding rate.

A different approach is obtained by evolving from turbo-codes to trellis coded modulated turbo-codes [12]. In this scheme, two TCM modules are concatenated in parallel. Extending this idea, we proposed a scheme that consists of a parallel concatenation of two systematic space-time codes (STC) [13]. We referred to this scheme as turbo space-time coded modulation (turbo-STCM). This scheme applies the principle of turbo processing directly to the STC modules. The outputs are punctured resulting in a full rate code. The scheme can be viewed as a true extension of the original turbo scheme [1] from the bit to the symbol level. In [14], we developed the upper bound for turbo-STCM with two transmit antenna over the additive white Gaussian noise (AWGN) channel. In this paper, we extend those results to the block fading channel.

The paper is organized as follows. In the section II, we present the signal model. Section III contains the theoretical performance analysis of turbo STC. Numerical results including simulations are found in section IV. Finally, conclusion are presented in section V.

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## II. Signal Model

The goal of turbo-STCM is to apply the basic structure of turbo codes to space-time coded modulation. The turbo-STCM encoder consists of two STC modules in systematic form operating in a parallel concatenation structure. The systematic recursive 8 state 4-PSK STC is shown in Fig. 1. Systematic component codes are often encountered in turbo-codes applications due to their good performance at low signal-to-noise ratios. With the parallel concatenated scheme proposed in [13] and analyzed here, the systematic structure is further motivated by the need to puncture the parity data of the turbo code such the data rate of the overall code is the same as that of the constituent codes. A block diagram of the turbo-STCM encoder with two transmit antennas is shown in Fig. 2. Further details on the encoder can be found in [13], [14].

We now proceed to formulate the system model used for the performance evaluation. Consider a wireless data system, which employs turbo-STCM to transmit data blocks length $K$ of 4-PSK symbols through $N$ transmit antennas. Turbo-STC is a multiple-input multiple-output (MIMO) system with the receiver utilizing an array of $M$ antennas. The signals at the turbo-STCM receiver input are modeled as a sequence of $M \times 1$ vectors, $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_K)$, where each component of $\mathbf{y}_k$ represents the signal received at time $k$, at one of the receive antennas. Then the received signal at the receiver array at time $k$ is

$$\mathbf{y}_k = \sqrt{E_s}\mathbf{H}\mathbf{s}_k + \mathbf{z}_k, \tag{1}$$

where $E_s$ is the energy per symbol. $\mathbf{H}$ is the $M \times N$ ma-

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trix of channel coefficients assumed fixed during an arbitrary time interval referred to as frame (block flat fading). Elements of the matrix $\mathbf{H}$ are independent and identically distributed complex Gaussian with zero-mean and unity variance (Rayleigh fading). The $i, j$ element of $\mathbf{H}$ represents the path gain from transmit antenna $j$ to receive antenna $i$. The $N \times 1$ vector $\mathbf{s}_k$ represents the streams of symbols transmitted by each antenna at the transmitter. Additive white Gaussian noise is modeled by the $M \times 1$ vector $\mathbf{z}_k$. The background noise is assumed independent and identically distributed complex-valued, Gaussian with zero-mean and variance $N_0/2$ for each real and imaginary components.

The performance bound analysis for turbo codes with a fixed interleaver is intractable. To circumvent the problem, the interleaver is assumed random and with uniform distribution (equally likely data permutations). Then the performance analysis is carried out by averaging over the interleavers.

## III. Turbo-STCM Performance Analysis

In this section, we revisit definitions and notations used for the union bound for the AWGN channel and introduced in [14], [15]. Subsequently, we proceed to develop the union bound for the fading channel. The specific system considered is 8 state 4-PSK turbo-STCM, with two transmit and one receive antenna. To derive the bound of turbo-STCM, we first classify the error sequences into different types from the point of view of error event probability. That takes into account all possible transmitted and received codewords in computing the union bound. Due to the uniform random interleaver, all the error sequences of the same error type have the same occurrence probability, while different error types have different occurrence probability. The computation of the union bound proceeds by finding the average number of error sequences in each type together with their probability. Finally, we compute the union bound by averaging all squared Euclidean distances of each error type and averaging over the fading channel realizations.

