Throughput Scaling of Wireless Ad Hoc Networks With No Side Information

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***Abstract*—This letter studies the throughput of wireless ad hoc networks in which neither the source nodes nor the destination nodes have channel state information. Work reported in [3] has shown that in the regime of a large network with a total number of nodes $n \to \infty$, the throughput scales at most as $\log\log n$. We present a scheme that achieves a throughput scaling of $\Theta(\log n)$ for the Gauss-Markov fading process and for network sizes of practical interest.**

***Index Terms*—Ad hoc networks, channel state information (CSI), throughput.**

I. Introduction

CONSIDER a wireless ad hoc network with n source-to-destination (S–D) pairs. Inspired by the seminal work of Gupta and Kumar [**?**], there has been significant progress in understanding the *throughput scaling of wireless ad hoc networks*. It turns out that the throughput depends critically on the assumptions of channel state information (CSI) availability. Under the Rayleigh fading channel, and in the limit of $n \to \infty$, when global CSI is available to all nodes, a linear throughput in n is achievable [**?**]; when CSI is only available to the receiver, the throughput scales as $\Theta(\log n)$ [**?**, Corollary 3]; when neither the source nodes nor the destination nodes know the CSI, the throughput is upper-bounded by $\Theta(\log\log n)$ [**?**, Corollary 1].[1] Furthermore, it is shown in [**?**] that, to achieve $\Theta(\log\log n)$, it suffices to allow only one S–D pair to communicate at a time.

Remark 1: The $\Theta(\log\log n)$ throughput in [**?**] follows from the fact that the capacity of ad hoc networks is upper-bounded by that of a point-to-point multiple-input multiple-output (MIMO) channel with the same power constraint (since the cooperation inherent in MIMO will result in higher capacity). The capacity of ad hoc networks when the CSI is not available at either the transmitter or the receiver, is then bounded by the capacity of a MIMO channel, which for the same CSI conditions, grows only double logarithmically with the signal-to-noise ratio (SNR) in the high SNR regime [**?**]. Interestingly, it is shown by Etkin and Tse [**?**] that the $\log\log \mathsf{SNR}$ capacity of [**?**] should be interpreted only in the regime where the SNR is very high. They show that in SNR regimes of practical interest, the capacity of $\log \mathsf{SNR}$ is preserved.

In this paper, exploiting the relations between MIMO systems and ad hoc networks [**?**], [**?**], [**?**], we show that under the Gauss-Markov fading process and with a system size of practical interest, the throughput of a wireless ad hoc network scales logrithmically with the number of nodes, even when neither the source nodes nor the destination nodes know the channel. Given the fact that $\log n$ is also the best throughput scaling even with CSI at the receivers, we conclude that in the Rayleigh fading channel, there is no throughput loss in the scaling sense, when no side information is available.

II. Channel Model

Assume that the channel connection from source node i to destination node j ($1 \le i, j \le n$) follows a flat-fading Rayleigh, discrete time, baseband, Gauss-Markov process as

$$h_{i,j}[\ell+1] = \sqrt{1-\epsilon}\, h_{i,j}[\ell] + \sqrt{\epsilon}\, w_{i,j}[\ell], \tag{1}$$

where ℓ denotes the symbol index, $w_{i,j}[\ell] \sim \mathcal{CN}(0,1)$ is circularly symmetric complex Gaussian with zero mean and unit variance, and $h_{i,j}[0]$ is also $\mathcal{CN}(0,1)$. Note that in the model (**??**), the fading process varies from symbol to symbol, and the coherence time of the model is controlled by the parameter ϵ. Throughout the work, we assume neither the source nor the destination nodes have knowledge of $\{h_{i,j}[\ell]\}$.

Assume that at time n, a subset $\mathcal{K}$ of the total S–D pairs is active. We can write the channel equation as,

$$y_j[\ell] = \textstyle\sum_{i\in\mathcal{K}} h_{i,j}[\ell]\, \sqrt{P}\, x_i[\ell] + z_j[\ell], \tag{2}$$

where $y_j[\ell]$ is the channel output at the jth destination, $\sqrt{P}\, x_i[\ell]$ is the channel input of the ith source with an *average* power constraint P, i.e., $\sum_{n=1}^{N} \mathbb{E}[|x_i[\ell]|^2] \le N$ for large N, and $z_j[\ell] \sim \mathcal{CN}(0,1)$.

