

Coherent and Non-coherent MIMO Radar Techniques for Target Localization

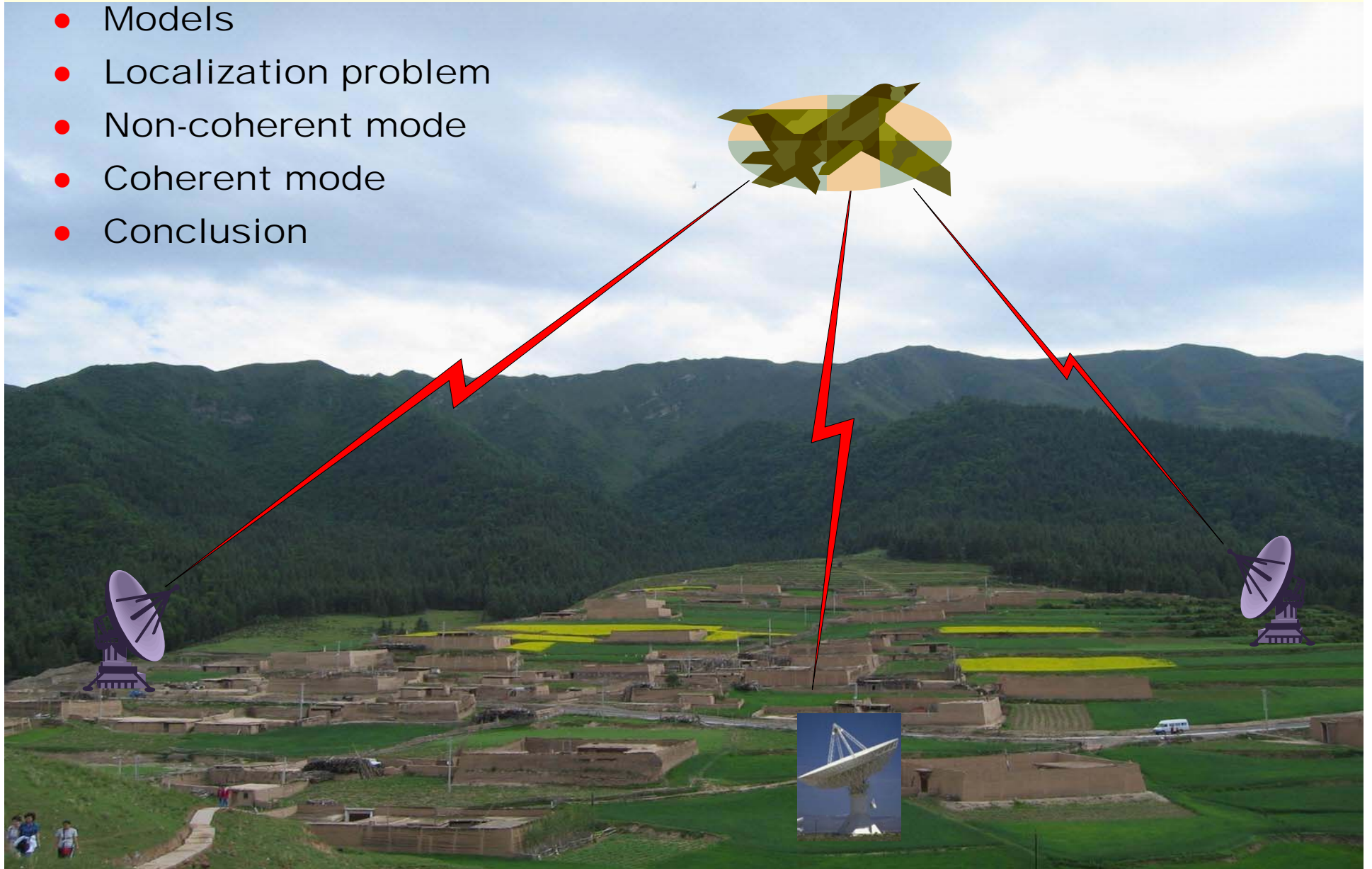
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**2008 ONR/GTRI Workshop on
Target Tracking & Sensor Fusion**

