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Performance Analysis of VPSK Digital Modulation

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Abstract. A digital modulation recently proposed and referred to as *variable phase-shift-keyeing* is analyzed. The autocorrelation function, power spectral density, and probability of error are computed. Our results show that claims of very high spectral efficiency are unjustified.

Keywords: digital modulations.

1. Introduction

Digital Audio Broadcasting (DAB) technology, expected to be implemented within the next few years requires the use of highly efficient digital modulation techniques. Several European countries have already adopted the Eureka 147 system. Various proposals are currently debated in the U.S. In an attempt to preserve the existing frequency spectrum allocation and channels, the National Association of Broadcasters, called for an in-band DAB system, which during the transition period could be broadcast on the same channel and simultaneously with the existing AM/FM signals [1]. Recently a digital modulation method termed *variable phase-shift-keying* (VPSK) was suggested for digital audio and video transmissions, claiming high spectral efficiencies of up to 15 bits/s/Hz [2]. VPSK is a patented delay modulation [3] that can be interpreted as a higher order Miller code [4]. It is implemented using a two-level waveform with variable pulse width (see next section). The purpose of this paper is to examine the properties of VPSK modulation. Section 2 presents the rules for generating VPSK and develops its state transition diagram. The autocorrelation function, power spectral density and bit error are computed in Section 3. We conclude the analysis by showing that the claims of high spectral efficiency are unjustified.

2. VPSK Modulation

VPSK modulation is a two-level waveform used to encode binary data. The modulation is a type of differential encoding in the sense that a state transition occurs when there is a change in the current bit from the previous bit. A change in the transmitted pulse polarity occurs with each data bit. Each data bit level change causes a stretching of the current transmitted pulse. The Miller code is an example of such modulation with a stretching equal to 1/2 bit [4]. For the purpose of this analysis a 6-VPSK variation of VPSK was considered. Following [2], the algorithm encoding binary data into a 6-VPSK modulated wave uses 2 variables: (1) PreviousBit $\in \{0, 1\}$ – is the value of the previous data bit, (2) SlipCounter $\in \{0, \ldots, 5\}$ – is the offset between the start of the data bit interval and the transition in the VPSK waveform. The SlipCounter increases its value by one with each data bit change.

To better understand the encoding procedure, imagine that the binary data stream is supplied to the encoder input through a FIFO buffer. The encoder pulls each bit out of the buffer as it is transmitted and it can probe the next bits in the queue when needed. VPSK encoding proceeds according to the following steps:

1. Initialize: PreviousBit = 1, SlipCounter = 0.
2. If SlipCounter < 4 then:
   - If current bit = previous bit – send a pulse of duration 6/6 T.
   - If current bit $\neq$ previous bit – send a pulse of duration 7/6 T, increment SlipCounter by 1.
3. If SlipCounter = 4, since the program was initialized with PreviousBit = 1 and as indicated by the SlipCounter there were an even number of transitions, PreviousBit is set again to 1. Then:
   - If current bit = 1 – send a pulse of duration 6/6 T.
   - If current bit = 0 – check the next bit (after 0) in the queue and then:
     - If the next bit = 1 – send 8/6 T pulse which represents both current 0 and next 1 bits. Set SlipCounter = 0, and PreviousBit = 1; continue at step 2.
     - If the next bit = 0 – send 7/6 T pulse, increment SlipCounter to 5, set PreviousBit = 0.
4. If SlipCounter = 5, (current bit to transmit must be 0), check the next bit in the queue and then:
   - If the next bit = 0 – send 6/6 T pulse.
   - If the next bit = 1 – send 7/6 T pulse which represents both current 0 and next 1 bits. Set SlipCounter = 0, and PreviousBit = 1; continue at step 2.

Figure 1 shows a stream of binary data which has been converted to 6-VPSK. Also shown for comparison are the corresponding NRZ and Miller waveforms. The number of different waveforms over an interval of T obtained using the rules listed above is 12. Assuming levels of +1 and –1, the 12 possible 6-VPSK waveforms are shown in Figure 2. Each waveform is assigned a system state and those are also listed in Figure 2. The matrix of transition probabilities $\mathbf{P}$, with elements $\mathbf{P}(i, j)$ = transition probability from state i to state j, is given below (to conserve space, the number of zeros in each row is listed as a multiplier):

