

Iterative Estimation and Cancellation of Clipping Noise for OFDM Signals

Hangjun Chen and Alexander M. Haimovich, *Senior Member, IEEE*

Abstract—Clipping is an efficient and simple method to reduce the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals. However, clipping causes distortion and out-of-band radiation. In this letter, a novel iterative receiver is proposed to estimate and cancel the distortion caused by clipping noise. The proposed method is applied to clipped and filtered OFDM signals. It is shown by simulation that for an IEEE 802.11a typical scenario the system performance can be restored to within 1 dB of the nonclipped case with only moderate complexity increase and with no bandwidth expansion.

Index Terms—Clipping noise, iterative method, orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) is one of the technologies considered for 4G broadband wireless communications due to its robustness against multipath fading and relatively simple implementation compared to single carrier systems. One of the main drawbacks of OFDM is its high peak-to-average power ratio (PAPR).

Deliberate digital clipping of the OFDM signal [1], [2], is a simple and efficient way of controlling the PAPR. The clipping process is characterized by the clipping ratio (CR), defined as the ratio between the clipping threshold and the rms level of the OFDM signal. Clipping is a nonlinear process that may lead to significant distortion and performance loss. In particular, clipping at the Nyquist sampling rate causes the clipping noise to fall in-band and suffers considerable peak regrowth after digital to analog (D/A) conversion. In [1], [2], it is shown that clipping the oversampled OFDM signal reduces the peak regrowth after D/A conversion and generates less in-band distortion. But, it causes out-of-band noise that needs to be filtered. It is this problem of distortion caused by intentional clipping with the additional constraint of out-of-band noise filtering that we are addressing in this letter.

Unlike additive white Gaussian noise (AWGN), clipping noise is generated by a process known and that can be recreated at the receiver, and subsequently be removed. Based on this observation and the analysis of the clipping process, a novel iterative clipping noise canceler is proposed for clipped and filtered OFDM signals.

Manuscript received January 18, 2003. The associate editor coordinating the review of this letter and approving it for publication was Prof. D. Taylor. This work was supported in part by the National Science Foundation under Award CCR 0085846 and by the New Jersey Center for Wireless Telecommunications.

The authors are with the Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102 USA (e-mail: hxc1170@njit.edu; haimovic@njit.edu).

Digital Object Identifier 10.1109/LCOMM.2003.814720

Schemes for mitigating the effect of clipping noise were also proposed in [3] and [4]. However, both schemes produce acceptable loss in SNR (less than 1 dB) only for CR ≥ 4 dB. Furthermore, the scheme in [3] only applies to nonfiltered signals, and the scheme in [4] also needs significant bandwidth expansion to work well. The scheme proposed in this letter, performs well at low CR and is applicable to filtered signals (i.e., it does not require bandwidth expansion). Another major difference is that while [3] and [4] attempt to *reconstruct* the “affected” or “lost” time domain signal samples, the proposed scheme instead regenerates and *cancels* the clipping noise samples in the frequency domain. We will demonstrate that the reconstruction of signals to their nonclipped form is inherently more error prone than the estimation and cancellation of the clipping noise only.

The rest of the letter is organized as follows. Section II introduces the system model. Numerical results are presented in Section III. Section IV draws conclusions.

II. SYSTEM MODEL

The low pass equivalent of an OFDM symbol can be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} C_k \exp(j2\pi k f_0 t) \quad 0 \leq t \leq T \quad (1)$$

where N is the number of subcarriers, f_0 is the subcarrier spacing, T is the symbol duration, C_k is the complex modulated symbol. An OFDM block consists of the sequence $\{C_k\}_{k=0}^{N-1}$. The PAPR of the transmitted OFDM symbol is defined as:

$$\text{PAPR} = \frac{\max_{0 \leq t \leq T} |s(t)|^2}{P_{av}} \quad (2)$$

where P_{av} is the average power of the transmitted symbol and the maximum is sought over the symbol duration. Note that the PAPR in (2) is defined for the average power P_{av} measured after clipping and filtering. Consider the OFDM signal of (1) sampled at time intervals $\Delta t = T/JN$, where J is the oversampling factor. An oversampled signal can be obtained by padding $\{C_k\}_{k=0}^{N-1}$ with $(J-1)N$ zeros and taking the inverse discrete Fourier transform (IDFT). The discrete-time OFDM signal sampled at time instant $t = n\Delta t$ is then expressed

$$s_n \triangleq s(n\Delta t), \quad n = 0, \dots, JN - 1. \quad (3)$$

Clipping and filtering are performed digitally on the OFDM symbol at the transmitter as described in [2]. To reduce peak power regrowth and distortion, the time domain signal is usually oversampled by a factor greater than two. Following oversampling, the amplitude of the time domain signal samples are