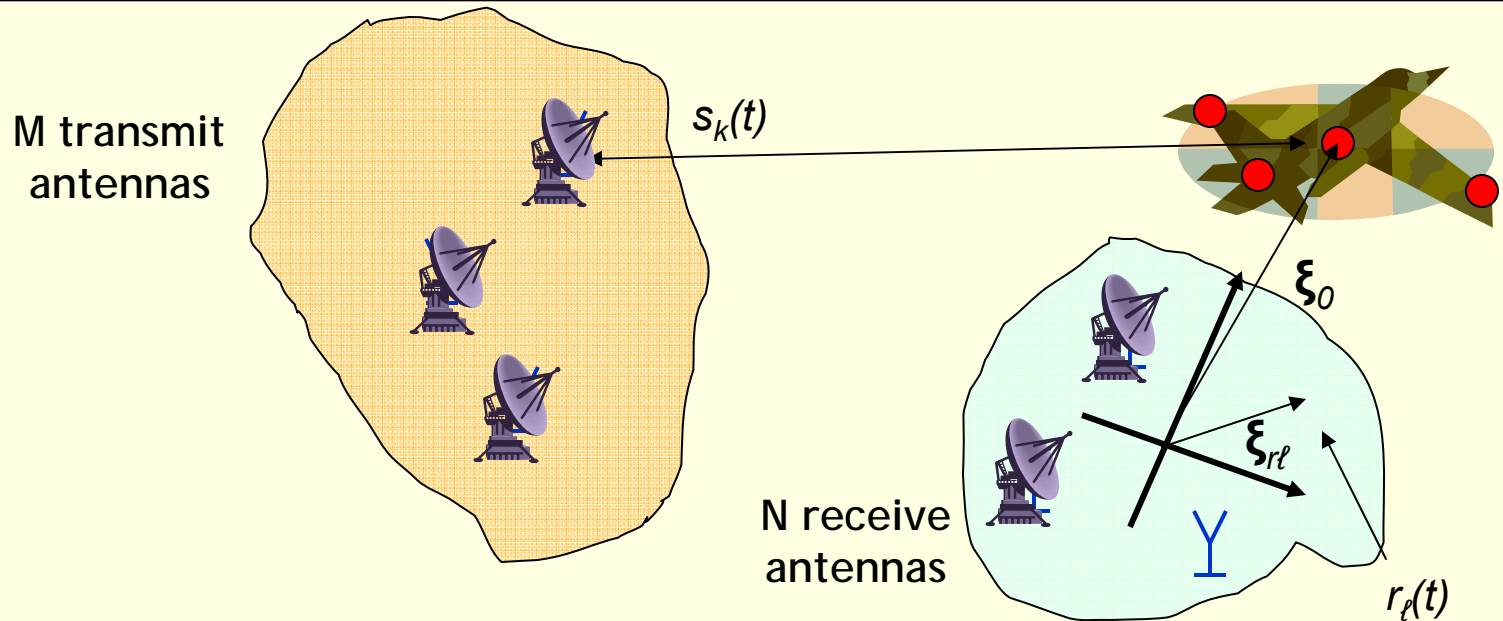
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Blum – Lehigh, Cimini – Delaware, Valenzuela, Chizhik – Bell Labs**

Overview

- Models
- Localization problem
- Non-coherent mode
- Coherent mode
- Conclusion



Basic Model



- Signal observed at sensor ℓ due to transmission from sensors $k = 1, \dots, M$ and reflection from scatterer at ξ_0

