A New Bandwidth Efficient Antenna Diversity Scheme Using Turbo Codes

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Abstract — **We suggest a new channel coded modulation scheme that utilizes concatenated space-time codes with multilevel modulation and multiple transmit/receive antennas. The proposed scheme is analogous to binary and to trellis coded modulation turbo codes and is referred to as *turbo space-time coded modulation* (turbo-STCM). Turbo-STCM is a new bandwidth efficient coded modulation designed to operate over fading channels. It combines the advantages of powerful turbo codes with the transmit/receive diversity of space-time coded modulation. The receiver consists of a symbol-by-symbol *maximum a posteriori* decoding algorithm is presented. Simulation results are provided for 4-PSK signal sets with a throughput of 2/b/s/Hz, achieving significant coding gain and the diversity advantage.**

I. Introduction

Recently there has been an explosive growth of personal and wireless services whose ultimate goal is to support universal voice and multimedia services without regards to mobility and location. For these services that provide image, video and local area network, to become reality, high-speed data will be expected to be transmitted reliably over multipath fading channels.

Various techniques have been proposed and studied for fast, reliable wireless data transmission. Channel coding, diversity combining and other techniques have been applied to improve the performance of receivers operating in severe signal environments. Most of the work published either uses temporal signal processing alone, such as error control coding, or spatial processing alone. Coding does not address the multipath fading problem. Conversely, spatial processing does not meet the needs for coding. Multiple antennas at the transmitter are an attractive diversity method to reduce the required transmission power over the multipath fading channel, since it does not impose bandwidth expansion like frequency division diversity. Recent results in information theory show that large gains in capacity can be achieved through the use of multiple antennas at the transmitter and the receiver [1],[2]. To achieve this projected capacity, new techniques are required that combine coding and space-time processing.

Although antenna diversity can be utilized either at the transmitter (transmit diversity) or at the receiver (receive diversity) or both, until recently, work has focused mainly on spatial diversity at the receiver. Simple combining methods are available for that, such as maximum ratio combining (MRC) or antenna selection. Recently, transmit diversity was recognized as alternative and complementary to receive diversity. Realizing the benefits of transmit diversity is particularly challenging because usually the transmitter does not have channel information. The effect of transmit diversity is to convert a flat fading channel to a frequency-selective channel, i.e., a channel capable of accommodating multiple signal paths and thus providing diversity gain.

In this paper, we propose a new class of codes that extends the concept of turbo coding to include space-time encoders as constituent building blocks of turbo codes. The codes are referred to as *turbo space-time coded modulation* (turbo-STCM). The motivation behind the turbo-STCM concept is to combine the important properties of turbo coding and space-time coded modulation (STCM) into a unified design framework. Turbo coding consists of a concatenation of simple codes yielding a single powerful code [3]. Turbo codes allow efficient decoding algorithms and have been shown to operate close to the Shannon limit when moderate to low bit error rates (as low as 10^{-5}) are required. Like the turbo codes in the temporal domain, STCM is a newly invented family of codes. It combines the diversity advantage of space-time processing with the bandwidth efficiency and error correction coding of trellis coded modulation (TCM) [4], [5], [6].

Turbo codes were originally proposed for binary modulation. Due to their remarkable performance, turbo codes became a subject of intensive current research in a wide range of telecommunications applications. The link between turbo coding and STCM is very interesting but not obvious. Recent publications have taken some steps in that direction. First, schemes were suggested that extend turbo coding to higher order modulations [7, 8]. These schemes combine binary turbo codes with higher order modulation or, alternatively, employ multilevel component codes. A serial combination between turbo coding and STCM was suggested in [9] with the turbo code serving as the outer code, and STCM serving as the inner code. Alternatively, turbo coding can be concatenated with spatial diversity [10].

