Throughput of Two-Hop Wireless Networks with Relay Cooperation

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Abstract— **In this paper, we consider the throughput problem of a wireless fading network with two-hop relaying, where n single antenna source-destination pairs communicate through a set of single antenna relays using *amplify-and-forward* strategy. Two different cooperation schemes at the relay nodes are considered, where the relays share either channel state information (CSI) or both CSI and received signals. The high level of cooperation is justified for a case where the relaying role is fulfilled by infrastructure nodes that can communicate through a wired backbone without an overhead on the wireless channel. We show that in the first case, at least n^2 relays are needed to achieve linear scaling of the system throughput versus n. In the second case, exchanging the received signals at the relays can reduce the needed number of relays to n in order to achieve linear scaling. It is also shown that the second cooperation scheme achieves a strictly positive per node throughput, where the total number of nodes accounted for includes the relays.**

I. Introduction

In their seminal work on the throughput of wireless ad-hoc networks [1], Gupta and Kumar showed that for an ad-hoc network with n identical randomly located nodes, the throughput per node scales as $\Theta\left(\frac{1}{\sqrt{n}}\right)$[1] thus vanishing with an increasing number of nodes in the network. Further analysis demonstrates that, even with the addition of K pure relay nodes that have no traffic of their own but only help the ongoing transmissions (through decode-and-forward point-to-point transmission), the per user throughput scales as $\Theta\left(\frac{n+K}{n\sqrt{(n+K)\log(n+K)}}\right)$, evidence that simply adding plain regenerative relays cannot overcome the linear scaling problem of ad-hoc networks.

The above results are drawn based on a geometric path loss model, where the channel gains are governed by a power law as a function of the source-destination separation. Recently, the throughput of a network with random connections was studied in [2]. In such a network, fading, rather than deterministic path loss, is the dominant radio propagation factor controlling the connectivity between nodes. The model is also relevant to wireless networks with small physical size and a rich scattering environment. The throughput of this random connection network is shown to be heavily dependent on the assumed channel gain distribution. In particular, it is proved that for a wireless network with n nodes subject to independent, but identical Rayleigh distributed channel gains between nodes, a multihop *decode-and-forward* scheme results in an achievable throughput that scales as $\Theta(\log n)$. The scheme analyzed in [2] chooses the multi-hop routing along "good paths," that is paths with signal to interference plus noise ratio (SINR) that exceeds a chosen threshold. It also imposes the condition that each node can only receive or transmit at most one message in any time slot (indicating each relay node is assigned to help the transmission of only one certain pair of source-destination nodes). Thus the scheme is affected by, but avoids exploiting, the interfering signals in the network.

As a further network model of interest, several papers considered wireless fading networks where n single antenna source-destination pairs communicate through K relay nodes with half-duplex, two-hop relaying protocols [3] [4] [5] [6]. In this model, relays are used in various configurations to orthogonalize and beamform the data streams between sources and destinations. Protocols of the *amplify-and-forward* type allow each relay to deliver data from multiple sources. The specific protocols differ in CSI awareness at the relays and/or the amount of cooperation allowed between the relay nodes, as discussed below in more detail. Common to the protocols based on this model is the result that for a fixed number n of source-destination pairs, a large number of relays K, and a perfectly synchronized network, the network aggregate throughput scales as $(n/2)\log K + O(1)$ with K, which coincides with the cut-set bound. Moreover, the scaling law is attained by all the protocols also in the case when n grows, but only as long as K satisfies certain conditions described below. Next, we briefly review three schemes that conform to the two-hop model discussed previously.

The scheme proposed in [3] (referred to as S1 in the following) requires that all the relays be partitioned into n clusters, each assigned to one of the n source-destination ad-

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[1] $f = O(g)$ if there are positive constants c and n_0 such that for all $n \geq n_0$, $f(n) \leq cg(n)$; $f = \Theta(g)$ if there are positive constants c_0, c_1, and n_0 such that for all $n \geq n_0$, $c_0 g(n) \leq f(n) \leq c_1 g(n)$.

hoc user node pairs. Relays in a cluster maintain CSI only to the associated source-destination pair. Thus, the relays in each cluster have the *backward* (source to relay) channel information for a specific source, and the forward (relay to destination) channel information to a specific destination. With this information, each cluster of relays is capable of co-phasing signals at the destination node, resulting in a form of distributed beamforming. This distributed beamforming accounts for the $\log K$ array gain in the expression of the throughput. There is no cooperation in the system either at the relays or at the source/destination nodes. Since cooperation between destination nodes is not allowed, the data decoding is per antenna. The processing at the relays and the use of clusters serve as a distributed orthogonalization scheme for the n channels linking sources and destinations. This accounts for the pre-log term n in the throughput expression. The 1/2 factor is due to the half-duplex two-hop protocol.