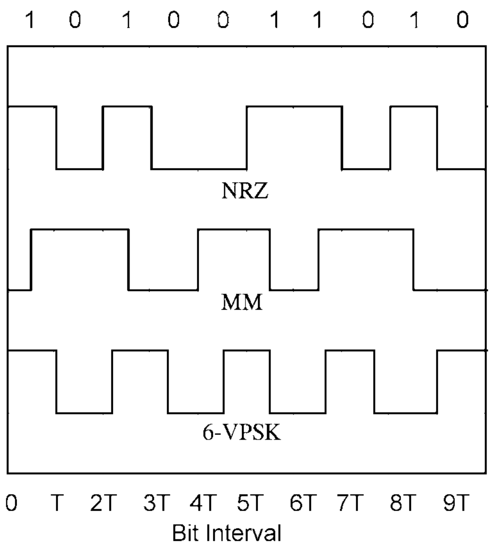

Figure 1. Digital modulation formats.

$$\mathbf{P} = \begin{pmatrix} 6\times 0 & 1/2 & 1/2 & 4\times 0 & \\ 7\times 0 & 1/2 & 1/2 & 3\times 0 & \\ 8\times 0 & 1/2 & 1/2 & 2\times 0 & \\ 9\times 0 & 1/2 & 1/2 & 0 & \\ 1/4 & 9\times 0 & 1/2 & 1/4 & \\ 1/2 & 9\times 0 & 0 & 1/2 & \\ 1/2 & 1/2 & 0 & 9\times 0 & \\ 0 & 1/2 & 1/2 & 9\times 0 & \\ 2\times 0 & 1/2 & 1/2 & 8\times 0 & \\ 3\times 0 & 1/2 & 1/2 & 7\times 0 & \\ 4\times 0 & 1/2 & 1/4 & 1/4 & 5\times 0 \\ 5\times 0 & 1/2 & 1/2 & 5\times 0 & \end{pmatrix}. \qquad (1)$$

The transition probabilities matrix is used next to evaluate the performance of the 6-VPSK modulation.

3. Analysis

To analyze the 6-VPSK waveform, each bit interval is divided into 6 equal parts. From observing the waveform in Figure 1, it can be observed that the autocorrelation function is completely specified by its values $R[(n+\frac{m}{6})T]$, where n is the bit interval index and $m = 0, \ldots, 5$. Values of the autocorrelation at any other arguments can be found by linear interpolation. Extending the analysis in [5] to 6-VPSK it can be shown that for any integer n and m

$$R\left[\left(n + \frac{m}{6}\right)T\right] = R^{(-)} + R^{(+)}, \qquad (2)$$

where $R^{(-)}$ and $R^{(+)}$ are respectively the early and late autocorrelations defined by

$$R^{(-)} = \sum_{i=0}^{11} p(i) \sum_{j=0}^{11} \mathbf{P}^n(i, j) w_{ij}^{(-)}(m) \qquad (3)$$

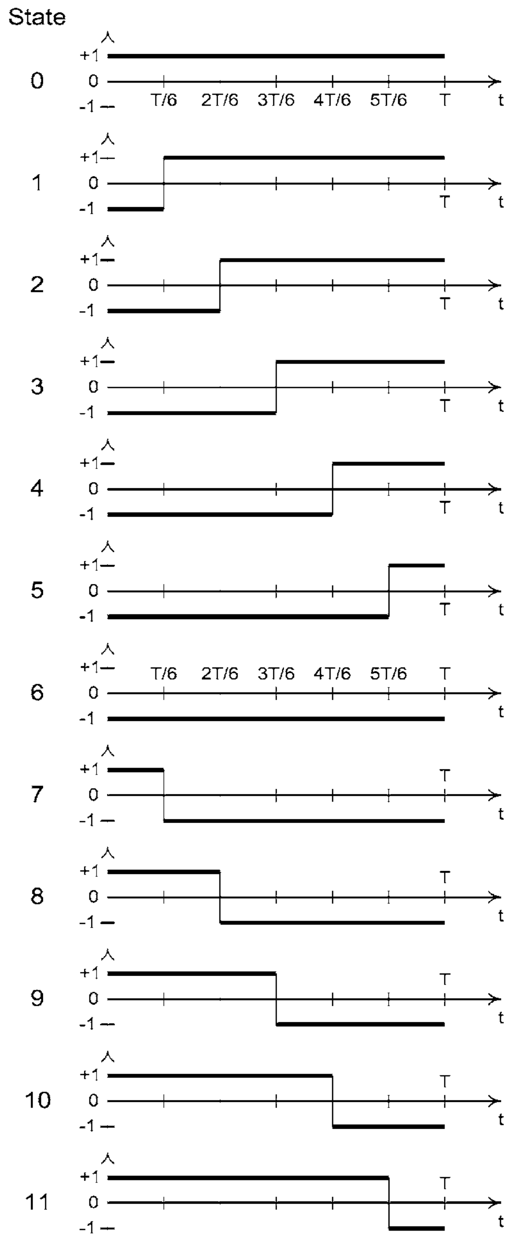

Figure 2. 6-VSPK waveforms.