In this work, we take a different approach, where turbo codes and STCM are not concatenated but rather *merged*. In this arrangement, STCM provides the constituent codes. The basic principle of turbo coding-decoding is applied by retaining the soft-decision and iterative decoding. The paper is organized as follows: in section II, we introduce the main concept of a turbo-STCM encoder based on a parallel architecture. In section III we derive an iterative *maximum a posteriori* soft-decoder, and present the principles of the sub-optimal algorithm. Section IV provides numerical results for

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a 2 b/s/Hz code. These include a comparison with other 2 b/s/Hz methods such as STCM and turbo-TCM. Finally, section V provides the conclusions.

II. Turbo-STCM Encoder

Turbo-STCM applies the basic structure of turbo codes to space-time coded modulation (STCM). The turbo-STCM constituent codes are two STCM modules in systematic form. A schematic of a 4-PSK systematic STCM encoder is shown in Fig. 1. The constituent codes are connected in parallel as shown in Fig. 3. A pseudorandom symbol-wise bit interleaver between the STCM encoding modules ensures that the transmitted data stream possesses random-like properties. The integration of STCM modules in turbo-STCM is similar to the integration of TCM in the turbo-TCM structure of [8]. In particular, there are some important differences between the binary turbo codes in [3] and turbo-STCM: (1) the constituent STCM codes need to be systematic at the symbol level rather than the bit level, (2) the interleaver operates on symbols, (3) the turbo-STCM output consists of multiple streams of symbols that are being transmitted through multiple transmit antennas.

Consider a wireless data system that employs turbo-STCM to transmit data sequences of length L symbols, through N transmit antennas. For clarity, we base the presentation on a specific example. A constituent 4-PSK, eight-state STCM encoder is shown in Fig. 2. The input to each STCM consists of pairs of bits and the output is formed by $N = 2$ streams of 4-PSK symbols. Referring to the figure, the two labels associated with each trellis branch represent the phases of the 4-PSK signals transmitted by each antenna. The signal mapping is also shown in Fig. 2. It can be easily verified that the STCM encoder is systematic in the sense that the symbol transmitted by the first antenna corresponds to the two input bits, while the other antenna transmits the two parity bits.

The architecture of the turbo-STCM encoder is shown in Fig. 3. To best illustrate the operation of the turbo-STCM encoder, we continue with the specific example. For clarity of presentation, we assume a short sequence of six pairs of bits. In practice, sequences will be much longer such that the output of the interleaver is pseudo-random with respect to its input symbol sequence. Assume that the data block to be transmitted consists of the following pairs of bits: $(00, 01, 11, 10, 00, 11)$. For brevity of notation, we use the signal mapping in Fig. 2 to list the input sequence in terms of four level symbols $(0, 1, 3, 2, 0, 3)$. Following the trellis transitions in Fig. 2, the output of encoder STCM1 consists of the sequence of pairs of 4-PSK symbols $(00, 10, 31, 23, 01, 32)$. Prior to being fed to STCM2, the input sequence is interleaved by a pairwise bit interleaver. Let the interleaver output be $(3, 3, 0, 1, 0, 2)$. When this sequence is fed to encoder STCM2, the output is given by $(30, 33, 02, 13, 00, 22)$. The output of STCM2 is then de-interleaved (IL^{-1}) to ensure matching of the systematic parts at the output of each STCM encoder (i.e., the first of each pair of symbols of each STCM encoder). With that in mind, transmission of the systematic components can be accomplished by STCM1 only. Both STCM1 and STCM2 transmit parity symbols. It follows that IL^{-1} can be applied only to the parity symbol at STCM2's output. Following the de-interleaver, the parity symbol sequence becomes: $(0, 3, 0, 2, 2, 3)$. Antenna 1, which transmits the systematic part of the codeword, is connected only to STCM1. A selector alternatively selects the output of encoder STCM1 or the de-interleaved output of encoder STCM2. Thus, the symbol sequences transmitted by the two antennas are $(0, 1, 3, 2, 0, 3)$ and $(0, 3, 1, 2, 1, 3)$, respectively. This arrangement ensures that the 4-PSK symbols are transmitted systematically and that the parity symbols are alternately chosen from STCM1 and STCM2.