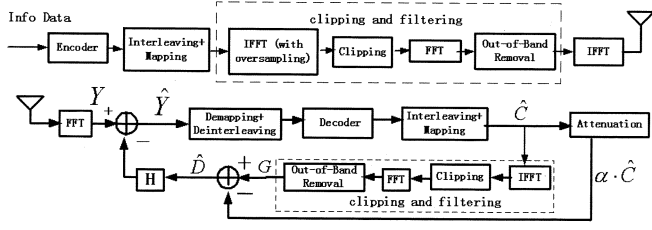


Fig. 1. OFDM transmitter with (top) clipping and filtering and (bottom) receiver with iterative distortion cancellation.

limited by a threshold A . Let \bar{s}_n be a clipped time sample with the phase left unchanged. Then

$$|\bar{s}_n| = \begin{cases} |s_n| & \text{if } |s_n| \leq A \\ A & \text{if } |s_n| > A. \end{cases} \quad (4)$$

It was shown in [2] that the clipped signal $\{\bar{s}_n\}_{n=0}^{JN-1}$ can be modeled as the aggregate of an attenuated signal component and clipping noise $\{d_n\}_{n=0}^{JN-1}$

$$\bar{s}_n = \alpha s_n + d_n \quad n = 0, \dots, JN - 1 \quad (5)$$

where the attenuation α is a function of the clipping ratio γ , defined as $\gamma = A/\sqrt{P_{\text{in}}}$, with P_{in} the average signal power before clipping, [2]:

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2} \text{erfc}(\gamma). \quad (6)$$

To remove the out-of-band components resulting from clipping, the time domain samples (5) are converted back to the frequency domain by applying the discrete Fourier transform (DFT) to the sequence $\{\bar{s}_n\}_{n=0}^{JN-1}$, to obtain the sequence $\{\bar{C}_k\}_{k=0}^{JN-1}$ (see Fig. 1). Using (5), the terms \bar{C}_k can be expressed

$$\bar{C}_k = \alpha C_k + D_k \quad k = 0, \dots, JN - 1 \quad (7)$$

where $\{C_k\}_{k=0}^{JN-1}$ and $\{D_k\}_{k=0}^{JN-1}$ are respectively, the DFT of $\{s_n\}_{n=0}^{JN-1}$ and $\{d_n\}_{n=0}^{JN-1}$ in (5). In particular, $\{D_k\}_{k=0}^{JN-1}$ is the sequence representing the clipping noise in the frequency domain. Out-of-band components are removed by processing only the in-band-components $\{\bar{C}_k\}_{k=0}^{N-1}$ through an N -point IDFT (see Fig. 1). For simplicity, adding a guard interval is ignored since it has no bearing on the analysis in this letter.

Assuming perfect synchronization and following DFT, the signal at the receiver is

$$Y_k = H_k(\alpha C_k + D_k) + Z_k \quad k = 0, \dots, N - 1 \quad (8)$$

where H_k is the complex channel gain of the k -th subcarrier assumed to be perfectly known, and Z_k is AWGN.

The main idea of the proposed clipping noise cancellation scheme is to recreate the clipping process at the receiver using detected symbols, then estimate and cancel the frequency domain clipping noise caused by it. The receiver works in an iterative fashion as described below with reference to Fig. 1.

- Channel observations $\{Y_k\}_{k=0}^{N-1}$ are decoded and detected. Let decisions of the transmitted sequence be denoted $\{\hat{C}_k\}_{k=0}^{N-1}$.
- The sequence $\{\hat{C}_k\}_{k=0}^{N-1}$ is then processed through two branches. One branch regenerates the attenuated

frequency domain samples of the nonclipped signals $\{\alpha \hat{C}_k\}_{k=0}^{N-1}$. The other branch regenerates the clipped signals at the receiver by passing $\{\hat{C}_k\}_{k=0}^{N-1}$ through the same clipping and filtering process as at the transmitter. Denote regenerated clipped samples by $\{G_k\}_{k=0}^{N-1}$. Similar to (7), these clipped signals can be represented as the sum of an attenuated nonclipped signal $\alpha \hat{C}_k$ and the clipping noise \hat{D}_k ,

$$G_k = \alpha \hat{C}_k + \hat{D}_k, \quad k = 0, \dots, N - 1. \quad (9)$$

Since G_k and \hat{C}_k are observable and α can be computed from (6), the clipping noise \hat{D}_k can be estimated as

$$\hat{D}_k = G_k - \alpha \hat{C}_k, \quad k = 0, \dots, N - 1. \quad (10)$$

- The estimated clipping noise terms \hat{D}_k are subtracted from the current channel observation to obtain the refined channel observation for the next iteration

$$\begin{aligned} \hat{Y}_k &= Y_k - H_k \hat{D}_k \quad k = 0, \dots, N - 1 \\ &= \alpha H_k C_k + H_k (D_k - \hat{D}_k) + Z_k \end{aligned} \quad (11)$$

where $(D_k - \hat{D}_k)$ is the residual clipping noise.

