An Iterative Method to Restore the Performance of Clipped and Filtered OFDM Signals

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Abstract— **Clipping is an efficient and simple method to reduce the peak-to-average power ratio (PAPR) of OFDM signals. However, clipping causes distortion and out-of-band radiation. In this paper, a novel iterative receiver is proposed to estimate and cancel the distortion caused by clipping noise. The proposed method is applied to clipped and filtered OFDM signals. It is shown by simulation that for an 802.11a system, the PAPR can be reduced to as low as 4 dB while the system performance can be restored to within 1 dB of the non-clipped case with only moderate complexity increase and with no bandwidth expansion.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the technologies considered for 4G broadband wireless communications due to its robustness against multipath fading and relatively simple implementation compared to single carrier systems. To preserve both amplitude and phase information, OFDM transmitters utilize linear power amplifiers. One of the main drawbacks of OFDM is the high cost of linear power amplifiers with high dynamic range. Such amplifiers are required due to the high peak-to-average power ratio (PAPR) when the OFDM signal consists of a large number of subcarriers.

Various schemes has been proposed to reduce the PAPR of OFDM signals [1], [2], [3] etc. Among these techniques, deliberate digital clipping of the OFDM signal before amplification is a simple and efficient way of controlling the PAPR. The clipping process is characterized by the clipping ratio (CR), defined as the ratio between the clipping threshold and the rms level of the OFDM signal. Clipping is a non-linear process, which may lead to significant distortion and performance loss. In particular, clipping at the Nyquist sampling rate will cause all the clipping noise to fall in-band and suffers considerable peak regrowth after digital to analog (D/A) conversion. In [3], [4], it was shown that clipping the oversampled OFDM signal reduces the peak regrowth after D/A conversion and generates less in-band distortion. But it causes out-of-band noise that needs to be filtered. It is this problem of distortion caused by intentional clipping with the additional constraint of out-of-band noise filtering that we are addressing in this paper. Clipping noise is the factor limiting performance in OFDM systems operating at high signal to noise ratio (SNR).

Unlike AWGN, clipping noise is generated by a process, which is known and can be recreated at the receiver and subsequently removed. Based on this observation and the analysis of the clipping process, a novel iterative clipping noise canceler is proposed for clipped and filtered OFDM signals.

Schemes for mitigating the effect of clipping noise were also proposed in [5] and [6]. However, both schemes produce acceptable loss in SNR (less than 1 dB) only for CR $\geq$ 4 dB, furthermore, the scheme in [6] also needs significant bandwidth expansion to work well. Also [5] only applies to Nyquist rate clipping. The scheme proposed in this paper, performs well at low CR and is applicable to filtered signals (i.e., it does not require bandwidth expansion). A major difference is that while [5] and [6], as well as other proposed methods, attempt to *reconstruct* the ”affected” or ”lost” *time* domain *signal* samples, the proposed scheme instead regenerates and *cancels* the *clipping noise* in the *frequency* domain. We will argue that the reconstruction of time domain signals is inherently more error prone than the estimation of the clipping noise only. The analysis and simulations in this paper are based on digital clipping with out-of-band filtering [3], but the scheme equally applies if the out-of-band radiation is not filtered.

The proposed clipping noise cancellation method is also applied to the repeated clipping scheme [7], which is an extension of deliberate clipping [3]. The resulting system can reduce the PAPR of a 64-subcarrier OFDM system to as low as 4 dB with only slight performance loss.

The rest of the paper is organized as follows. Section II introduces the system model based on the deliberate clipping method. Numerical results are presented in Section III. In section IV we present the application of the proposed method to repeated clipping. Section V draws conclusions.