III. The MAP Decoder

The turbo-STCM system is multiple-input multiple-output (MIMO) with the receiver utilizing an array of M antennas. The signals at the turbo-STCM receiver input are modeled as a sequence of $M \times 1$ vectors, $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_L)$, where each component of $\mathbf{y}_k$ represents the signal received at time k, at one of the receive antennas. Then the received signal at the receiver array at time k is given by

$$\mathbf{y}_k = \mathbf{H}\mathbf{u}_k + \mathbf{z}_k, \tag{1}$$

where $\mathbf{H}$ is the $M \times N$ matrix of channel coefficients assumed fixed during an arbitrary time interval referred to as frame (quasi-static flat fading). The i, j element of $\mathbf{H}$ represents the path gain from transmit antenna j to receive antenna i. The $N \times 1$ vector $\mathbf{u_k}$ represents the symbol streams transmitted by each antenna at the transmitter. Additive white Gaussian noise is modeled by the $M \times 1$ vector $\mathbf{z_k}$. The noise is assumed complex-valued, Gaussian distributed with zero-mean and variance $N_0/2$ for each real and imaginary components. The elements of the noise vector are assumed independent and identically distributed.

Decoding of turbo-STCM is based on the *maximum a posteriori* (MAP) criterion. The decoder consists of an iterative receiver structure as shown in Fig. 4. A sub-optimal MAP algorithm is derived for a receiver with multiple antennas.

The optimal MAP solution for the N transmitted symbols (one symbol for each transmit antenna) at time k is given by:

$$\widehat{\mathbf{u}}_k = \arg\max_i P(\mathbf{u}_k = i/\mathbf{y}), \qquad k = 1, 2, \ldots L, \tag{2}$$

where $P(\mathbf{u}_k = i/\mathbf{y})$ is the *a posteriori* probability of the signal $\mathbf{u}_k$, $i \in \left\{1, \ldots, m^N\right\}$ denotes the symbol vector index, and m represents the number of symbols in the transmitter output alphabet (for example for 4-PSK, $m = 2$). Turbo codes are block codes with a block length equal to the length of the interleaver. This makes the analysis of turbo codes very difficult if approached from the point of view of very long block codes. But a major advantage of turbo codes is the availability of relatively simple sub-optimal algorithms based on decoders associated with the constituent codes. Each STCM APP decoder estimates its own *a posteriori probability* (APP). Data is shared between STCM APP decoders and an iterative process is applied to refine the soft decisions.

The soft decoder is based on Bahl's algorithm [11]. The decoder consists of two constituent STCM decoders each computing the APP associated with a constituent STCM encoder. The turbo-STCM decoder is shown schematically in Fig. 4. It features an extension of the iterative decoder for turbo-TCM suggested in [8] to the MIMO case. Unlike classical binary turbo codes, with turbo-STCM, systematic and parity symbols are transmitted through different transmit antennas, but are received in a combined form at the receiver. Hence, the systematic and parity components can not be separated as it is done for the binary case. This requires modifications in the design of the turbo-STCM receiver. As shown in Fig. 4, the decoder features two parallel MAP decoders denoted APP1 and APP2, respectively. The output of each APP decoder is split into two components: (1) a-priori and (2) systematic and extrinsic. Each APP decoder will pass only the systematic and extrinsic component to the other APP decoder. That ensures that the systematic information is not be used more than once in each corresponding STCM APP decoder.

Let $i \in \{1, \cdots, m^N\}$ be the input symbol associated with the trellis branch transition at time k between states s' and s. The notation refers to the state at time $k-1$ as being indexed by s'. The states at k and $k+1$ are indexed by s and s'', respectively. The received signal consists of a sequence of $M \times 1$ symbol vectors $\mathbf{y} = \mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_L$. The MAP decoding algorithm computes and maximizes the APP at time k:

$$\max_i P(\mathbf{u}_k = i/\mathbf{y}) = \max_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s/\mathbf{y}), \quad (3)$$

where the sums in (3) are taken over all trellis branch transitions from states s' to states s when $\mathbf{u}_k = i$.