Although mainly focused on the power efficiency of *amplify-and-forward* schemes for sensory and ad-hoc networks, [4] also investigates a two-hop scheme, referred to here as S2. The more demanding assumption is made that all relays have local CSI of the backward and forward channels. The relays exploit this information to co-phase signals at the destinations, again orthogonalizing the data streams and simultaneously supporting a distributed array gain. It is established in [6] that the throughput of S1 can scale with n (i.e., n does not have to be fixed as in [3]) as long as $K \geq n^3$; for S2, $K \geq n^2$ is needed to obtain the same result. Note that although S1 and S2 achieve linear scaling with number of source-destination pairs n, the per node throughput still vanishes to zero[2].

Another scheme is also mentioned in [4] (referred to as S3 in the following), where cooperation is introduced at the relay nodes by letting them exchange the local CSI so that global (transmit and receive) CSI is available at all the relays. Under this assumption, by performing the inter-stream interference cancellation and orthogonalizing the channels between source and destination nodes (zero-forcing), it is shown that, similar to S2, the throughput scales linearly with n as long as the number of relays $K \geq n^2$. Not focusing on the scaling law, and accounting for the values embedded in the $O(1)$ terms, the global CSI available at the relays in S3 leads to an actual throughput that is higher than that of S2.

Main contributions and relation to previous work

In this paper, we study the benefits of cooperation in two-hop wireless networks. Similar to [3] [4] [5], n source-destination pairs communicate through K relays. We focus on two levels of cooperation among the relays. In the first type of cooperation, local CSI is exchanged among the relays enabling them to have global CSI, as in S3. An even higher level of cooperation is also studied in which the relays are allowed to exchange not only CSI, but also the actual received signals and perform zero-forcing processing as in [4]. The latter scheme is referred to as S4 and was first studied in [7]. It is well understood that cooperation exacts a price in the sense that the overhead required to implement the cooperation may drastically reduce the useful throughput [8] [9]. Nevertheless, the cooperation assumed here is not among source or destination nodes, but rather among relay nodes. We are motivated in this approach by a type of networks known as *hybrid networks*, in which relay nodes are infrastructure nodes connected to a wired backbone [10] [11]. Hybrid networks act as a bridge between the concepts of infrastructure networks (such as cellular systems) and ad hoc networks. Our goal is to study the throughput scaling laws in such networks.

Main contributions of the paper are summarized as follows:

1. A lower bound is developed for S3 by evaluating the rate of an achievable scheme. This lower bound adds to the results in [4] to provide additional insight into the performance of S3.

2. Lower and upper bounds are developed for the scheme S4, where relays have global information of the received signals.

3. The main result of the paper is to prove that in the case of an S4 scheme, linear scaling with the number of source-destination pairs can be attained with a number of relays $K \geq n$. Recounting that for S1, $K \geq n^3$, for S2, $K \geq n^2$, and for S3, $K \geq n^2$, the S4 result is more favorable than the other schemes, and it means, in fact, that S4 achieves a non-vanishing throughput per node (where the number of nodes is the aggregate of the source, destination, and relay nodes). To sum up, different levels of relay cooperation trade off the number of relay nodes needed to achieve a strictly positive per user throughput scaling.

The rest of the paper is organized as follows. In the following section, the basic network model and the half-duplex two-hop relaying protocol are introduced. In Section III, we briefly review the results in [4] and we derive a tight lower bound of the throughput of S3, and analyze the per user and per node throughputs scaling characterization. The scheme S4 is presented in Section IV. Lower and upper bounds of the throughput are calculated. The per user and per node throughputs are also analyzed. Numerical results are provided in Section V as examples for the concepts discussed in the paper, and concluding remarks are offered in Section VI.