III. Achievable Throughput

In this section, the $\Theta(\log n)$ throughput is shown to be achievable by a specific scheme. The scheme involves two stages: decentralized random access and communication. The random access stage supports in a decentralized fashion and with probability of $\Theta(1)$, i.e., probability independent of the number of S–D pairs, only one active S–D transmission at a time. The communication stage consists of the operations of the source and destination. In this section, we discuss also the achievable throughput and the regime of validity.

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[1]The following notation is used throughout this paper. For two functions $f(n)$ and $g(n)$, the notation $f(n) = O(g(n))$ means that $|f(n)/g(n)|$ remains bounded as $n \to \infty$. We write $f(n) = \Theta(g(n))$ to denote that $f(n) = O(g(n))$ and $g(n) = O(f(n))$.

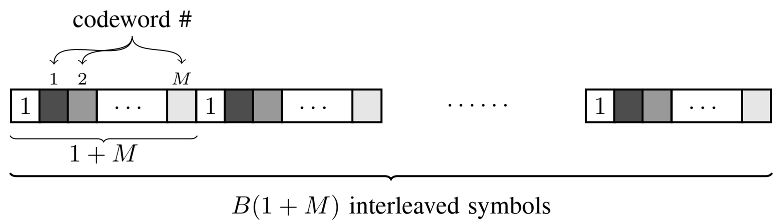

Fig. 1. Schematic representation of the transmission scheme. The scheme interleaves M codewords, each has B symbols (symbols of the same codeword are represented with the same shading). At the beginning of each block, the training symbol "1" is sent.

A. Random Access Stage

It is assumed that source nodes have knowledge of the number of nodes in the system. At each frame (defined as $B(1+M)$ interleaved symbols as shown in Fig. **??**), each source node transmits with probability $1/n$. Then the total number of concurrent transmissions, denoted by K, in each frame follows a binomial distribution. It can be trivially shown that the probability of only one active S–D transmission is of the order of one, i.e., $\Pr[K=1]=\Theta(1)$, as follows:

$$\Pr[K=1] = \binom{n}{1}\left(\frac{1}{n}\right)\left(1-\frac{1}{n}\right)^{n-1} = \left(1-\frac{1}{n}\right)^{n-1}$$

$$\approx e^{-\frac{n-1}{n}} \tag{3}$$

$$\to 1/e, \tag{4}$$

where (**??**) follows from the fact that $(1-x)^n \to e^{-nx}$ for $x \to 0$. In what follows, we focus on the event $\{K=1\}$, i.e., there is only a single active S–D transmission. Since each source transmits during only a fraction of time $1/n$, its instantaneous power can be as high as nP while still satisfying the average (long-term) power constraint. From (**??**), we have $y_j[\ell] = h_{j,j}[\ell]\sqrt{nP}\,x_j + z_j[\ell]$, whenever the jth S-D pair is active. Focusing on a single S–D transmission, and for notational brevity, from here on we drop the node index. Furthermore, with a slight abuse of notation of $y[\ell]$, we rewrite the input-output equation as

$$y[\ell] = h[\ell]\,x[\ell] + \sqrt{\tfrac{1}{nP}}\,z[\ell]. \tag{5}$$

B. Communication Stage

The communication stage consists of the operations at the transmitter and the receiver. The transmitted frame of $B(1+M)$ symbols consists of M codewords of B symbols each and B training symbols. An interleaving scheme organizes the frame such that the a training symbol is transmitted each M symbols. The scheme is illustrated in Fig. **??**. The first symbol of each of the M codewords is transmitted following the first training symbol, the second symbol of each of the codewords is transmitted following the second training symbol, etc. The scheme is a simplified version of a more general MIMO setup proposed in [**?**]. The training symbol enables the destination node to obtain an estimate of $h[\ell]$ at times $\ell = k(1+M)+1$ for $k=0,1,\ldots,B-1$.