Bayes rule can be applied to obtain the APP expression

$$P(\mathbf{u}_k = i, s', s/\mathbf{y}) = \frac{P(\mathbf{u}_k = i, s', s, \mathbf{y})}{\sum_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s, \mathbf{y})}. \quad (4)$$

Note that the denominator of the expression is $\sum_i \sum_s \sum_{s'} P(\mathbf{u}_k = i, s', s, \mathbf{y}) = P(\mathbf{y})$. Next, we concentrate on computing the joint probability $P(\mathbf{u}_k = i, s', s, \mathbf{y})$.

Utilizing a procedure similar to Bahl's for decoding convolutional codes [11], it is possible to express the joint probability $P(\mathbf{u}_k = i, s', s, \mathbf{y})$ as the product:

$$P(\mathbf{u}_k = i, s', s, \mathbf{y}) = \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s), \quad (5)$$

where the recursive expression for the forward probability from state s' to state s is

$$\begin{aligned} \alpha_k(s) &= P(s, \mathbf{y}_{t<k}, \mathbf{y}_k) \\ &= \sum_{s'} \alpha_{k-1}(s')\gamma(s', s, \mathbf{y}_k) \end{aligned} \quad (6)$$

The backward probability $\beta_k(s)$ in recursive form in terms of the reverse transitions from state s'' at time $k+1$ to state s at time k is:

$$\begin{aligned} \beta_k(s) &= P(\mathbf{y}_{t>k}/s) \\ &= \sum_{s''} \beta_{k+1}(s'')\gamma(s, s'', \mathbf{y}_{k+1}). \end{aligned} \quad (7)$$

If we assume no parallel transitions between states, then the branch transition probability $\gamma(s', s, \mathbf{y}_k)$ (the probability of transitioning from s' to s with output $\mathbf{y}_k$) is determined by the joint probability of s and $\mathbf{y}_k$ given s',

$$\begin{aligned} \gamma(s', s, \mathbf{y}_k) &= P(s, \mathbf{y}_k/s') \\ &= \sum_i P(\mathbf{y}_k/\mathbf{u}_k = i, s, s')P(\mathbf{u}_k = i/s, s')P(s/s') \\ &= \sum_i \gamma_i(s', s, \mathbf{y}_k). \end{aligned} \quad (8)$$

By combining (6),(7) and (8), the APP can be expressed

$$\begin{aligned} P(\mathbf{u}_k = i/\mathbf{y}) &= \sum_s \sum_{s'} P(\mathbf{u}_k = i, s' s/y) \\ &= \frac{\sum_s \sum_{s'} \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s)}{\sum_s \sum_{s'} \sum_j \alpha_{k-1}(s')\gamma_j(s', s, \mathbf{y}_k)\beta_k(s)} \\ &= \frac{\sum_s \sum_{s'} \alpha_{k-1}(s')\gamma_i(s', s, \mathbf{y}_k)\beta_k(s)}{P(\mathbf{y})}, \end{aligned} \quad (9)$$

where the above equation comprises a-priori, systematic and extrinsic recursive terms. In practice, the computation of the probability $P(\mathbf{y})$ is not necessary since at each iteration, APP's $P(\mathbf{u}_k = i/\mathbf{y})$ in (9) can be normalized such that $\sum_i P(\mathbf{u}_k = i/\mathbf{y}) = 1$.