Notation: Throughout the paper, we denote matrices and vectors with bold face type, using capital letters for matrices and lower case letters for vectors. For any matrix $\mathbf{A}$, the superscript ^{T} denotes transpose and * complex conjugate transpose. We write the determinant of $\mathbf{A}$ as $|\mathbf{A}|$ and its trace as $\mathrm{tr}(\mathbf{A})$. $\|\cdot\|$ indicates the Frobenius norm. $E[\cdot]$ denotes the expected value of the expression in brackets. For any vector $\mathbf{a}$, diag($\mathbf{a}$) is a diagonal matrix with the main diagonals the components of vector $\mathbf{a}$. For any complex number z, we write its absolute value as $|z|$. The Kronecker (or tensor) product [12] of two matrices is denoted by $\otimes$. The vector of stacked columns of a matrix $\mathbf{A}$ is denoted $\mathrm{vec}(\mathbf{A})$, while the function $\mathrm{vecd}(\mathbf{D}) = [d_1, \ldots, d_K]^T$, extracts the main diagonal of a matrix $\mathbf{D}$.

[2]Accounting for both source-destination pairs and relays, per node throughput is defined as $\lim_{n\to\infty} \frac{C}{2n+K}$.

II. System Model

Consider a wireless network where n source nodes engage in simultaneous transmission of signals that are intended to reach n destination nodes. The n transmit/receive pairs are aided by K relays. The communication protocol consists of two time slots. In the first time slot, the relay nodes $k \in 1, \ldots, K$ receive data transmitted from the n transmitting source nodes (backward channel transmission). After processing of the received signals, in the second time slot (forward channel transmission), the relay nodes forward the processed data simultaneously to all the destination nodes. We further assume that there are no direct path between source nodes and destinations nodes such that all source nodes need to communicate with destinations through relay nodes.

We assume a homogeneous model, where all the channels in the network experience independent and identically distributed (i.i.d.), frequency flat, "block" fading. Accordingly, channels are assumed to be constant during each of the stages of the two-hop protocol. Denote the channel complex gain from source node i ($i = 1, \ldots, n$) to the k-th ($k = 1, \ldots, K$) relay, g_{ki}, and the gain from k-th relay to destination node j ($j = 1, \ldots, n$), h_{jk}.

Throughout the paper, perfectly synchronized transmissions and receptions are assumed. The transmission/reception between the source nodes and the relay nodes (first time slot) can be expressed

$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \ldots, s_n]^T$ is the transmit signal vector, consisting of realizations of zero mean, complex Gaussian random variables with power P ($E[\|s_i\|^2] = P$); $\mathbf{G} \in \mathbb{C}^{K \times n}$ is the channel matrix corresponding to the backward (source-relay) channel with entries g_{ki} modeled as realizations of i.i.d., circularly symmetric, complex Gaussian process with zero mean and unity variance; $\mathbf{w} \in \mathbb{C}^{K \times 1}$ is noise circularly symmetric, complex Gaussian noise, with zero mean and covariance matrix $E[\mathbf{w}\mathbf{w}^*] = \sigma_w^2 \mathbf{I}_K$; $\mathbf{r}$ is the signal vector received at the relays.

In the second time slot, the relay nodes process the received signal vector $\mathbf{r}$ to produce the signal vector $\mathbf{u}$ transmitted on the forward channel. The signal received at the destination nodes is then given by:

$$\mathbf{y} = \mathbf{H}\mathbf{u} + \mathbf{z}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{n \times K}$ with entries h_{jk} having statistics similar to the g_{ki}'s. The vector $\mathbf{u}$ is normalized to meet the power constraint $E\left[\|\mathbf{u}\|^2\right] = P_R$, where the average is taken over a block of data (during which the channels are assumed fixed). The vector $\mathbf{z} \in \mathbb{C}^{n \times 1}$ represents additive white Gaussian noise, circularly symmetric, with zero mean and covariance matrix $E[\mathbf{z}\mathbf{z}^*] = \sigma_w^2 \mathbf{I}_n$

We then investigate the following two schemes:

- Scheme S3 that performs *amplify-and-forward* transmission with global CSI available at the relays as proposed in [4]:

In this case, the k-th relay transmits a scaled version of the signal it has received in the first time slot:

$$\mathbf{u} = \mathbf{D}_A \mathbf{r}, \tag{3}$$

where $\mathbf{D}_A = \text{diag}(d_A^1, \ldots, d_A^K) \in \mathbb{C}^{K \times K}$ is a diagonal matrix with diagonal entries corresponding to the complex gains applied by the K relays. In choosing the scalar d_A^k, the k-th relay can exploit the global CSI assumed available at the relay, i.e., knowledge of the backward channel $\mathbf{G}$ and the forward channel $\mathbf{H}$. Fig. 1 illustrates the S3 scheme.

- Scheme S4 that performs *amplify-and-forward* transmission with global CSI and global received signal information available at the relays (as in [7]).