II. SYSTEM MODEL

The low pass equivalent of an OFDM can be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} C_k \exp(j2\pi k f_0 t) \qquad 0 \leq t \leq T, \quad (1)$$

where N is the number of subcarriers, f_0 is the subcarrier spacing, T is the symbol duration, C_k is the complex modulated symbol. Keeping with the IEEE Std 802.11a, the modulated symbols are obtained by mapping an encoded bit stream. An OFDM block consists of the sequence of symbols

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$\{C_k\}_{k=0}^{N-1}$. The PAPR of the transmitted OFDM signal is defined as:

$$\mathrm{PAPR} = \frac{\max_{0 \le t \le T} |s(t)|^2}{P_{av}}, \tag{2}$$

where P_{av} is the average power of the transmitted symbol and the maximum is sought over the symbol duration. Note that the PAPR in (2) is defined for the average power P_{av} measured after clipping and filtering. Consider the OFDM signal of (1) sampled at time intervals $\Delta t = T/JN$, where J is the oversampling factor. An oversampled signal can be obtained by padding $\{C_k\}_{k=0}^{N-1}$ with $(J-1)N$ zeros and taking the inverse discrete Fourier transform (IDFT). The discrete-time OFDM signal sampled at time instant $t = n\Delta t$ is then expressed

$$s_n \triangleq s(n\Delta t) \qquad n = 0, ..., JN-1. \tag{3}$$

The clipping and filtering operation is performed digitally at the transmitter as described in [3]. To reduce peak power regrowth and distortion, the time domain signal is usually oversampled by a factor greater than two. Following oversampling, the amplitude of the time domain signal samples are limited by a threshold A. Let $\overline{s}_n$ be a clipped time sample with the phase left unchanged. Then,

$$|\overline{s}_n| = \begin{cases} |s_n| & \text{if } |s_n| \le A \\ A & \text{if } |s_n| > A \end{cases} \tag{4}$$

It was shown in [3] and [8] that the clipped signal $\{\overline{s}_n\}_{n=0}^{JN-1}$ can be modeled as the aggregate of an attenuated signal component and clipping noise $\{d_n\}_{n=0}^{JN-1}$

$$\overline{s}_n = \alpha s_n + d_n \qquad n = 0, ..., JN-1, \tag{5}$$

where the attenuation factor α is a function of the clipping ratio γ, defined as $\gamma = A/\sqrt{P_{in}}$, with P_{in} the average signal power before clipping, [3]:

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2}\,\mathrm{erfc}(\gamma). \tag{6}$$

To remove the out-of-band components resulting from clipping, the time domain samples (5) are converted back to frequency domain by applying the discrete Fourier transform (DFT) to the sequence $\{\overline{s_n}\}_{n=0}^{JN-1}$, to obtain the sequence $\{\overline{C_k}\}_{k=0}^{JN-1}$. Using (5), the terms $\overline{C_k}$ can be expressed

$$\overline{C}_k = \alpha C_k + D_k \qquad k = 0, ..., JN-1, \tag{7}$$

where $\{C_k\}_{k=0}^{JN-1}$ and $\{D_k\}_{k=0}^{JN-1}$ are respectively, the DFT of $\{s_n\}_{n=0}^{JN-1}$ and $\{d_n\}_{n=0}^{JN-1}$ in (5). In particular, $\{D_k\}_{k=0}^{JN-1}$ is the sequence representing the clipping noise in the frequency domain. Out-of-band components are removed by processing only the in-band-components $\{\overline{C_k}\}_{k=0}^{N-1}$ through an N-point IDFT. The resulting sequence is transmitted. For simplicity, adding a guard interval is ignored since it has no bearing on the analysis in this paper. A schematic diagram of the proposed OFDM transmitter is shown in the top part of Fig. 1.

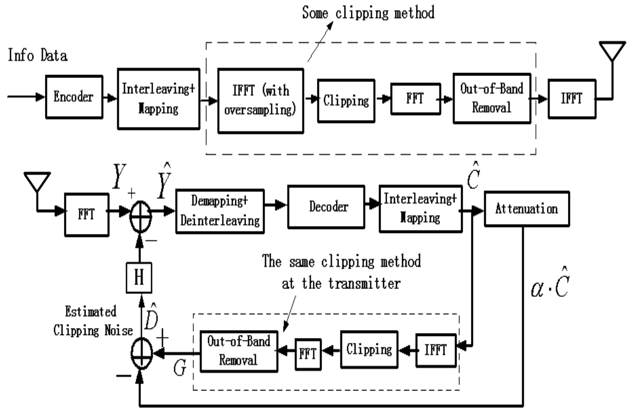

Fig. 1. Stucture of the proposed iterative distortion cancellation receiver and the OFDM transmitter with some possible clipping method.