Sub-optimal Log-MAP Algorithm

Expression (9) provides the APP for each of the possible transmitted symbol vectors. As a fact, there are two practical problems that affect this computation: (1) it is computationally intensive requiring a large number of multiplications per symbol, (2) it is sensitive to numerical round-off errors. To overcome these problems, a sub-optimal logarithmic form of the algorithm is adopted. The best benefit of executing the MAP algorithm in the logarithmic domain is that multiplications becomes additions and that computations of exponentials is avoided. A drawback is however, that sums of exponentials are not simpler to calculate. An approximation to a sum of exponentials in the log-domain can be obtained as follows: consider the *Jacobian logarithm*,

$$\log\left(e^x + e^y\right) = \max\left(x, y\right) + \log\left(1 + \exp\left(-\left|y - x\right|\right)\right). \quad (10)$$

This suggests that addition in the log-domain becomes a maximization operation combined with a correction function. Furthermore, when y and x are not equal, that correction function is close to zero. Thus a reasonable approximation to the above equation is

$$\log\left(e^x + e^y\right) \approx \max\left(x, y\right). \quad (11)$$

Relation (11) is used to convert (3)-(9) to the log-demain. For example, (8) becomes in the log-domain:

$$\begin{aligned} \overline{\gamma}(s', s, \mathbf{y}_k) &= \log[\sum_i \gamma_i(s', s, \mathbf{y}_k)] \\ &= \max_i \log[\gamma_i(s', s, \mathbf{y}_k)]. \end{aligned} \quad (12)$$

It is of interest to take a close look at (12). The branch transition probability in the log-domain can be written

$$\begin{aligned} \log[\gamma(s', s, \mathbf{y}_k)] &= \log P(\mathbf{y}_k/\mathbf{u}_k = i, s', s) \\ &+ \log P(\mathbf{u}_k = i/s', s) + \log P(s/s') \\ &= -\frac{1}{2}\log(\frac{\pi N_0}{E_s}) - \frac{E_s}{N_0}\|\mathbf{y}_k - \mathbf{H}\mathbf{u}_k\|^2 + \log P(\mathbf{u}_k = i). \end{aligned} \quad (13)$$

Since $s' \rightarrow s$ occurs only for $\mathbf{u}_k = i$, we have $P(s/s') = P(\mathbf{u}_k = i)$. If there does not exist an i such that $P(\mathbf{u}_k = i/s', s) = 1$, then $\overline{\gamma}(s', s, \mathbf{y}_k) = -\infty$. Note that the second term of the branch metric computation in (13) is exactly the same as the branch metric used by the classical Viterbi algorithm. The forward probability α_k and backward probability β_k follow processing similar to (13).

Let the STCM APP1 output at time k be

$$M_1(\mathbf{u}_k) = \log P(\mathbf{u}_k = i/\mathbf{y}_k^{(1)}), \quad (14)$$

where $\mathbf{y}_k^{(1)}$ is the input data to APP1. Ideally, $\mathbf{y}_k^{(1)}$ would consists of the noisy and faded signal transmitted by STCM1.

However, due to the encoder architecture, the input to APP1 consists of: (1) correct systematic data, (2) parity data that is alternately provided by STCM1 or STCM2, (3) noise. Thus, the input to APP1 consists of a sequence that contains correct STCM1 symbols punctured by STCM2 symbols. To overcome this difficulty, at the times corresponding to the punctured symbols, the data input to APP1 is set to zero, and the only input to APP1 is the *a priori* information. When not set to zero, the input to APP1 is given by the metric computation $\left\|\mathbf{y}_k^{(1)} - \mathbf{H}\mathbf{u}_k\right\|^2$, where $\mathbf{y}_k^{(1)}$ is a vector of non-punctured values corresponding to decoder 1, the norm is Euclidean, and $\mathbf{u}_k$ is the hypothesized transmitted vector. Subsequently, APP1 is computed from this metric value and the *a priori* information. A distinguishing feature of turbo-STCM iterative decoding is that the *a priori* input to APP1 is provided by the output of APP2. There is a caveat, however. Only new information generated by APP2 can be fed to APP1. The APP2 output $M_2\left(\mathbf{u_k}\right)$ can be written