We are interested in error sequences of length $4K$ bits such that the systematic part of the STC code $w \leq 2K$ and a punctured parity part $z \leq 2K$. Given a systematic block code $C$ of length $4K$ bits, the set of transmitted codewords, and the set of received codewords, the function $a(4K, w, z)$ denotes the number of *error* sequences of length $4K$ that have Hamming weight $w$ for the information bits and $z$ for the parity bits. We refer to such error sequences as $(w, z)$ sequences. The overall Hamming weight of these sequences is $w+z$. The sum is taken over all sequences such that $w + z \leq 4K$. A method for computing the coefficients $a(4K, w, z)$ (known as input-output weight enumerating function coefficients) for STC with puncturing is given in [14]. Here we assume that these coefficients are available. In [14] (and references therein) it is also shown that given the coefficients of the constituent STC codes, say $a_1(4K, w, z_1)$ and $a_2(4K, w, z_2)$, the turbo-STCM coefficients can be found from

$$\tilde{a}(2K, w, z) = \sum_{z_1+z_2=z} \frac{a_1(4K, w, z_1)a_2(4K, w, z_2)}{\binom{2K}{w}}, \quad (2)$$

where $w, z \geq 0$ and $w + z \leq 4K$. This expression is used to evaluate the input-output weight enumerator (IOWE) of turbo-STCM given the IOWE's of the constituent codes. The denominator in the expression (2) is suitable for a bit interleaver and is not easily extended to the symbol interleaver employed by turbo-STCM. However, this effect is minimal, since the IOWE will be used later to express the union bound to the bit error probability, and the bound is dominated by terms with small weight $w$. So that use of the bit interleaver in the denominator of (2) introduces only a negligible error in the computation of the IOWE.

$M$-PSK turbo-STCM accepts $m$ binary symbols at a time and transforms them into $N$ (number of transmit antenna) blocks of $m = \log_2 M$ binary symbols that are fed into a memoryless mapper $\sigma(\cdot)$. With our scheme, symbols emitted by antenna 1 at time index $k$, represent systematic information, $s_k^{sys} = \sigma(\mathbf{c}_k^{sys})$, while symbols transmitted by antenna 2 are parity data, $s_k^{par} = \sigma(\mathbf{c}_k^{par})$, where $\mathbf{c}_k^{sys}$, $\mathbf{c}_k^{par}$ are respectively, the binary labels ($m$ bits each) associated with each symbol. The signal at the receiver is constituted by the sum of systematic and parity symbols and the effect of the channel. The received symbol is equivalent to a mapping $f(\cdot)$,

$$f(\mathbf{c}_k) = h_1 s_k^{sys} + h_2 s_k^{par} \quad (3)$$

where $\mathbf{h}^T = [h_1 h_2]$ is substituted for $\mathbf{H}$ in the signal model (1) and the $2m$-tuple $\mathbf{c}_k = (\mathbf{c}_k^{sys} \mathbf{c}_k^{par})$ is the binary label of the equivalent symbol $f(\mathbf{c}_k)$. In the rest of this analysis, symbols are understood to be defined as in (3).

Let $\underline{\mathbf{c}}_K = (\mathbf{c}_1, \ldots, \mathbf{c}_K)$ be a sequence of $K$ binary labels associated with the transmitted symbols. An error event occurs when the demodulation chooses $f(\underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K)$, where $a \oplus b = (a + b) \bmod 2$ and $\underline{\mathbf{e}}_K$ is a non-zero sequence of binary error vectors.

The pairwise error probability of choosing $\underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K$ instead of $\underline{\mathbf{c}}_K$ is denoted $P(\underline{\mathbf{c}}_K \to \underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K)$ and is given by

$$P(\underline{\mathbf{c}}_K \to \underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K) = Q\left(\sqrt{\frac{R_c E_b d^2(\underline{\mathbf{c}}_K, \underline{\mathbf{e}}_K)}{2N_0}}\right) \quad (4)$$

where $R_c = 1/N$ is the turbo-STCM code rate and $d^2(\underline{\mathbf{c}}_K, \underline{\mathbf{e}}_K) = \|f(\underline{\mathbf{c}}_K) - f(\underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K)\|^2$ is the cumulative squared Euclidean distance (SED) associated with the output sequence $\underline{\mathbf{c}}_K$ and the error sequence $\underline{\mathbf{e}}_K$. The pairwise error probability can be upper bound by the Bhattacharyya bound

$$P(\underline{\mathbf{c}}_K \to \underline{\mathbf{c}}_K \oplus \underline{\mathbf{e}}_K) \leq \frac{1}{2} Z^{d^2(\underline{\mathbf{c}}_K, \underline{\mathbf{e}}_K)}, \quad (5)$$

where $Z = e^{-R_c E_b / 4N_0}$.