Assuming perfect synchronization and following DFT, the signal at the receiver is

$$Y_k = H_k(\alpha C_k + D_k) + Z_k \qquad k = 0, ..., N-1, \tag{8}$$

where H_k is the complex channel gain of the k-th subcarrier assumed to be perfectly known and Z_k is AWGN.

The main idea of the proposed clipping noise cancellation scheme is to recreate the clipping process at the receiver using the detected symbols, then estimate and cancel the frequency domain clipping noise caused by it. The receiver works in an iterative fashion as described below with reference to Fig. 1.

(a) The channel observations $\{Y_k\}_{k=0}^{N-1}$ are decoded and detected. The decisions of the transmitted sequence are denoted $\{\widehat{C}_k\}_{k=0}^{N-1}$.

(b) Sequence $\{\widehat{C}_k\}_{k=0}^{N-1}$ is then processed through two branches. One branch regenerates the attenuated frequency domain samples of the non-clipped signals $\{\alpha\widehat{C}_k\}_{k=0}^{N-1}$. The other branch regenerates the clipped signals at the receiver by passing the $\{\widehat{C}_k\}_{k=0}^{N-1}$ through the same clipping and filtering process as in the transmitter. Denote regenerated clipped samples $\{G_k\}_{k=0}^{N-1}$. Similar to (7), the clipped signals can be represented as the sum of an attenuated non-clipped signal $\alpha\widehat{C}_k$ and the clipping noise $\widehat{D}_k$,

$$G_k = \alpha\widehat{C}_k + \widehat{D}_k \qquad k = 0, ..., N-1. \tag{9}$$

Since G_k and $\widehat{C}_k$ are observable and α can be computed from (6), the clipping noise $\widehat{D}_k$ can be estimated as

$$\widehat{D}_k = G_k - \alpha\widehat{C}_k \qquad k = 0, ..., N-1. \tag{10}$$

(c) The estimated clipping noise terms $\widehat{D}_k$ are subtracted from the current channel observation to obtain the refined channel observation for the next iteration

$$\begin{aligned} \widehat{Y}_k &= Y_k - H_k\widehat{D}_k \qquad k = 0, ..., N-1 \\ &= \alpha H_k C_k + H_k(D_k - \widehat{D}_k) + Z_k, \end{aligned} \tag{11}$$

where $(D_k - \widehat{D}_k)$ is the residual clipping noise.

(d) Go back to step (a) and replace $\{Y_k\}_{k=0}^{N-1}$ with $\{\widehat{Y}_k\}_{k=0}^{N-1}$.

The whole loop (a) - (d) continues for a few iterations. As the iteration proceeds, the estimation of the clipping noise components $\{\widehat{D}_k\}_{k=0}^{N-1}$ becomes more and more accurate and the receiver performance is improved.

From Fig. 1 and the discussion above, each iteration for clipping noise estimation and cancellation requires a single pair of IFFT/FFT operations and a decoding. In the numerical results provided in next section it is shown that no more than two of these iterations are required, implying that the proposed method incurs only a moderate increase of complexity at the receiver.