$$M_2\left(\mathbf{u}_k\right) = \Psi_2\left(\mathbf{y}_k^{(2)}/\mathbf{u}_k\right) + L_2\left(\mathbf{u}_k\right), \tag{15}$$

where $\Psi_2\left(\mathbf{y}_k^{(2)}/\mathbf{u}_k\right)$ is the *extrinsic* information derived by APP2 at the current stage of decoding, which will be used as the a-priori information of the APP1 decoder in the next iteration, and $L_2\left(\mathbf{u_k}\right) = \log P\left(\mathbf{u}_k = i\right)$ is the *a-priori* information into APP2. To preclude multiple uses of the same information, only $\Psi_2\left(\mathbf{y}_k^{(2)}/\mathbf{u}_k\right)$ is fed to APP1 as *a priori* information. Similarly, the APP1 output is separated into *extrinsic* and *a priori* parts:

$$\Psi_1\left(\mathbf{y}_k^{(1)}/\mathbf{u}_k\right) = M_1\left(\mathbf{u}_k\right) - L_1\left(\mathbf{u}_k\right). \tag{16}$$

Then $\Psi_1\left(\mathbf{y}_k^{(1)}/\mathbf{u}_k\right)$ is fed to APP2 as *a priori* information.

IV. Simulation Results

In this section, we present simulation results on the performance of the new turbo-STCM scheme for multiple transmit and receive antennas. The simulation was performed for frames of 1024 information bits and 4-PSK modulation transmitted over a quasi-static, flat Rayleigh fading channel. The quasi-static assumption means that the channel was assumed constant during a frame, but independent frame-to-frame. For illustration purposes, performance is also provided for a 4-PSK, eight-state conventional STCM scheme with $N = 2$ transmit and $M = 2$ receive antennas as given in [4], and for a 8-PSK, eight-state turbo-TCM single-input single-output scheme [12]. The interleaver/de-interleaver is chosen to be pseudo-random and the STCM trellis diagram has no parallel transitions. Note that all schemes correspond to a throughput of 2 b/s/Hz per transmit antenna.

Fig. 5 and Fig. 6 shows the bit error rate (BER) performance of turbo-STCM with two transmit and two receive antennas for various number of iterations. In Fig. 5, the BER is plotted versus the ratio E_b/N_0 dB per receive antenna. It is observed that at BER $= 10^{-3}$, the gain achieved by additional iterations is respectively, 3 dB, 2.7 dB, and 1.7 dB, where the last figure is for the gain obtained by adding four iterations for a total of eight iterations. In Fig. 6, the BER is plotted versus the number of iterations for various E_b/N_0 values. From the two figures, it can be seen that the decoding methods converge, i.e., little is gained by performing more than four iterations. Four decoding iterations are used in the comparisons with other schemes.

Fig. 7 provides a comparison of the following methods: (1) 4-PSK eight-state turbo-STCM with 2 transmit antennas and 2 receive antennas (2T2R), (2) 4-PSK four-state turbo-STCM 2T2R, (3) 4-PSK four-state turbo-STCM 2T1R, (4) conventional 4-PSK eight-state 2T2R STCM, and (5) 8-PSK eight-state turbo-TCM 1T1R. It is observed that at BER $= 10^{-4}$, 2T2R eight-state turbo-STCM has the following advantages: 2.8 dB over 2T2R four-state turbo-STCM, 4.7 dB over eight-state 2T2R STCM, 10 dB over 2T1R turbo-STCM, and 20 dB over single antenna turbo-TCM.

V. Conclusions

In this paper we introduced a new class of turbo-codes with space-time codes as the constituent codes. The new turbo-STCM codes combine the high temporal performance of turbo-coding with the spatial diversity advantage of space-time processing and the bandwidth efficiency of coded modulation under a *single* framework. We outline a parallel turbo-STCM encoder scheme with two constituent systematic space-time codes. The bit-wise interleaver is replaced by an interleaver operating on groups of bits. A sub-optimal MAP iterative decoder is also presented. The decoder operates in the logarithmic domain to avoid numerical problems and to reduce the MAP decoding complexity. Using simple STCM component encoders (four or eight states), system performance is studied for a 4-PSK turbo-STCM code with 2 transmitter antennas and 2 receiver antennas. It is shown that significant gains can be obtained in performance over a Rayleigh fading channel. In particular, an advantage of 4.7 dB is achieved over conventional STCM.