and

$$R^{(+)} = \sum_{i=0}^{11} p(i) \sum_{j=0}^{11} \mathbf{P}^{n+1}(i, j) w_{ij}^{(+)}(m)\,, \tag{4}$$

where $p(i)$ is the probability of state i which can be determined from (1), $\mathbf{P}^n(i, j)$ is the i, j element of the matrix $\mathbf{P}^n$ (the matrix $\mathbf{P}$ raised to power n), $w_{ij}^{(-)}(m) = 1/T \int_0^{(1-\frac{m}{6})T} w_i(t) w_j(t + \frac{m}{6}T) dt$, $w_{ij}^{(+)}(m) = 1/T \int_0^{\frac{m}{6}T} w_i[t + (1 - \frac{m}{6})T] w_j(t) dt$, and $w_i(t)$ is the waveform of state i.

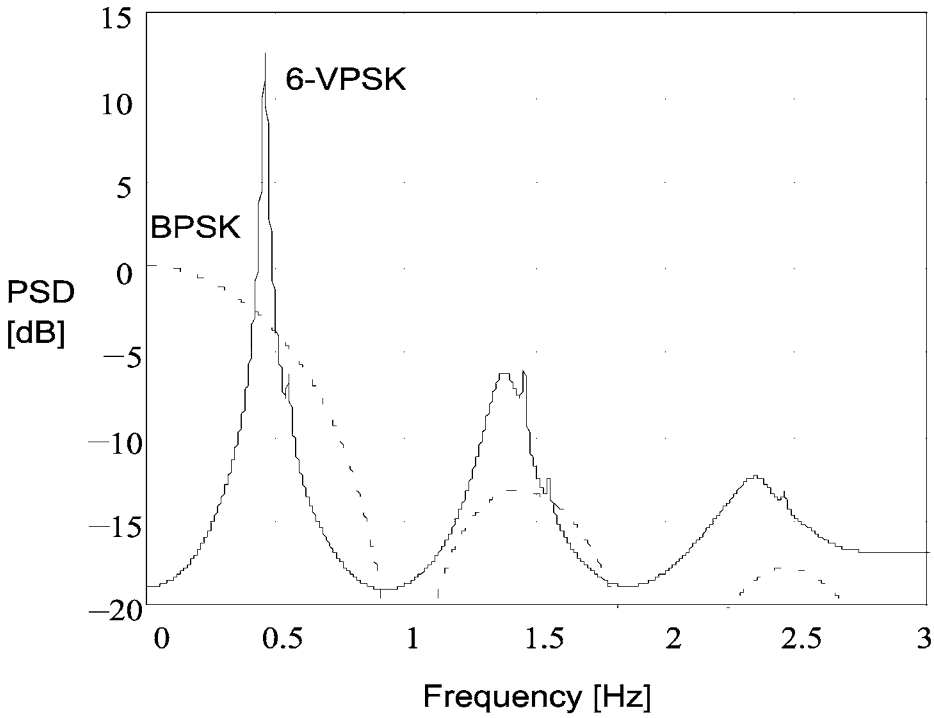

Figure 3. Power spectral density of BPSK and 6-VPSK.

Performing these calculations numerically, is equivalent to sampling the autocorrelation function with the sampling period of $T/6$. Calculations were done for time lags up to 500 T resulting in $500 \times 6 = 3000$ autocorrelation values. The corresponding power spectral density is shown in Figure 3. Also shown in the figure is the spectrum of BPSK. It can be seen from the figure that the spectrum of 6-VPSK has a narrow peak around frequency 0.45 Hz and smaller peaks at odd harmonics of this frequency. The frequency range of 0 to 0.8 Hz contains only 82% of the total signal energy, while [2] erroneously claims that the main peak contains over 99% of energy. Later in the paper, it is shown that removing the harmonic components by passing the signal through a low-pass filter with cut-off frequency 0.8 Hz, causes a significant deterioration in the bit error rate (BER).