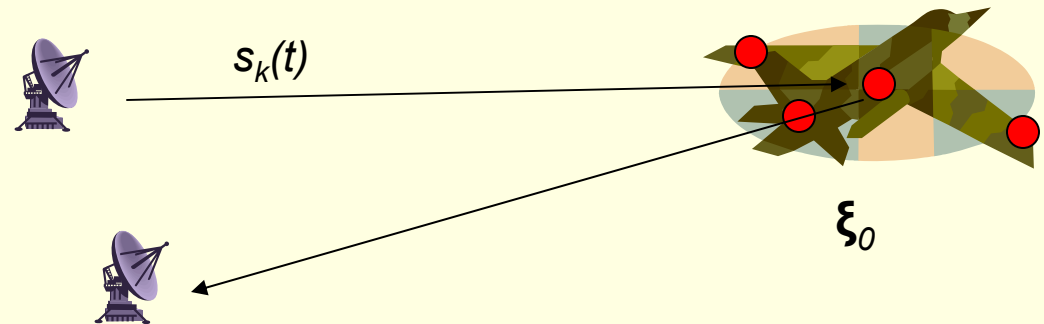
$$r_\ell(t) = \sum_{k=1}^M h_{\ell k}(\xi_0) s_k(t - \tau_{tk}(\xi_0) - \tau_{r\ell}(\xi_0)) + z_\ell(t)$$

$h_{\ell k}(\xi_0)$ = target response from transmitter k to receiver ℓ

$z_\ell(t)$ = noise

$s_k(t)$ = waveform transmitted by sensor k

“Channel” Model



- For a complex target, $h_{\ell k}$ is a random variable
- For a sufficiently large number of scatterers, and sufficiently separated antennas, the channel elements $h_{\ell k}$ are complex-valued, i.i.d., jointly Gaussian [Fishler, Haimovich, Blum...2004]
- $h_{\ell k}$ are unknown, statistics are known
- For single point target,

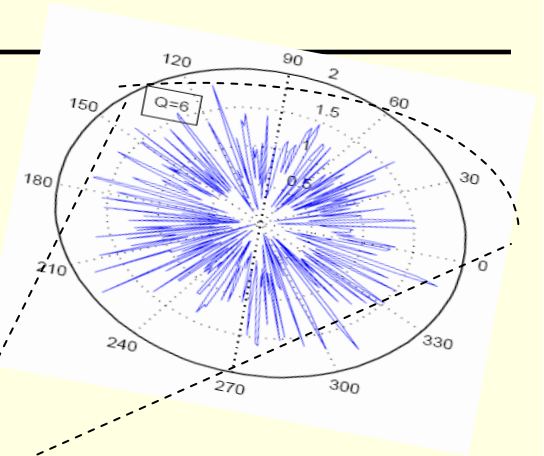
$$h_{\ell k}(\xi_0) = \zeta_0 e^{-j\omega_c[\tau_{tk}(\xi_0) + \tau_{r\ell}(\xi_0)]}$$

$\tau_{tk}(\xi_0)$ = time delay from transmit sensor k to scatterer

$\tau_{r\ell}(\xi_0)$ = time delay from scatterer to receive sensor ℓ

Extended Target Model

- Simplest target model: point target, uniform RCS vs. aspect angle.
- Model not suitable for complex target/targets and a radar system with distributed antennas.
- Physical targets comprise many individual scatterers
- A common target model in radar: a complex target, such as an airplane, comprises many independent, spherical scatterers [Delano53, Swerling60].
- Swerling models: statistical description of RCS
 - RCS fluctuations in time
 - RCS fluctuations in aspect angle



Features

- Features of MIMO radar with M transmit and N receive antennas
- Non-coherent (statistical) mode
 - MN diversity paths to overcome RCS fading
 - Geometric advantage of radar footprint
- Coherent mode
 - High localization resolution
 - Spurious peaks mitigation
 - Localization of multiple targets
 - Flexible modes of operation