Such turbo-STCM codes can be easily extended to higher efficiency modulation, larger interleaver size and more than two parallel constituent STCM encoders for additional coding and diversity gain. These codes have the potential to facilitate high-rate data transmission over fading channels in future wireless networks.

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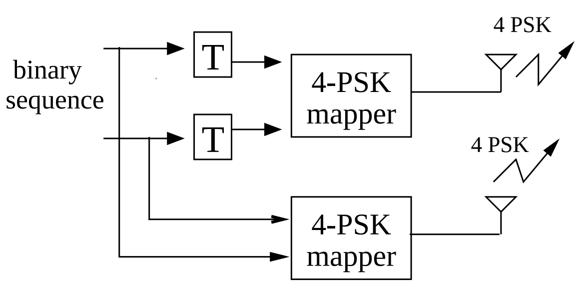

Fig. 1: Systematic STCM encoder for 4-PSK, four-state, 2 transmit antennas, 2b/s/Hz.

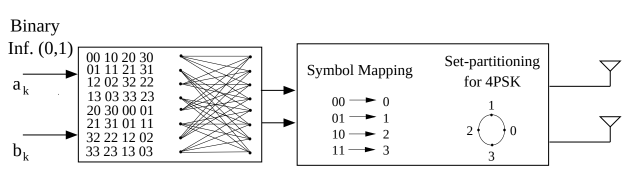

Fig. 2: Recursive systematic STCM encoder for 4-PSK, eight-state, 2 transmit antennas, 2b/s/Hz.

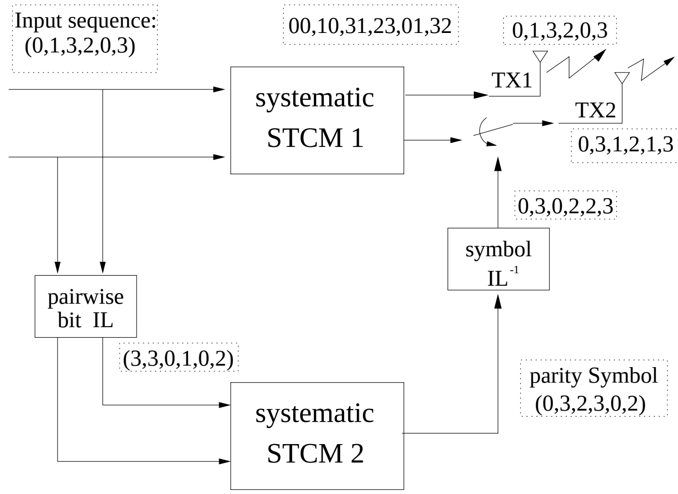

Fig. 3: Turbo-STCM encoder, 2 transmit antennas, 2 b/s/Hz.