In this case, all relays know the matrices $\mathbf{G}$ and $\mathbf{H}$ as well as the received signal vector $\mathbf{r}$. The transmitted signal in the second time slot is

$$\mathbf{u} = \mathbf{D}_B \mathbf{r},$$

where $\mathbf{D}_B \in \mathbb{C}^{K \times K}$ has K^2 entries denoted d_B^{pq} ($p, q = 1, \ldots, K$). Fig. 2 is an illustration of scheme S4.

Using a generic transformation $\mathbf{D}$ to denote the processing at the relays, both schemes S3 and S4 can be expressed in the general form

$$\mathbf{y} = \mathbf{HDGs} + \mathbf{HDw} + \mathbf{z} \tag{4}$$

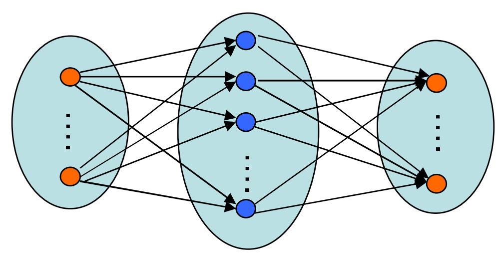

Fig. 1. S3 scheme: *amplify-and-forward* transmission with global CSI available at the relays

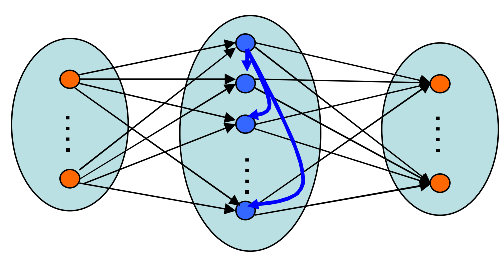

Fig. 2. S4 scheme: *amplify-and-forward* transmission with global CSI and received signals information available at the relays

III. *Amplify-and-forward* with Global CSI

In this section, we investigate the scheme S3 defined above, providing more insight into the performance of this scheme by computing a lower bound for the system throughput.

Letting $\mathbf{D}_A$ be the matrix representing the processing at the relays as in (3), and substituting it in (4), the equivalent channel matrix between the source nodes and the destination nodes for S3 is $\mathbf{H}\mathbf{D}_A\mathbf{G}$. As noted in [4], if the number of relays K is sufficiently large, we can diagonalize the channel matrix $\mathbf{H}\mathbf{D}_A\mathbf{G}$ through appropriate design of the diagonal matrix $\mathbf{D}_A$. We require

$$\mathbf{H}\mathbf{D}_A\mathbf{G} = \alpha_A \mathbf{I}_n, \tag{5}$$

where $\mathbf{I}_n$ is the identity matrix of order n, and α_A is some scalar. The matrix $\mathbf{D}_A$ also has to meet the power constraint,

$$P\,\mathrm{tr}(\mathbf{G}^*\mathbf{D}_A^*\mathbf{D}_A\mathbf{G}) + \sigma_w^2\,\mathrm{tr}(\mathbf{D}_A^*\mathbf{D}_A) \le P_R. \tag{6}$$

In particular, since (5) is a system of n^2 equations and K unknowns, at least one solution for $\mathbf{D}_A$ is guaranteed if $K \ge n^2$. With such matrix $\mathbf{D}_A$, the link between the n sources and destinations consists of n parallel, interference free channels. The signal received at the j-th destination node is

$$y_j = \alpha_A s_j + z_j + \sum_{k=1}^{K} h_{jk} d_A^k w_k, \tag{7}$$

where the terms z_j and w_k are respectively, elements of the vectors $\mathbf{z}$ and $\mathbf{w}$. In this expression, the signal term is $\alpha_A s_j$, while the other terms are noise.

An upper bound on the throughput of S3 can be developed by neglecting the extra noise term due to noise forwarding at the relays. From (4),

$$\begin{aligned} C &\le \frac{n}{2} E\left\{\log\left(1 + \frac{|\alpha_A|^2 P}{\sigma_w^2}\right)\right\} \\ &\le \frac{n}{2}\log\left(1 + \frac{E\left[|\alpha_A|^2\right] P}{\sigma_w^2}\right), \end{aligned} \tag{8}$$

where the second inequality is obtained via Jensen's inequality. The expectation is over the ensemble of possible channels that determine the value of $|\alpha_A|^2$. It is shown in [4], that $E\left[|\alpha_A|^2\right]$ subject to the definition (5) and the constraint (6) is upper bounded $E\left[|\alpha_A|^2\right] \le KP_R/n\sigma_w^2$. Therefore, we finally get the upper bound:

$$C \le \frac{n}{2}\log\left(1 + \frac{KP_R P}{n\sigma_w^4}\right).$$

We now derive a lower bound on the throughput C. The lower bound is identified as an achievable rate obtained by enforcing a specific solution $\hat{\mathbf{D}}_A$ which satisfies (5). Towards this goal, we apply the definitions of the vec and vecd operators defined at the end of the Introduction section. We have

$$\mathrm{vec}(\mathbf{H}\hat{\mathbf{D}}_A\mathbf{G}) = \hat{\alpha}_A \mathrm{vec}(\mathbf{I}_n). \tag{9}$$