Non-coherent MIMO Radar

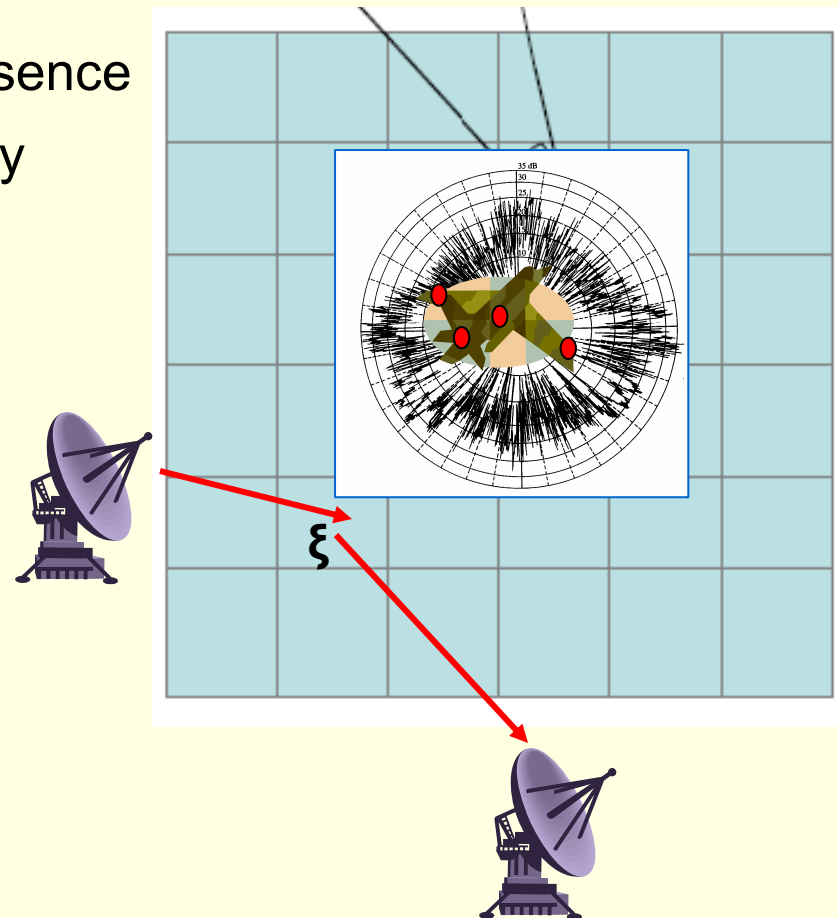
- Search surveillance area for target presence
- Resolution of search grid determined by bandwidth of transmitted signals
- Processing based only on time delay measurements

Neyman-Pearson detector for target at coordinates $\boldsymbol{\xi} = (x, y)$:

- at each sensor, form the correlation

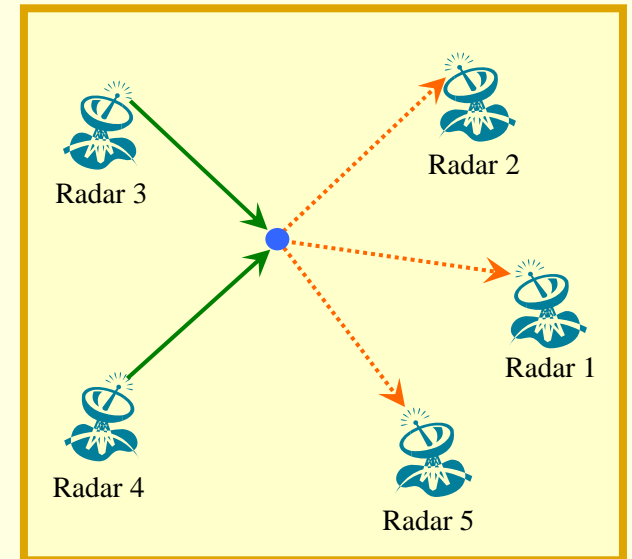
$$[\mathbf{y}(\boldsymbol{\xi})]_{\ell k} = \int r_{\ell}(t) s_k(t - \tau_{\ell k}(\boldsymbol{\xi})) dt$$

- $r_{\ell}(t)$ observation is function of:
target scattering gain at aspect angle
relative to transmitter k and receiver ℓ