The *type* of an error sequence is defined as the vector $\mathbf{n}$ with elements $n_{ij}$ denoting the number of symbols in the error sequence that have $i$ systematic bit errors and $j$ parity bit errors. For the 2 bits/s/Hz, 4-PSK 2 transmit antennas we have $(i,j) \in \{(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$. The information bits weight and the parity bits weight in an error sequence can be found from the relations

$$\begin{aligned} n_{1,0}+n_{1,1}+n_{1,2}+2n_{2,0}+2n_{2,1}+2n_{2,2} &= w \\ n_{0,1}+2n_{0,2}+n_{1,1}+2n_{1,2}+n_{2,1}+2n_{2,2} &= z. \quad (6) \end{aligned}$$

The mapping $\mathbf{n} \to (w,z)$ is many-to-one. Let $\Phi(w,z,\mathbf{n})$ denote the probability that $(w,z)$ error sequence is also type $\mathbf{n}$. We index with $\beta$ all types of error sequences $(w,z)$, i.e., $\bigcup_\beta \mathbf{n}_\beta(w,z)$ is the set of $(w,z)$ error sequences. Using steps similar to [14], [15], the conditional upper bound on the bit error probability is given by

$$\begin{aligned} P(e|\mathbf{h}) &\le \sum_{w=1}^{2K}\sum_{z=0}^{2K}\sum_{\mathbf{n}_\beta(w,z)} \frac{w}{4K}\widetilde{a}(4K,w,z) \\ &\cdot\Phi(w,z,\mathbf{n}_\beta)\,E\left[Z^{D^2_{\mathbf{n}_\beta}}\right] = B(e|\mathbf{h}) \qquad (7) \end{aligned}$$

where the term $E\left[Z^{D^2_{\mathbf{n}_\beta}}\right]$ is the expected value of over SED's associated with error sequences of type $\mathbf{n}_\beta(w,z)$, and we used the notation $B(e|\mathbf{h})$ to denote the bound. It follows that the average upper bound to the bit error probability is given by

$$\begin{aligned} B(e) &= E_\mathbf{h}[B(e|\mathbf{h})] = \qquad (8) \\ \sum_{w=1}^{2K}\sum_{z=0}^{2K}&\sum_{\mathbf{n}_\beta(w,z)}\frac{w}{4K}\widetilde{a}(4K,w,z)\Phi(w,z,\mathbf{n}_\beta)\,E_D\left\{E_\mathbf{h}\left[Z^{D^2_{\mathbf{n}_\beta}}\right]\right\}, \end{aligned}$$

where we switched the order of the expectation operations with respect to $D$ and $\mathbf{h}$. We now focus on evaluating the term $E_\mathbf{h}\left[Z^{D^2_{\mathbf{n}_\beta}}\right]$.

To simplify notation a bit, let the binary labels associated with the information and systematic data $\mathbf{c}_k^{sys} = \mathbf{c}_k^{(1)}$ and $\mathbf{c}_k^{par} = \mathbf{c}_k^{(2)}$. Then the SED for a single epoch of an error path can be expressed

$$\begin{aligned} d^2(\mathbf{c}_k,\mathbf{e}_k) &= \left\|\sum_{n=1}^{2} h_n\left(\sigma\left(\mathbf{c}_k^{(n)}\right)-\sigma\left(\mathbf{c}_k^{(n)}\oplus\mathbf{e}_k^{(n)}\right)\right)\right\|^2 \\ &= \left\|\sum_{n=1}^{2} h_n\delta(c_{n,k},e_{n,k})\right\|^2 \qquad (9) \end{aligned}$$

where $\mathbf{e}_k^{(n)}$, $n=1,2$, is the bit pattern error associated with the systematic (antenna 1) and parity data (antenna 2), respectively. Subsequently, the cumulative SED associated with an error sequence can be expressed in terms of the channel as follows:

$$\begin{aligned} d^2(\underline{\mathbf{c}}_K,\underline{\mathbf{e}}_K) &= \sum_{k=1}^{K}\left\|\sum_{n=1}^{2} h_n\delta(c_{n,k},e_{n,k})\right\|^2 \\ &= \mathbf{x}^H\mathbf{x} \qquad (10) \end{aligned}$$

where the definition of the $K\times 1$ vector $\mathbf{x}$ is obvious from the context, and the superscript denotes complex and transposed. Since $h_n$ is complex Gaussian, the vector $\mathbf{x}$ is a complex-valued Gaussian multivariate with zero mean and covariance matrix $\mathbf{M}_x = \mathrm{cov}(\mathbf{x}) = (\mathbf{m}_{ij})$. It is easy to show that

$$(m_{ij}) = \begin{cases} i=j & \sum_{n=1}^{2}\delta^2(c_{n,i},e_{n,i}) \\ i\neq j & 0 \end{cases}, \quad 1\le i,j\le K \qquad (11)$$