- Go back to step (a) and replace $\{Y_k\}_{k=0}^{N-1}$ with $\{\hat{Y}_k\}_{k=0}^{N-1}$. The whole loop (a)–(d) continues for a few iterations. As the iteration proceeds, the estimation of the clipping noise components $\{\hat{D}_k\}_{k=0}^{N-1}$ is found to become increasingly accurate and the receiver performance is improved.

From Fig. 1 and the discussion above, each iteration for clipping noise estimation and cancellation requires a single pair of IFFT/FFT operations. In the numerical results provided in Section III it is shown that no more than two of these iterations are required, implying that the proposed method incurs only a moderate increase in the complexity of the receiver.

The scheme described above estimates and cancels the clipping noise. An alternative signal reconstruction approach, which attempts to restore the clipped signal to its nonclipped form, is more sensitive to decision errors. Indeed, using earlier notation, define the estimated difference between the frequency domain samples of the nonclipped and clipped signals $\Delta C_k = \hat{C}_k - G_k$. The reconstructed frequency-domain samples are by definition

$$\hat{Y}_k^{(R)} = Y_k + H_k \Delta C_k, \quad k = 0, \dots, N - 1. \quad (12)$$

Substituting $\Delta C_k = \hat{C}_k - G_k$ and G_k from (9), and applying the first relation in (11), we obtain

$$\begin{aligned} \hat{Y}_k^{(R)} &= Y_k - H_k \hat{D}_k + (1 - \alpha) H_k \hat{C}_k \\ &= \hat{Y}_k + (1 - \alpha) H_k \hat{C}_k, \quad k = 0, \dots, N - 1. \end{aligned} \quad (13)$$

The contrast between the two approaches that mitigate clipping effects are evident: $\hat{Y}_k^{(R)}$, the reconstructed observation with the signal restored to its nonclipped form, has an extra term $(1 - \alpha) H_k \hat{C}_k$ compared to \hat{Y}_k , the corrected observation with the clipping noise removed. Note that \hat{C}_k is the decision at the previous iteration and should not be directly passed to the next iteration as part of the refined channel observation. Hence, (12) contains an additional term, which will propagate decision errors. Only for large clipping ratios $\alpha \approx 1$, this error term is negligible.

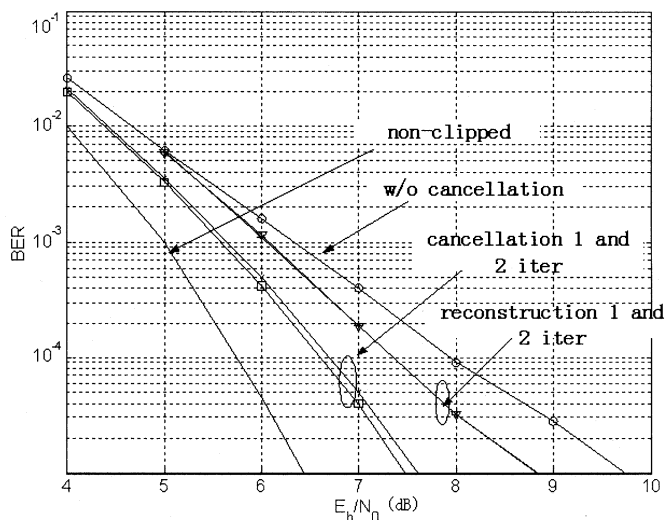


Fig. 2. BER performance of the proposed receiver over AWGN channel and comparison with "signal reconstruction" approach.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical results are presented for the proposed clipping noise estimation and cancellation scheme over both AWGN and fading channels. Transmitted signals are clipped to a clipping ratio of $\gamma = 0$ dB and filtered. The simulation model was designed to match IEEE Std. 802.11a. The convolutional encoder used was a rate 1/2 with generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$. The number of subcarriers was $N = 64$, and the modulation was 16-QAM. Decoding was carried out using a soft Viterbi algorithm. In the figures, E_b/N_0 is measured after signal clipping and filtering.