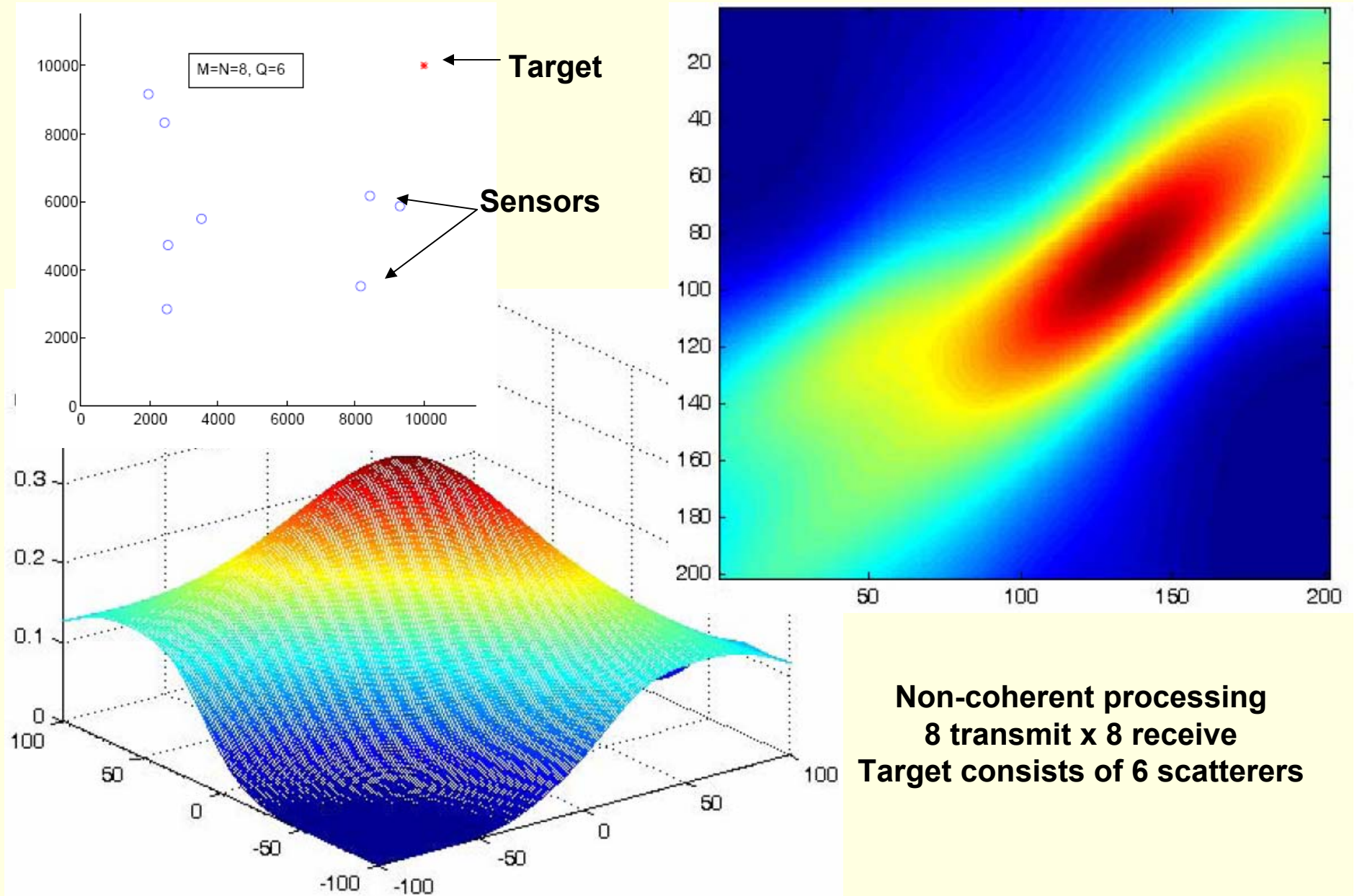
Non-coherent MIMO Radar

- Each $y_{\ell k}(\boldsymbol{\xi})$ depends on the target response transmit antenna k and receive antenna ℓ
- $s_k(t)$ need to be orthogonal in order to compute $y_{\ell k}(\boldsymbol{\xi})$
- Target response is a fading process and $y_{\ell k}(\boldsymbol{\xi})$ are response of diversity paths
- Fusion center; sensor synchronization
- Detection test: non-coherent combination of sensor outputs [Fishler, Haimovich, Blum...2006]



$$\sum_{k=1}^M \sum_{\ell=1}^N |y_{\ell k}(\boldsymbol{\xi})|^2 \underset{H_0}{\overset{H_1}{>}} \gamma$$