From (5), (10) and the definition of $Z$, evaluation of

$$\begin{aligned} E_\mathbf{h}\left[Z^{D^2_{\mathbf{n}_\beta}}|D^2_{\mathbf{n}_\beta} = d^2(\underline{\mathbf{c}}_K,\underline{\mathbf{e}}_K)\right] &= E_\mathbf{h}\left[e^{-\gamma d^2(\underline{\mathbf{c}}_K,\underline{\mathbf{e}}_K)}\right] \\ &= E_\mathbf{h}\left[e^{-\gamma\mathbf{x}^H\mathbf{x}}\right], \qquad (12) \end{aligned}$$

where $\gamma = R_cE_b/4N_0$, amounts to the computation of the Laplace transform with respect to the quadratic form of a complex Gaussian random variable.

Using [16], we obtain

$$\begin{aligned} E_\mathbf{h}\left[e^{-\gamma\mathbf{x}^H\mathbf{x}}\right] &= \det(\mathbf{I}_K+\gamma\mathbf{M}_\mathbf{x})^{-1} \\ &= \prod_{k=1}^{K}\left(1+\gamma\delta_k^2\right)^{-1} \qquad (13) \end{aligned}$$

where $\delta_k^2 = \sum_{n=1}^{2}\delta^2(c_{n,k},e_{n,k})$ and $\mathbf{I}_K$ is the $K\times K$ identity matrix. The last expression can be recast in terms of equivalent SED (averaged over fading)

$$\prod_{k=1}^{K}\left(1+\gamma\delta_k^2\right)^{-1} = \exp\left(-\sum_{k=1}^{K}\Delta_k^2\right) \qquad (14)$$

where $\Delta_k^2 = \log\left(1+\gamma\delta_k^2\right)$. The computation of the bound can now proceed as in the AWGN case, with the cumulative SED for a type $\mathbf{n}_\beta(w,z)$ error sequence being represented by the random variable $\widetilde{D}^2_{\mathbf{n}_\beta}$, where $\widetilde{D}^2_{\mathbf{n}_\beta}$ has already been averaged over the fading channel. Thus the final expression for the bit error bound is given by

$$P_e \le \sum_{w=1}^{2K}\sum_{z=0}^{2K}\sum_{\mathbf{n}_\beta(w,z)}\frac{w}{4K}\widetilde{a}(4K,w,z)\Phi(w,z,\mathbf{n}_\beta)\,E\left[e^{\widetilde{D}^2_{\mathbf{n}_\beta}}\right] \qquad (15)$$

*Extension to Multiple Receive Antennas*

We now extend the union bound to the case of multiple receive antennas. It is also noted that while, consistent with our turbo scheme, we used $N=2$ transmit antennas in expressing the bound, extension to an arbitrary number of transmit antennas is straightforward with $N$ being substituted for the numeral 2 in all summations over index $n$. We will do that below and at the same time assume that the number of receive antennas is $M$.

With the channel matrix $\mathbf{H}$ in the signal model of (1) consisting of elements $h_{m,n}$, the SED in (9) can be rewritten

$$d^2(\mathbf{c}_k,\mathbf{e}_k) = \sum_{m=1}^{M}\left\|\sum_{n=1}^{N} h_{m,n}\left(\sigma\left(\mathbf{c}_k^{(n)}\right)-\sigma\left(\mathbf{c}_k^{(n)}\oplus\mathbf{e}_k^{(n)}\right)\right)\right\|^2$$

$$= \sum_{m=1}^{M} \left\| \sum_{n=1}^{N} h_{m,n} \delta\left(c_{n,k}, e_{n,k}\right) \right\|^2 \quad (16)$$

It follows that the cumulative SED can be expressed

$$\begin{aligned} d^2(\underline{\mathbf{c}}_K, \underline{\mathbf{e}}_K) &= \sum_{k=1}^{K} d^2\left(\mathbf{c}_k, \mathbf{e}_k\right) \\ &= \sum_{k=1}^{K} \sum_{m=1}^{M} \left\| \sum_{n=1}^{N} h_{m,n} \delta\left(c_{n,k}, e_{n,k}\right) \right\|^2 \\ &= \overline{\mathbf{x}}^H \overline{\mathbf{x}} \end{aligned} \quad (17)$$