1) Channel Estimation: Consider the first time unit when the training symbol is transmitted, and assume that the training symbol is "1",

$$y[1] = h[1]\cdot 1 + \sqrt{\tfrac{1}{nP}}\,z[1]. \tag{6}$$

The optimal MMSE estimate of $h[1]$ is the conditional mean $\mathbb{E}\big[h[1]|y[1]\big]$. To simplify the analysis, we use the suboptimal estimate $\hat{h}[1]=y[1]$, which is a good estimate for large n. The corresponding estimation error is given by $\sqrt{1/nP}\,z[1]$ with variance $1/(nP)$. In general, we set $\hat{h}[k(1+M)+1] = y[k(1+M)+1]$. Also, for decoding the first codeword, we set,

$$\hat{h}[k(1+M)+2] \triangleq \hat{h}[k(1+M)+1],\ k=0,\ldots,B-1.$$

Due to the channel variations (cf. eq.(**??**)), the estimation error at time $k(1+M)+2$ contains an additional independent, $\mathcal{CN}(0,\epsilon)$ term.[2] Therefore, the overall estimation error of the first codeword is circularly symmetric complex Gaussian with variance $1/nP+\epsilon$. That is, $h[k(1+M)+2] = \hat{h}[k(1+M)+2] + \tilde{h}[k(1+M)+2]$ where $\tilde{h}[k(1+M)+2] \sim \mathcal{CN}(0, 1/nP+\epsilon)$. The estimation of the channel at other times is decisions-directed as discussed in the sequel.

2) Coding Rate: To decode the first codeword, we write the input–output relation as shown at the bottom of the page. Rearranging the relation, $y[k(1+M)+2] = \hat{h}[k(1+M)+2]x[k(1+M)+2] + z'[k(1+M)+2]$ with $z'[k(1+M)+2] = \tilde{h}[k(1+M)+2]x[k(1+M)+2] + \sqrt{\frac{1}{nP}}\,z[k(1+M)+2] \sim \mathcal{CN}(0, 2/nP+\epsilon)$. As shown in [**?**], the capacity of this system can be lower-bounded by the capacity of a fading channel with known fading coefficients $\{\hat{h}[k(1+M)+2]\}$ and Gaussian noise $\{z'[k(1+M)+2]\}$. Therefore, the capacity of the communication scheme is lower-bounded by

$$C \geq \mathbb{E}\left[\log\left(1+\frac{|\hat{h}|^2}{2/nP+\epsilon}\right)\right]$$

[2]We have made the approximation $\sqrt{1-\epsilon}\approx 1$ when computing the variance. This approximation is valid for $\epsilon \to 0$.

$$y[k(1+M)+2] = \Big(\underbrace{\hat{h}[k(1+M)+2] + \tilde{h}[k(1+M)+2]}_{h[k(1+M)+2]}\Big)x[k(1+M)+2] + \sqrt{\tfrac{1}{nP}}\,z[k(1+M)+2], \tag{7}$$

for $k=0,\ldots,B-1$.

$$\geq \log\left(\frac{1}{2/nP+\epsilon}\right)+\mathbb{E}[\log(|\hat{h}|^2)]\,. \quad (8)$$

When $\frac{2}{nP} \geq \epsilon$, i.e., $n \leq \frac{2}{P\epsilon}$, we have from (**??**),

$$C \geq \log\left(\frac{1}{4/nP}\right)+\mathbb{E}[\log(|\hat{h}|^2)]$$
$$\geq \log n + \log\tfrac{P}{4} + \mathbb{E}[\log(|\hat{h}|^2)]. \quad (9)$$

Therefore, one can operate the communication scheme by setting the coding rate as $\log n + \log\frac{P}{4} + \mathbb{E}[\log(|\hat{h}|^2)]$. When B is sufficiently long, the decoding error probability can be arbitrarily small.