Example



Advantages of NC MIMO Radar

- Non-coherent radar mode is suitable for detection and localization of a target with complex RCS or a group of targets
- Diversity paths overcome target RCS fading
- Total number of diversity terms is MN [Fishler, Haimovich,...2006]
- MIMO radar advantage when $MN > M+N$
- Target localization is essentially a multilateration process that takes advantage of the system footprint.
- It can be shown that for M transmitters and N receivers, target localization error is reduced by a factor (geometric dilution of position – GDOP)

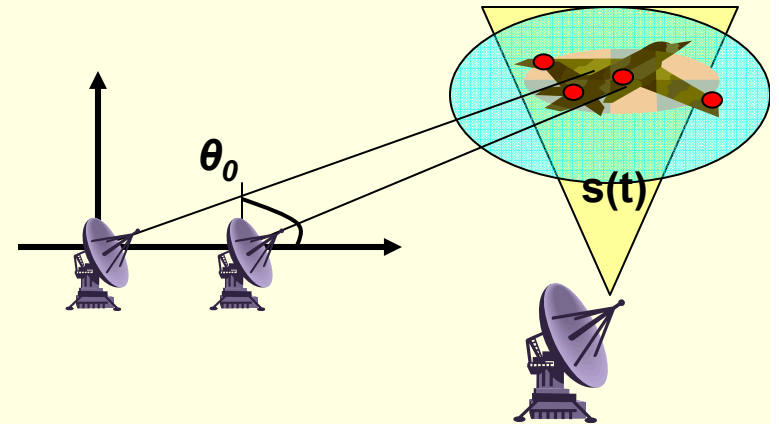
$$g \geq \sqrt{\frac{2}{MN}}$$

Other Applications

- Other applications of non-coherent MIMO radar
- Distributed transmit antennas support RCS diversity, colocated receive antennas to support direction finding [Fishler, Haimovich... 2004, Lehman, Fishler, Haimovich, ...2007]
- Distributed transmit and receive antennas with joint processing support moving target detection [Lehmann, Haimovich, ... 2006]

Target Localization: Far Field

- Typical phased arrays operate in far field mode: target is at the same angle with respect to all array elements.
- Linear array: sensor located at $\lambda/2$ spacing
- Direction of arrival: estimated from phase differences between adjacent sensors
- Range estimated from time delay measurements.



Large Arrays – Grating Lobes

- Pattern of linear array phased in direction θ in the far field

$$f(\theta) = \sum_{\ell=1}^N e^{-j\omega_c \frac{x_{r\ell} \sin\theta}{c}}$$

- Properties of the pattern:

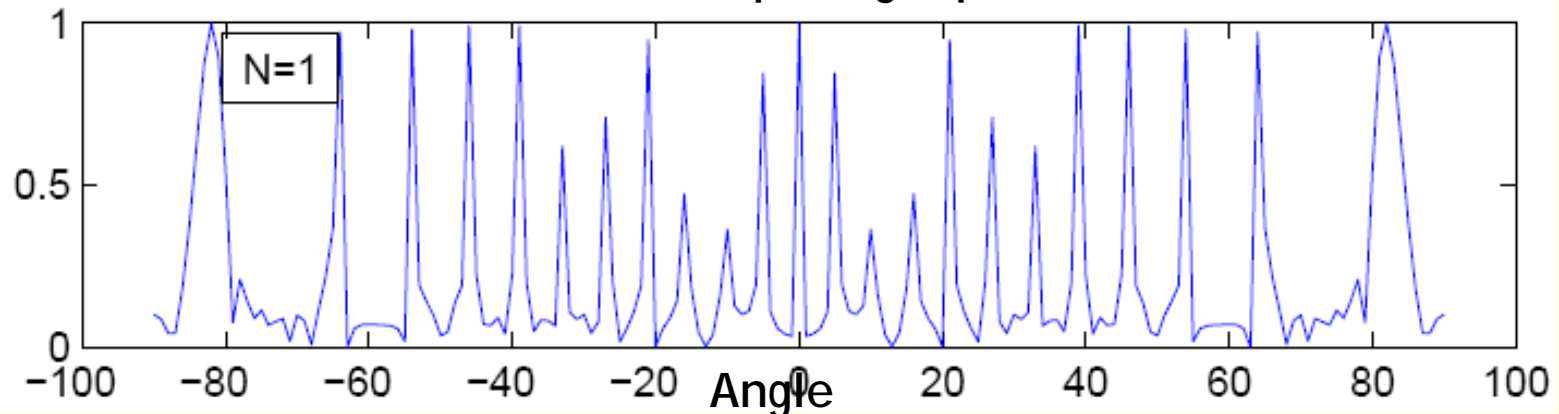
(1) Pattern peak at $\theta = 0$

(2) Mainlobe width $\approx \lambda/L$, where L is dimension of array

(3) Grating lobes at $\sin^{-1} \frac{\lambda}{L}$;

for spacing $x_{r\ell} - x_{r,\ell-1} = x \gg \lambda$, there are many grating lobes

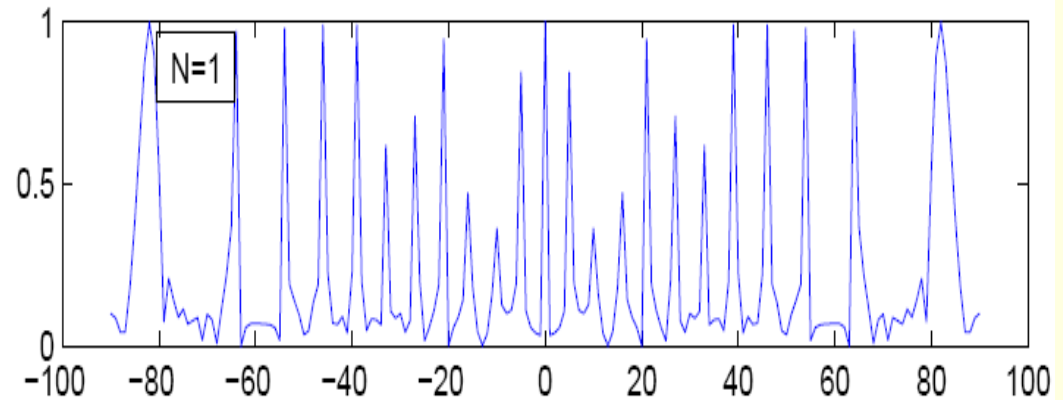
10 elements, uniform spacing, aperture = 100λ



Example: Grating Lobes

- Linear array with uniformly spaced elements $x_{r\ell} = \ell x$

$$\begin{aligned} f(\theta) &= \left| \sum_{\ell=1}^N e^{-j\omega_c \frac{x_{r\ell} \sin \theta}{c}} \right| \\ &= \left| \sum_{\ell=1}^N e^{-j\frac{2\pi}{\lambda} \ell x \sin \theta} \right| \\ &= \left| \frac{\sin\left(\frac{\pi N x \sin \theta}{\lambda}\right)}{\sin\left(\frac{\pi x \sin \theta}{\lambda}\right)} \right| \end{aligned}$$