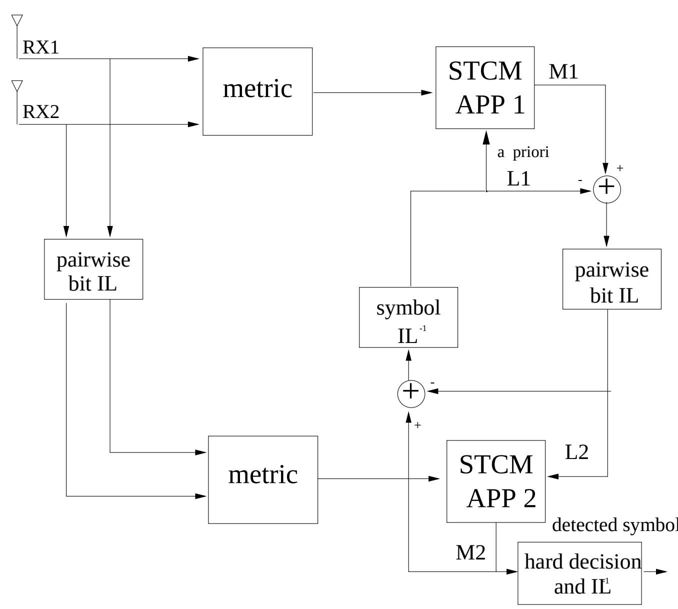

Fig. 4: Turbo-STCM decoder.

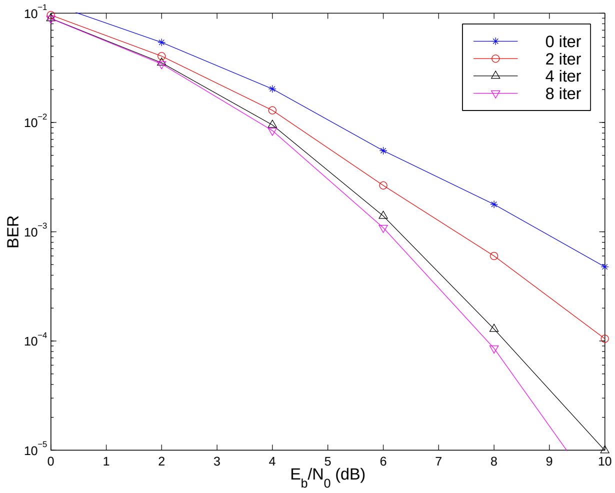

Fig. 5: Performance of turbo-STCM decoder, 2 transmit and 2 receive antennas.

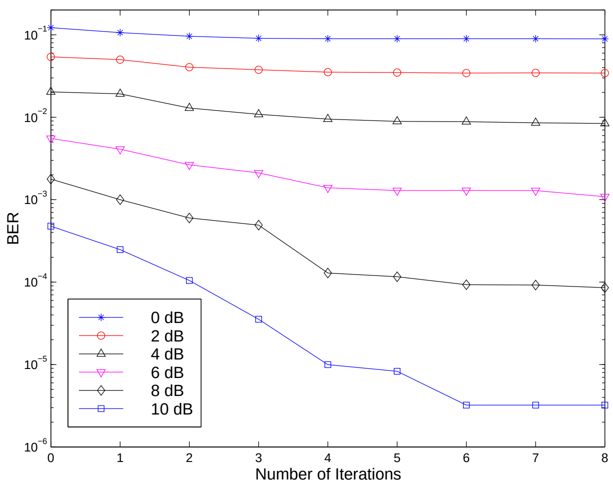

Fig. 6: Convergence of turbo-STCM iterative decoding.

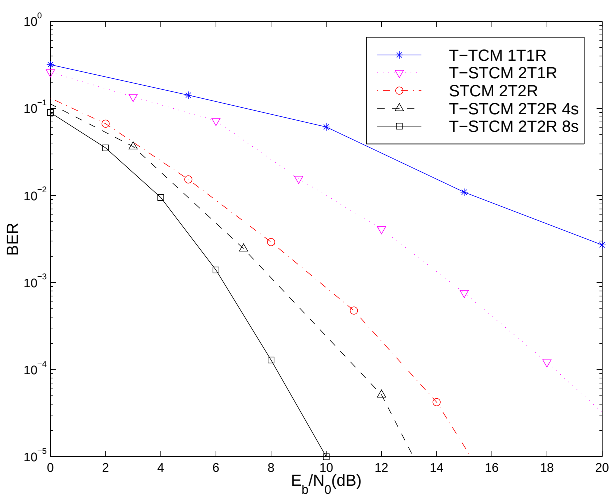

Fig. 7: BER for flat Rayleigh fading, 2 b/s/Hz, input block length $L = 1024$ bits.