The $MK \times 1$ vector $\overline{\mathbf{x}}$ is a complex Gaussian multivariate with zero mean and covariance matrix

$$\mathbf{M}_{\overline{\mathbf{x}}} = \mathbf{I}_M \otimes \mathbf{M}_{\mathbf{x}}$$

where $\mathbf{M}_{\mathbf{x}}$ was defined in (11), $\otimes$ is the Kronecker product and $\mathbf{I}_M$ is the $M \times M$ identity unity matrix. It follows that similar to (13), we get

$$\begin{aligned} E_{\mathbf{h}}\left[e^{-\gamma \overline{\mathbf{x}}^H \overline{\mathbf{x}}}\right] &= \det\left(\mathbf{I}_{MK} + \gamma \mathbf{M}_{\overline{\mathbf{x}}}\right)^{-1} \\ &= \prod_{k=1}^{K} \left(1 + \gamma \delta_k^2\right)^{-M}, \end{aligned} \quad (18)$$

resulting in a set of equivalent distances $\Delta_k^2 = M \log\left(1 + \gamma \delta_k^2\right)$. Finally a bound similar to (15) is obtained, except that the SED averaged over fading $\widetilde{D}^2_{\mathbf{n}_\beta}$, *incorporates the effect of* $M$ receive antennas.

## IV. Numerical Results

In this section, we demonstrate several examples of the performance analysis derived in this paper. For comparison, we also plot results obtained from simulation. The simulation is based on Monte Carlo runs with 10,000 channel realizations for each $E_b/N_0$. The channel is assumed block-fading, i.e., constant during a frame, but independent frame-to-frame. Two cases are analyzed for turbo-STCM with two transmit and one or two receive antennas, corresponding to data frame lengths $F = 130$ symbols and different interleavers of size $K$ as specified in the figures. The turbo-STCM consists of two parallel concatenated STC encoders with throughput 2 bits/s/Hz, 8 state 4-PSK STC, as shown in Fig. 1.

Fig. 3 provides the performance analysis for a single receive antenna. The curves in Fig. 4 show results for 2 receive antennas. Simulations results are shown after eight iterations of the turbo decoder. Each figure is divided in two parts corresponding to the interleaver sizes of $K$ =130 and 5200 symbols, respectively. Results displayed are consistent with the expectation of performance improving with the length of interleaver.

Note that the union bound is obtained by summing over all error sequences with systematic and parity weights, $w$ and $z$, respectively, from 1 to the block length $2K$. Main contributions to the bound are provided by error sequences with low weights $w$ and $z$. Low weights result in small SED's, which in turn are the dominant terms in the union bound. To generate the union bound curves in the figures, we summed over error sequences with systematic part weight $w$ as indicated in the figure and with parity weight $0 \leq z \leq 15$. Comparison of the simulation results with the curves for $w = 1$ show that when only error sequences with a single systematic bit are incorporated, the bound is not representative of the performance. However, an accurate upper bound is obtained with as few as $w = 2$ systematic bit errors accounted for. As the signal-to-noise ratio increases, the curves for the different systematic weights coalesce, which indicates that the lower weights are indeed dominant. These results justify simplifying the bound evaluation by neglecting terms with large weights.

## V. Conclusion

In this paper, we derive performance bounds for turbo-STCM over the Rayleigh block-fading channel with multiple transmit and receive antennas. This is an extension of previous results for the AWGN channel. The union bound is expressed in terms of parameters of the individual space-time codes that make up the turbo-STCM code, interleaver length, number of transmit/receive antennas, the SED spectra, and the statistical properties of the channel. The theoretical expressions are demonstrated by a close match with simulation results. It is shown that accounting for errors paths with just a few systematic errors suffices to obtain reasonably accurate results. This can simplify considerably the computation of the bound. The bounds can be used as tools in the analysis of turbo-STCM schemes.