Fig. 2 shows the bit error rate (BER) performance of the proposed receiver over the AWGN channel, with comparison to that of a receiver without clipping noise cancellation and to a receiver with signal reconstruction as discussed in Section II. It is observed that for the proposed scheme, an improvement of about 2.5 dB relative to the case without cancellation is obtained after only one iteration at $\text{BER} = 10^{-5}$, which is only about 1-dB loss compared to the nonclipped case. Signal reconstruction performs worse by about 1.2 dB than the proposed method.

Fig. 3 shows the BER performance of the proposed receiver over a Rayleigh fading channel with an exponentially decaying power delay profile, with normalized delay spread equal to 2 [5]. It can be observed that after one iteration, the performance of the clipped and filtered signals can be restored to within 1 dB of the nonclipped case at $\text{BER} = 10^{-5}$. This represents an improvement of about 3 dB compared to the case without processing for the mitigation of clipping effects. In this case, signal reconstruction performs worse by about 2 dB than the proposed method.

These simulations demonstrate that the proposed clipping noise cancellation scheme can significantly restore the system performance. Incremental gains diminish after the first iteration. This can be explained by the reasoning that some OFDM blocks are too badly damaged by clipping for the iterative process to converge and more iterations will not help.

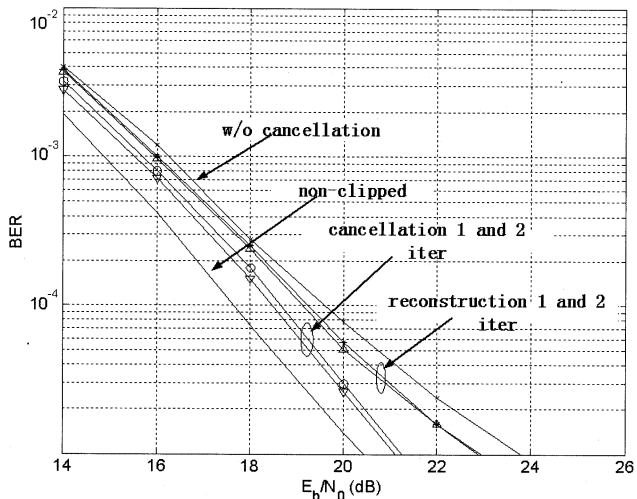


Fig. 3. BER performance of the proposed receiver over Rayleigh fading channel and comparison with "signal reconstruction" approach.

In the simulation for the fading channel case, we assumed the channel gain perfectly known at the receiver. That is a reasonable assumption since with IEEE 802.11a, the PAPR of the training symbols is designed to be only 3 dB and clipping is not required. It follows that clipping has no impact on channel estimation. Pilots inserted in the data symbols for phase tracking are distorted by clipping noise. In our scheme, however, the clipping noise is estimated and these pilots can be restored before being processed. Algorithms for iterative channel estimation, which can be combined with the proposed algorithm, are found in [6] and [7], etc.

IV. CONCLUSION

In this letter, we propose a novel iterative distortion cancellation receiver for clipped and filtered OFDM signals. It is shown the performance of a clipped and filtered OFDM system can be significantly improved with only moderate complexity increase at the receiver. This receiver is especially suitable for IEEE 802.11a wireless LAN, since it allows signals to be significantly clipped with only slight performance degradation.

REFERENCES

- [1] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," in *Proc. IEEE Vehicular Technology Conf. (VTC)*, May 1997, pp. 1634–1638.
- [2] H. Ochiai and H. Imai, "Performance analysis of deliberately clipped OFDM signals," *IEEE Trans. Commun.*, vol. 50, pp. 89–101, Jan. 2002.
- [3] D. Kim and G. L. Stuber, "Clipping noise mitigation for OFDM by decision-aided reconstruction," *IEEE Commun. Lett.*, vol. 3, pp. 4–6, Jan. 1999.
- [4] H. Saedi, M. Sharif, and F. Marvasti, "Clipping noise cancellation in OFDM systems using oversampled signal reconstruction," *IEEE Commun. Lett.*, vol. 6, pp. 73–75, Feb. 2002.
- [5] N. Chayat, "Tentative criteria for comparison of modulation methods," Doc. IEEE 802.11-97/96, 1997.
- [6] B. Lu, X. Wang, and Y. Li, "Iterative receivers for space-time block-coded OFDM systems in dispersive fading channels," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 213–225, Apr. 2002.
- [7] E. Jaffrot and M. Siala, "Turbo channel estimation for OFDM systems on highly time and frequency selective channels," in *Proc. 2000 IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP '00*, vol. 5, 2000, pp. 2977–2980.