The proposed scheme described above estimates and cancels the clipping noise. An alternative signal reconstruction approach, which attempts to restore the clipped signal to its non-clipped form, is more sensitive to decision errors. Indeed, using earlier notation, define the estimated difference between the frequency domain samples of the non-clipped and clipped signals $\Delta C_k = \widehat{C}_k - G_k$. The reconstructed frequency domain samples are by definition

$$\widehat{Y}_k^{(R)} = Y_k + H_k \Delta C_k \qquad k = 0, ..., N-1. \qquad (12)$$

Substituting $\Delta C_k = \widehat{C}_k - G_k$ and G_k from (9), and applying the first relation in (11), we obtain

$$\begin{aligned} \widehat{Y}_k^{(R)} &= Y_k - H_k \widehat{D}_k + (1-\alpha) H_k \widehat{C}_k \\ &= \widehat{Y}_k + (1-\alpha) H_k \widehat{C}_k \qquad k = 0, ..., N-1. (13) \end{aligned}$$

Differences between two approaches to mitigating the clipping effects are evident: $\widehat{Y}_k^{(R)}$, the reconstructed observation with the signal restored to its non-clipped form, has an extra term $(1-\alpha)H_k\widehat{C}_k$ compared to $\widehat{Y}_k$, the corrected observation with the clipping noise removed. Note that $\widehat{C}_k$ is the decision at the previous iteration and should not be directly passed to the next iteration as part of the refined channel observation. Hence, (12) contains an additional term, which will propagate decision errors. Only for large clipping ratios $\alpha \approx 1$, this error term is negligible.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical results are presented for the proposed clipping noise estimation and cancellation scheme for clipped and filtered OFDM signals over both AWGN and fading channels. The clipping method used in the simulations was the one proposed in [3] with a clipping ratio set to $\gamma = 1$ and the out-of-band radiation removed. The simulation model was designed to match IEEE Std. 802.11a. The convolutional encoders used were the industry standard constraint length 7, rate 1/2 with generator polynomials $\mathrm{g}_0 = 133_8$ and $\mathrm{g}_1 = 171_8$. The number of subcarriers was $N = 64$, and the modulation was16-QAM. Decoding was carried out using a soft Viterbi algorithm. The system performance is measured as packet error rate (PER) where each packet consists of 16 OFDM symbols. In the figures, E_b/N_0 is measured after signal clipping and filtering.

Fig. 2 shows the complementary cumulative density function (CCDF) of the PAPR of digitally clipped OFDM signals with out-of-band radiation removed. For clipping ratio $\gamma = 1$, the PAPR is reduced from 12 dB to 6 dB.

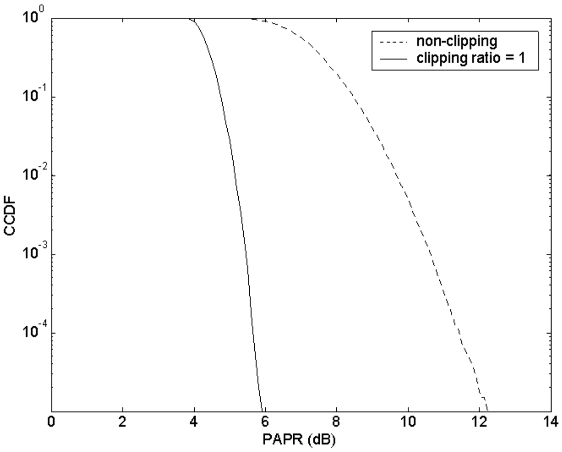

Fig. 2. CCDF of the PAPR of digitally clipped OFDM signal with clipping ratio equal to 1.

Fig. 3 shows the packet error rate (PER) performance of the proposed receiver over the AWGN channel. Performance is compared with that of a receiver without clipping noise cancellation and to a receiver with signal reconstruction as discussed in Section II. For reference, the performance of a system without clipping is also provided. For the proposed scheme, an gain of about 2 dB relative to the case without cancellation is achieved after only one iteration at PER = 0.01 and the system performance is restored to within 1 dB of the non-clipped case after two iterations. Notice that the performance gain over the non-cancellation case increases as SNR increases. This is because at high SNR the AWGN noise becomes relatively small and clipping noise begins to dominate. It is also observed that signal reconstruction performs worse by about 1.5 dB than the proposed method, proves our analysis in the last section.