The 12 waveforms in Figure 2 are all the waveforms possible with 6-VPSK modulation. It is easy to observe that the associated signal space can be spanned by six (6) basis functions. The basis functions chosen are rectangular pulses of height $\sqrt{6/T}$ and duration $T/6$, shifted in time by a multiples of $T/6$. The symbol probability of error can be determined by noticing that the two closest neighbors for each symbol are at distance $\sqrt{2E/3}$, where E is the VPSK symbol energy. Assuming that all symbols have equal probability, and neglecting contributions from more distant constellation points, it follows that the probability of symbol error in additive Gaussian noise with power spectral density N_0 is given by

$$P_s \simeq 2Q\left(\sqrt{\frac{E}{3N_0}}\right). \tag{5}$$

From the encoding rules it results that VPSK symbols generally represent single data bits, with the exception of occasional 01 sequences being encoded by a single VPSK symbol. Thus

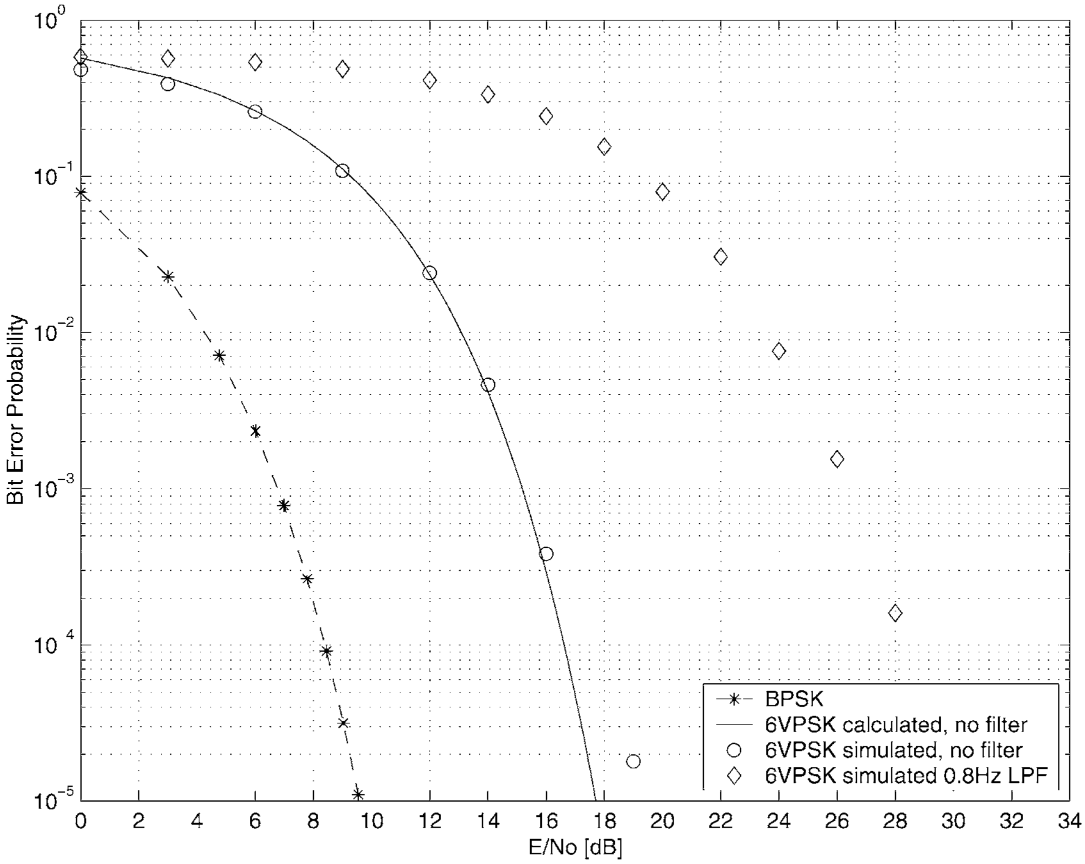

Figure 4. Comparisons of BPSK and 6-VPSK performance.

(5) can be used as approximation to the bit error as well. Indeed, following the encoding rules carefully and working out a detailed analysis of the bit error probability (not shown here due to space constraints) yields the expression

$$P_e = \frac{24}{11} Q\left(\sqrt{\frac{E}{3N_0}}\right), \tag{6}$$

which agrees closely with (5).

The bit error probability is plotted in Figure 4. The four curves shown represent the bit error probabilities for: (1) BPSK modulation, (2) from (6), (3) Monte Carlo simulations of 6-VPSK, and (4) Monte Carlo simulations with a low-pass linear phase FIR filter with cut-off frequency 0.8 Hz. With the lowpass filter the signal power loss is about 12 dB at 10^{-5} probability of error. For a data rate of 1 bit/s and a signal bandwidth limited to 0.8 Hz, the simulation results shown in Figure 4 allow to estimate spectral efficiency of 6-VPSK modulation to be about 1.25 bit/s/Hz with a signal to noise ratio of approximately 30 dB at probability of error of 10^{-5}. Thus this analysis shows that the performance of VSPK is significantly worse than claimed in [2] and that the method is clearly inferior even to BPSK.

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