With the help of a Khatri-Rao Product property [13], we have

$$\mathrm{vec}(\mathbf{H}\hat{\mathbf{D}}_A\mathbf{G}) = (\mathbf{G}^T \odot \mathbf{H})\mathrm{vecd}(\hat{\mathbf{D}}_A), \tag{10}$$

where $\odot$ is the operator for Khatri-Rao Product[3]. Thus under the assumption $K \ge n^2$, a diagonal matrix $\hat{\mathbf{D}}_A$ can be obtained from (9) and (10) as

$$\begin{aligned} \hat{\mathbf{D}}_A &= \mathrm{diag}\Big\{(\mathbf{G}^T \odot \mathbf{H})^* \left((\mathbf{G}^T \odot \mathbf{H})(\mathbf{G}^T \odot \mathbf{H})^*\right)^{-1} \\ &\qquad \times \hat{\alpha}_A \mathrm{vec}(\mathbf{I}_n)\Big\} \\ &= \hat{\alpha}_A \mathbf{\Phi}_A, \end{aligned} \tag{11}$$

where $\mathbf{\Phi}_A = \mathrm{diag}\Big\{(\mathbf{G}^T \odot \mathbf{H})^* \left((\mathbf{G}^T \odot \mathbf{H})(\mathbf{G}^T \odot \mathbf{H})^*\right)^{-1} \times \mathrm{vec}(\mathbf{I}_n)\Big\}$. The matrix $\mathbf{\Phi}_A$ is a function of the backward channel matrix $\mathbf{G}$ and the forward channel matrix $\mathbf{H}$. The constant $\hat{\alpha}_A$ is found from substituting $\hat{\mathbf{D}}_A = \hat{\alpha}_A \mathbf{\Phi}_A$ in the expression for the power constraint (6):

$$|\hat{\alpha}_A|^2 = \frac{P_R}{P\,\mathrm{tr}(\mathbf{G}^*\mathbf{\Phi}_A^*\mathbf{\Phi}_A\mathbf{G}) + \sigma_w^2\,\mathrm{tr}(\mathbf{\Phi}_A^*\mathbf{\Phi}_A)}.$$

Finally, by identifying the signal and noise terms in (7), a lower bound on the system throughput is found as

$$C \ge \frac{1}{2}\sum_{j=1}^{n} E\left\{\log\left(1 + \frac{P|\hat{\alpha}_A|^2}{\sigma_w^2\left(1 + |\sum_{k=1}^{K} h_{jk}\hat{d}_A^k|^2\right)}\right)\right\},$$

where $\hat{d}_A^k$ are the elements on the diagonal of $\hat{\mathbf{D}}_A$.

The upper and lower bounds both scale linearly with n for $K \ge n^2$. The transmission rate per node in the network is thus $\Theta(\frac{n}{2n+K})$, which asymptotically scales at most as $\Theta(\frac{1}{\sqrt{K}})$ when $n \sim \Theta(\sqrt{K})$. Therefore, the scheme features a vanishing per node throughput scaling.

IV. *Amplify-and-forward* with Global CSI and Signal Exchange

In this part, we study scheme S4, in which relays exchange the CSI (as in S3) as well as the signals received during the first time slot of the two-hop communication protocol. We focus on relay processing that aims at creating parallel channels between source-destination pairs. Thus we have the conditions

$$\mathbf{H}\mathbf{D}_B\mathbf{G} = \alpha_B \mathbf{I}_n, \tag{12}$$

and

$$P\,\mathrm{tr}(\mathbf{G}^*\mathbf{D}_B^*\mathbf{D}_B\mathbf{G}) + \sigma_w^2\,\mathrm{tr}(\mathbf{D}_B^*\mathbf{D}_B) \le P_R, \tag{13}$$

where α_B is a complex gain factor of the equivalent channels between source and destination nodes. The values of α_B are capped by the power constraint (13). The main difference between S3 and S4 is that in S3 the processing matrix $\mathbf{D}_A$ is constrained to be a diagonal matrix. Allowing the relays to exchange the received signals means that this limitation may be removed and $\mathbf{D}_B$ becomes a fully populated matrix with K^2 elements. With K^2 degrees of freedom to available diagonalize the channel, only $K \ge n$ relays are sufficient to guarantee at least one solution for $\mathbf{D}_B$ in (12).