Fig. 4 shows the PER performance of the proposed receiver over a Rayleigh fading channel with an exponentially decaying power delay profile, with normalized delay spread equal to 2 [9]. The channel is assumed to be constant over one packet and changes independently from packet to packet. It can be observed that after one iteration the performance of the clipped and filtered signals can be restored to within 1 dB of the non-clipping case. This represents an gain of about 2 dB at PER=0.01 compared to the case without processing for the mitigation of clipping effects. At PER=0.006, the gain becomes 2.8 dB. It is also observed that signal reconstruction performs worse by about 2 dB than the proposed method.

From the simulation results we can see that the proposed clipping noise cancellation scheme can significantly restore the performance. Also we can see that more than 2 iterations supplies diminishing benefit. The reason is that there exist some OFDM symbols which are too badly damaged by clipping for the iterative process to converge. This performance gap may be further narrowed by combining the proposed scheme with the bit or symbol interleaving method proposed in [2]. Since the transmitter knows exactly how the OFDM symbol is affected by clipping, the badly damaged symbols (according to some criterion) can be interleaved and re-clipped, which may result in less clipping noise.

In the simulation for the fading channel case, we assumed the channel gain perfectly known at the receiver. That is a reasonable assumption since with IEEE 802.11a, the PAPR of the training symbols is designed to be only 3 dB and

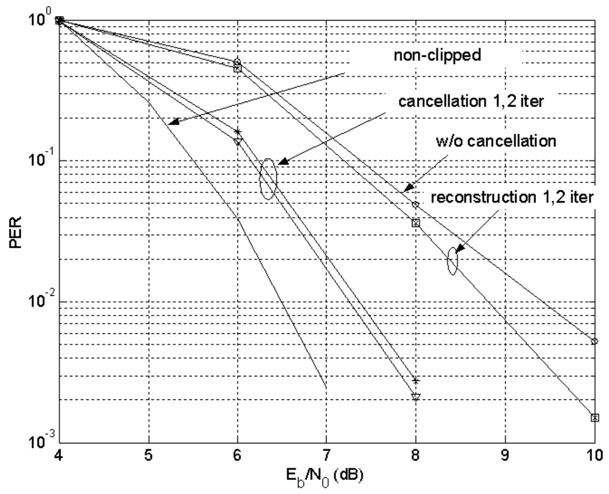

Fig. 3. PER performance of the proposed receiver over AWGN channel and comparison with "signal reconstruction" approach.

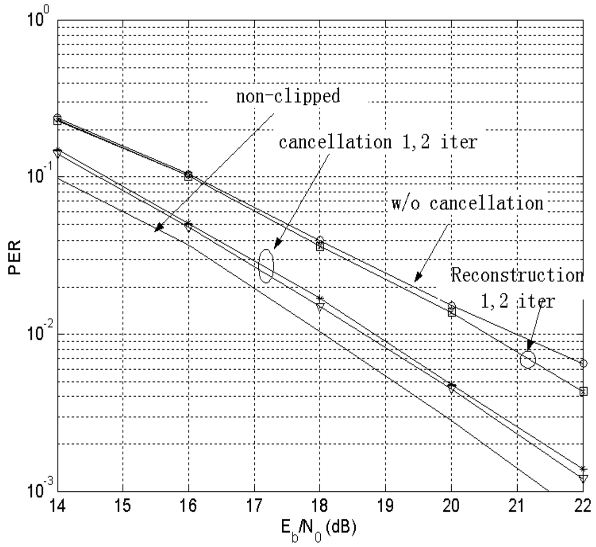

Fig. 4. PER performance of the proposed receiver over Rayleigh fading channel and comparison with "signal reconstruction" approach.

clipping is not required. It follows that clipping has no impact on channel estimation. Pilots inserted in the data symbols for phase tracking are distorted by clipping noise. In our scheme, however, the clipping noise is estimated and these pilots can be restored before being processed.