Intuitively, by transmitting only a fraction $1/n$ of the time, the source nodes can transmit with power nP. That is, each communication link has an average SNR that scales as n. When $\frac{2}{nP} \geq \epsilon$, the noise dominates the channel variation, and the system capacity behaves as though there is perfect CSI at the receiver. The average SNR at the receiver is nP, and the therefore the capacity is of the order of $\log n$. The term $\mathbb{E}[\log(|\hat{h}|^2)] = -0.83$ bits/s/Hz (for small ϵ) is the price we have to pay for operating over a fast fading channel [**?**, eq. (5.39)].

3) Iterative Channel Estimation and Decoding: The successfully decoded codeword can be used as a training symbol at time $k(1+M)+2$, $k = 0,\ldots,B-1$. Specifically, the successfully decoded $x[k(1+M)+2]$, together with $y[k(1+M)+2]$ and $\hat{h}[k(1+M)+2]$, can be used to estimate the channel at time $k(1+M)+3$. This is a Kalman filtering problem [**?**]. A simple, suboptimal estimate of $h[k(1+M)+3]$ is $y[k(1+M)+2]/x[k(1+M)+2]$ (cf. (**??**)). This is exactly the same problem as time instants $k(1+M)+2$ for $k = 0,\ldots,B-1$. Continuing in this way, at each step, the scheme uses the previously decoded codeword to update the estimates of the channel coefficients, which are then used in the decoding process of the next codeword. We can make the fraction of time spent in the transmission of the training symbol arbitrarily small by making M large enough.

C. Achievable Throughput

While the discussion has focused so far on a single S–D transmission, due to the random access protocol, there will be cases in which more than one S–D is active. However, in proving that the throughput scales as $\Theta(\log n)$, it suffices to ignore such events. More specifically, by focusing on $\{K=1\}$ case and combining the random access stage (cf. (**??**)) and the communication stage (cf. (**??**)), we summarize the achievable throughput of the scheme as follows:

Theorem 1: Under the Gauss-Markov channel model, the proposed scheme achieves a throughput $\geq \frac{1}{e}(\log n + \log\frac{P}{4} - 0.83)$, when $n \leq \frac{2}{P\epsilon}$.

D. Regime of Validity

Typical wireless channels are *underspread*, meaning $\epsilon \ll 1$. For example, ϵ may be as small as 10^{-4} in an outdoor environment [**?**]. For a better appreciation of the condition $n \leq \frac{2}{P\epsilon}$ in Theorem **??**, suppose $P = 1$ (0 dB average SNR), which yields $n \leq 2\times 10^4$. In an 1 km^2 area, the average distance is 7 meters. However, the system throughput may behave differently than $\log n$ when the condition on n is not met. For example, in the limit of $n \to \infty$, the throughput eventually scales as $\log\log n$. It is shown in [**?**] that in the point-to-point link, one needs $\mathsf{SNR} \geq \exp(1/\epsilon)$ to have $\log\log\mathsf{SNR}$ capacity. According to Remark **??**, in our ad hoc network context, this translates to the condition of $nP \geq \exp(1/\epsilon)$, which is out of range in practical channels.

IV. Discussion of Channel Models

In this work, we assume that all S–D pairs are subject to Rayleigh fading. Generalization of the Rayleigh fading model to others is of interest. By the Jensen's inequality, it is not hard to conclude that the proposed scheme has a $O(\log n)$ throughput upper bound (even with CSI at the destination nodes) for a class of fading channels with finite average power, i.e., $\mathbb{E}[\log(1+nP\,|h|^2)] \leq \log(1+nP\mathbb{E}[|h|^2]) = O(\log n)$. Interestingly, it has been shown in [**?**, Theorem 1] that for a class of fading channels with finite mean and variance, the throughput is upper-bounded by $O(n^{1/3})$, which is superior to the $O(\log n)$. Therefore, it is left as an interesting future research topic to understand the gap.

V. Concluding Remarks

Previous information-theoretic analysis has shown that, in the limiting case of $n \to \infty$, the throughput of wireless ad hoc network scales as $\log\log n$ when the CSI is unavailable either at the source or destination nodes. In this work, a specific scheme is shown to achieve a throughput scaling of $\Theta(\log n)$ for systems with a smaller and more practical number of nodes. A condition on the number of nodes to guarantee the $\Theta(\log n)$ achievability has been determined.

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