[3]Let $\mathbf{A} \in \mathbb{C}^{I\times T}$ and $\mathbf{B} \in \mathbb{C}^{J\times T}$ are two matrices with the same number of columns, the Khatri-Rao product, $\mathbf{A} \odot \mathbf{B}$, is defined as $[\mathbf{a}_1 \otimes \mathbf{b}_1 \cdots \mathbf{a}_T \otimes \mathbf{b}_T]$, where $\mathbf{a}_t$ and $\mathbf{b}_t$ are the tth column of $\mathbf{A}$ and $\mathbf{B}$, respectively.

TABLE I

SCHEMES AND THEIR METRICS

metric\scheme	S1	S2	S3	S4
network throughput scaling	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
number of relays required	$K \geq n^3$	$K \geq n^2$	$K \geq n^2$	$K \geq n$
per node throughput scaling	vanishing	vanishing	vanishing	$\Theta(1)$

After diagonalization of the channels, the received signal at the destination node j ($1 \leq j \leq n$), can be expressed

$$y_j = \alpha_B s_j + z_j + \sum_{p=1}^{K}\sum_{q=1}^{K} d_B^{pq} h_{jq} w_p. \tag{14}$$

An upper bound of the capacity under the full cooperation conditions is given by

$$\begin{aligned} C &= \frac{n}{2} E\left\{\log\left(1 + \frac{|\alpha_B|^2 P}{\sigma_w^2}\right)\right\} \\ &\leq \frac{n}{2}\log\left(1 + \frac{E\left[|\alpha_B|^2\right] P}{\sigma_w^2}\right). \end{aligned} \tag{15}$$

In Appendix A it is shown that for S4, we have $E\left[|\alpha_B|^2\right] \leq K^2 P_R / n\sigma_w^2$. It follows that an upper bound on the throughput is given by

$$C \leq \frac{n}{2}\log\left(1 + \frac{K^2 P_R P}{n\sigma_w^4}\right). \tag{16}$$

A lower bound of the throughput for this two hop communication scheme is obtained, through a pseudo-inverse solution $\hat{\mathbf{D}}_B$ in (12):

$$\hat{\mathbf{D}}_B = \hat{\alpha}_B \mathbf{H}^*(\mathbf{H}\mathbf{H}^*)^{-1}(\mathbf{G}^*\mathbf{G})^{-1}\mathbf{G}^*.$$

Substitute this result in the expression of ergodic capacity

$$C \geq \frac{1}{2}\sum_{j=1}^{n} E\left\{\log\left(1 + \frac{P|\hat{\alpha}_B|^2}{\sigma_w^2\left(1 + |\sum_{p=1}^{K}\sum_{q=1}^{K} h_{jq}\hat{d}_B^{pq}|^2\right)}\right)\right\},$$

where the corresponding $|\hat{\alpha}_B|^2$ is

$$|\hat{\alpha}_B|^2 = \frac{P_R}{P\,\mathrm{tr}\left(\mathbf{H}\mathbf{H}^*\right)^{-1} + \sigma_w^2\,\mathrm{tr}\left(\mathbf{\Psi}\right)},$$

and $\mathbf{\Psi} = \mathbf{G}(\mathbf{G}^*\mathbf{G})^{-1}(\mathbf{H}\mathbf{H}^*)^{-1}\left(\mathbf{G}^*\mathbf{G}\right)^{-1}\mathbf{G}^*$.

As explained above, with global CSI and cooperation among relays, only $K \geq n$ relays are necessary to assure parallel and non-interfering channels between the n pairs of source-destination nodes. As a result, the throughput of the whole network still scales as $\Theta(n)$, but the per node rate of this ad hoc network scales as $\Theta(\frac{n}{2n+K}) \sim \Theta(1)$ since $n \sim \Theta(K)$.

V. NUMERICAL RESULTS

We demonstrate the performance of S3 and S4 schemes through numerical examples. Fig. 3 plots achievable throughputs of the two schemes for $n = 2$ and 4 source-destination pairs and as a function of the number of relays K. Other parameters are as follows: power for each source node $P = 5$, noise variances $\sigma_w^2 = 0.5$, and relay power constraint $P_R = 10$.