IV. APPLICATION TO REPEATED CLIPPING

In this section we apply the iterative clipping noise cancellation scheme to the repeated clipping proposed in [7]. It is shown in [7] that by repeating the digital clipping and filtering process as described in section II, but with higher clipping ratio at each time, the signals can be clipped to the same PAPR with less distortion. Without clipping noise cancellation, even with three or four times clipping and filtering the PAPR of the OFDM signal can be reduced only moderately (to 7 dB in [7]). Applying the proposed receiver, we show that the PAPR of the 64-subcarrier OFDM signal can be reduced to 4 dB and the clipping noise cancellation scheme restores the systems performance to within 1 dB of the non-clipping case.

The proposed receiver for repeated clipping has a similar structure as in Fig. 1, with the only difference that clipping and filtering are repeated to match the transmitter. In the simulation, the clipping and filtering the transmitter are repeated three times, with clipping ratios set to 1.5, 1.3 and 1.35 respectively. Fig. 5 shows the PAPR distribution (CCDF) of the 64-subcarrier OFDM signals with this set of clipping ratios. It is observed that the PAPR is reduced to 4 dB, an 8 dB reduction compared to the non-clipped case. The clipping ratios used in the simulation was chosen empirically and further research is required to determine how to choose optimal clipping ratio values.

Fig. 6 shows the PER performance of the system with repeated clipping. The channel model is the same Rayleigh fading channel as described in last section. It is observed that proposed receiver restores the system performance to within 1 dB of the non-clipped case after two iterations. This represents an improvement of about 2.5 dB at PER=0.01.

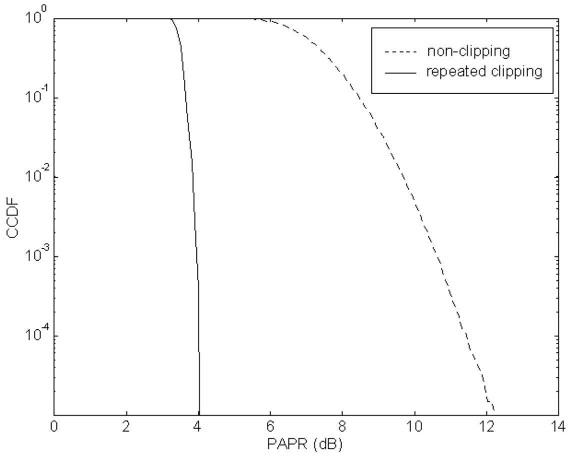

Fig. 5. CCDF of the PAPR of repeatedly clipped OFDM signal with clipping ratios equal to 1.5, 1.3 and 1.35.

V. CONCLUSION

In this paper, we propose a novel iterative distortion cancellation receiver for clipped and filtered OFDM signals. It is shown the performance of a clipped and filtered OFDM system can be significantly improved with only moderate complexity increase at the receiver. In other words, with the proposed receiver, the PAPR of the transmitted signal can be reduced more with acceptable performance loss. This receiver is especially suitable for IEEE 802.11a wireless LAN since it allows signals to be significantly clipped with only slight performance degradation.

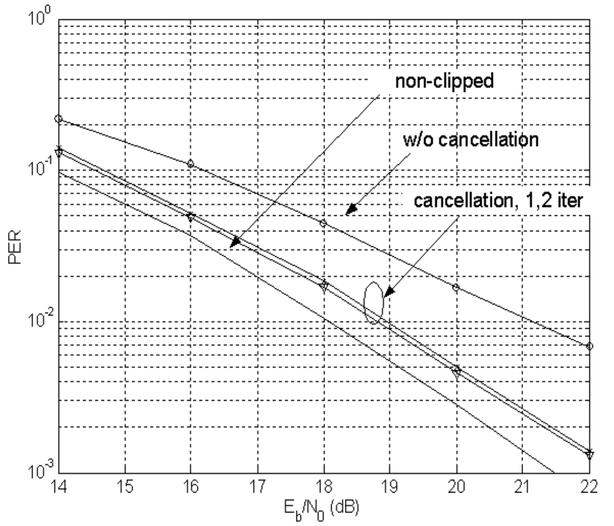

Fig. 6. PER performance of the proposed receiver for repeated clipping over Rayleigh fading channel with PAPR reduced to 4dB.

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