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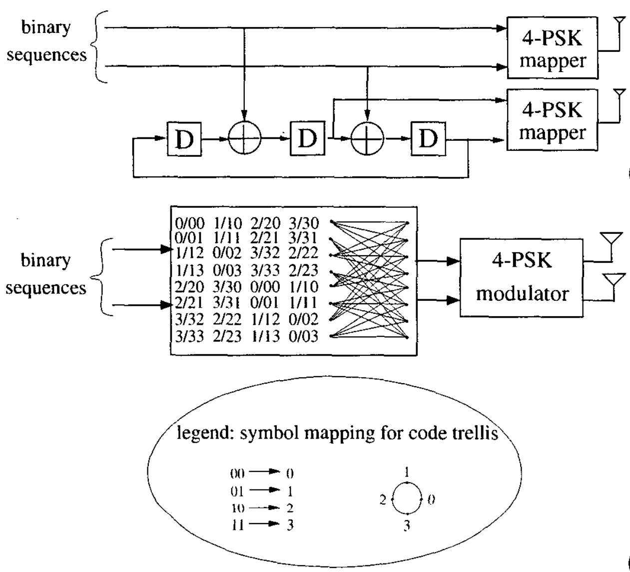



(a)

(b)

Fig. 1. Recursive systematic form of space-time code: (a) code implementation, (b) code trellis.

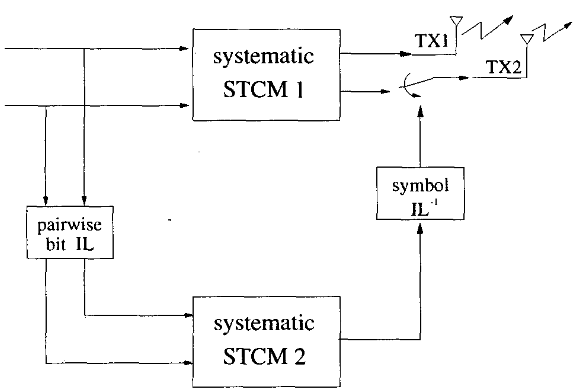
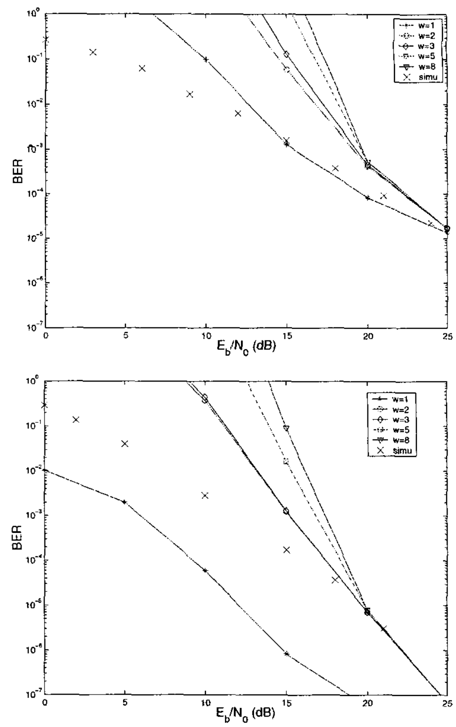


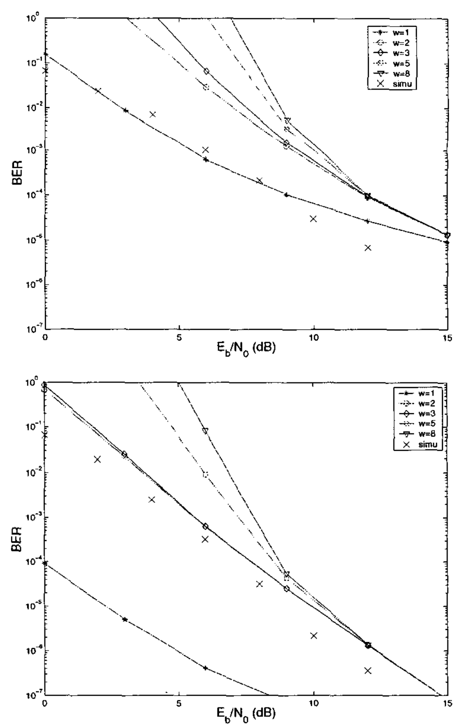

Fig. 2. Turbo-STCM encoder, 2 transmit antennas, 2 bits/s/Hz




(a)

(b)

Fig. 3. Union bound for turbo-STCM (2T1R) over block fading channel with interleaver size (symbols): (a) $K = 130$, (b) $K = 5200$.




(a)

(b)

Fig. 4. Union bound for turbo-STCM (2T2R) over block fading channel with interleaver size (symbols): (a) $K = 130$, (b) $K = 5200$.