In previous sections, we demonstrated that the throughputs of both schemes scale linearly with n. This is evidenced by the similar slopes of the two curves (one for S3 and the other for S4) for each of the cases $n = 2$ and $n = 4$. While the throughput scaling is the same for S3 and S4, the higher level of cooperation enabled by S4 results in higher throughputs. This can be seen in the offset between the curves for S3 and S4. An interesting point is that with a small number of relays, $n = 2$ source-destination pairs can in fact achieve better throughput than $n = 4$ pairs (conditions $K \geq n^2$ for S3, and $K \geq n$ for S4 still need to hold).

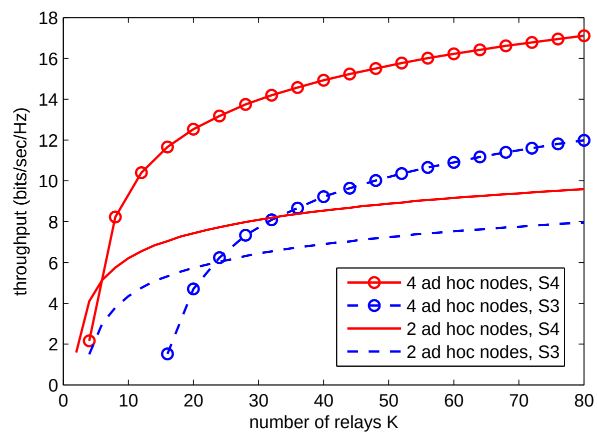

Fig. 3. Network throughput versus the number of relays K for S3 and S4

VI. CONCLUDING REMARKS

In this paper, we study the impact of different levels of relay cooperation on the throughputs of two-hop amplify-and-forward relaying schemes. This work complements earlier work in the references by focusing on schemes with a high level of cooperation. We show that, cooperation among relays at the level of exchanging information on the received signals, does not have an impact on the throughput scaling law, but it does increase the actual throughput for a given set of parameters. More importantly, the higher level of cooperation enables to significantly reduce the number of relays required to orthogonalize the source-destination channels. Finally, we have shown that it leads to a strictly positive *per node* throughput scaling. These observations are summarized in the table above. For context, we added information relevant to schemes S1 and S2 discussed in the references and in the paper.

APPENDIX

A. Derivation of (16)

Let the matrix $\mathbf{D}_B$ consists of K columns $\mathbf{d}_B^k$. Then define the concatenated vector $\text{vec}(\mathbf{D}_B) \triangleq [\mathbf{d}_B^{1\,T}, \ldots, \mathbf{d}_B^{K\,T}]^T$. We seek to determine an upper bound on $E\left[|\alpha_B|^2\right]$. To that end, take an average on (13) and with tedious, but straightforward calculation, (12) and (13) can be rewritten in the form

$$\begin{aligned} \mathbf{G}^T \otimes \mathbf{H} \cdot \text{vec}(\mathbf{D}_B) &= \alpha_B \text{vec}(\mathbf{I}_n) \\ E\left[\text{vec}(\mathbf{D}_B)^* \mathbf{\Lambda} \text{vec}(\mathbf{D}_B)\right] &= P_R, \end{aligned} \tag{17}$$

where $\otimes$ is defined as Kronecker Product; $\mathbf{\Lambda} \in \mathbb{C}^{K^2 \times K^2}$ is

$$\mathbf{\Lambda} = \mathbf{I}_K \otimes \text{diag}\left(P\|\mathbf{g}_1\|^2 + \sigma_w^2, \ldots, P\|\mathbf{g}_K\|^2 + \sigma_w^2\right),$$

and $\mathbf{g}_1, \ldots, \mathbf{g}_K$ denote the row vectors of the channel matrix $\mathbf{G}$.

Suggested by a similar problem in [4], apply the QR decomposition to $\mathbf{\Omega} = \mathbf{G}^T \otimes \mathbf{H}\mathbf{\Lambda}^{-\frac{1}{2}}$ to obtain,

$$\mathbf{\Omega} = \mathbf{G}^T \otimes \mathbf{H}\mathbf{\Lambda}^{-\frac{1}{2}} = \left[\begin{array}{cc} \mathbf{L} & \mathbf{0} \end{array}\right] \mathbf{Q}, \tag{18}$$

where $\mathbf{L}$ is a $n^2 \times n^2$ lower triangular matrix, $\mathbf{0}$ is the $n^2 \times \left(K^2 - n^2\right)$ matrix with all zero elements, and $\mathbf{Q}$ is a $K^2 \times K^2$ unitary matrix. Equating (18) with (17), we have

$$\begin{aligned} \mathbf{G}^T \otimes \mathbf{H}\text{vec}(\mathbf{D}_B) = \left[\begin{array}{cc} \mathbf{L} & \mathbf{0} \end{array}\right] \mathbf{Q}\mathbf{\Lambda}^{\frac{1}{2}} \text{vec}(\mathbf{D}_B) &= \alpha_B \text{vec}(\mathbf{I}_n) \\ \text{E}\left[\text{vec}(\mathbf{D}_B)^* \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{Q}^* \mathbf{Q} \mathbf{\Lambda}^{\frac{1}{2}} \text{vec}(\mathbf{D}_B)\right] &= P_R. \end{aligned} \tag{19}$$

Partition

$$\mathbf{Q}\mathbf{\Lambda}^{\frac{1}{2}} \text{vec}(\mathbf{D}_B) = \left[\begin{array}{c} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2, \end{array}\right] \tag{20}$$

where $\boldsymbol{\xi}_1$ is an $n^2 \times 1$ vector and $\boldsymbol{\xi}_2$ is a $(K^2 - n^2) \times 1$ vector. We have from (19)

$$\boldsymbol{\xi}_1 = \mathbf{L}^{-1} \alpha_B \text{vec}(\mathbf{I}_n), \tag{21}$$

and

$$\begin{aligned} & E\left[\text{vec}(\mathbf{D}_B)^* \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{Q}^* \cdot \mathbf{Q}\mathbf{\Lambda}^{\frac{1}{2}} \text{vec}(\mathbf{D}_B)\right] \\ = \;& E\left[\|\boldsymbol{\xi}_1\|^2 + \|\boldsymbol{\xi}_2\|^2\right] \\ = \;& E\left[|\alpha_B|^2 \text{vec}(\mathbf{I}_n)^* (\mathbf{L}\mathbf{L}^*)^{-1} \text{vec}(\mathbf{I}_n) + \|\boldsymbol{\xi}_2\|^2\right] \\ = \;& E\left\{|\alpha_B|^2 \text{vec}(\mathbf{I}_n)^* \left(\mathbf{\Omega}\mathbf{\Omega}^*\right)^{-1} \text{vec}(\mathbf{I}_n) + \|\boldsymbol{\xi}_2\|^2\right\} \\ = \;& P_R \end{aligned} \tag{22}$$

In the previous expression, we can upper bound $|\alpha_B|^2$ by designing the matrix $\mathbf{D}_B$ such that $\boldsymbol{\xi}_2 = \mathbf{0}$. With that,

$$E\left[|\alpha_B|^2\right] = \frac{P_R}{E\left[\text{vec}(\mathbf{I}_n)^* \left(\mathbf{\Omega}\mathbf{\Omega}^*\right)^{-1} \text{vec}(\mathbf{I}_n)\right]},$$

where from (20) and (21), we used

$$\text{vec}(\mathbf{D}_B) = \mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{Q}^* \left(\begin{array}{c} \mathbf{L}^{-1} \alpha_B \text{vec}(\mathbf{I}_n) \\ \mathbf{0} \end{array}\right).$$

With a bit more algebra and using the inequality $(\mathbf{x}^* \mathbf{A}\mathbf{x})(\mathbf{x}^* \mathbf{A}^{-1} \mathbf{x}) \geq (\mathbf{x}^* \mathbf{x})^2$, for any vector $\mathbf{x}$ and positive definite matrix $\mathbf{A}$, we have

$$\begin{aligned} E\left[|\alpha_B|^2\right] &\leq \frac{P_R \cdot E\left[\text{vec}(\mathbf{I}_n)^* \left(\mathbf{\Omega}\mathbf{\Omega}^*\right) \text{vec}(\mathbf{I}_n)\right]}{\left(\text{vec}(\mathbf{I}_n)^* \text{vec}(\mathbf{I}_n)\right)^2} \\ &\leq \frac{P_R}{n^2 \sigma_w^2} \text{E}\left[\text{vec}(\mathbf{I}_n)^* \left(\mathbf{H} \otimes \mathbf{G}^T\right)\right. \\ &\qquad \left.\left(\mathbf{H} \otimes \mathbf{G}^T\right)^* \text{vec}(\mathbf{I}_n)\right] \\ &= \frac{K^2 P_R}{n \sigma_w^2} \end{aligned} \tag{23}$$

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