- Main peak at $\theta = 0$; next pattern peak at

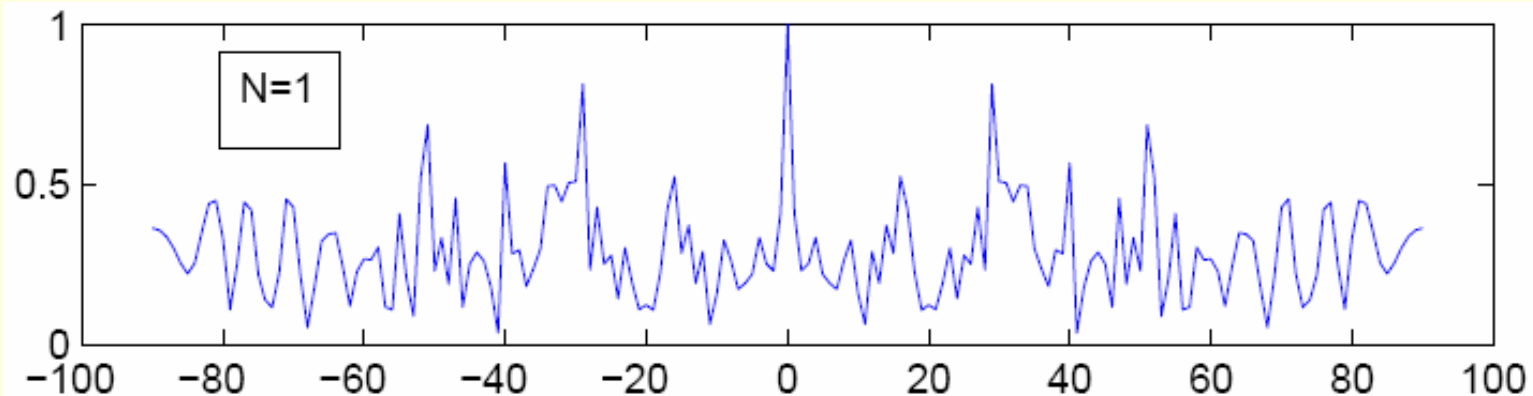
$$\frac{\pi x \sin \theta}{\lambda} = \pi \Rightarrow \theta_{peak} = \sin^{-1} \frac{\lambda}{x}$$

- For $x \leq \lambda/2$, there are no further peaks (grating lobes).
- For $x = \lambda$, $\theta_{peak} = \pi/2$

Large, Thinned Array

- The regular pattern of the grating lobes can be broken up by randomizing element location
- Random array have been studied in the past [Lo64, Steinberg76]
- Salient features [Steinberg81]:
 1. Resolving power λ/L
 2. Spurious sidelobe peaks
 3. Mean sidelobe level proportional to $1/N$

10 elements, uniform spacing, aperture = 100λ

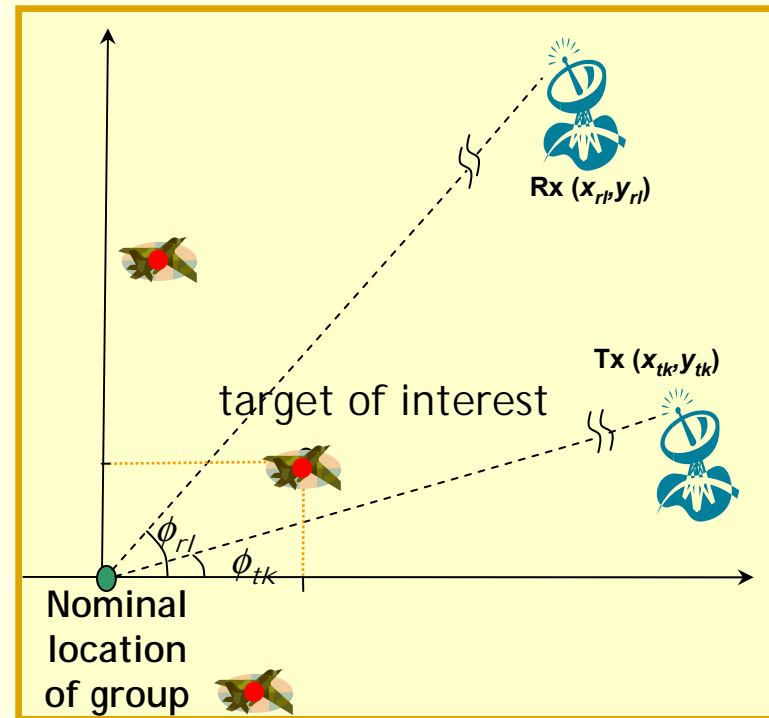


MIMO Radar: Coherent Mode

- Non-coherent mode identified a target of interest
- Now, switch to high resolution, coherent mode to investigate the target
- Goal is to obtain resolution beyond possible with the radar waveform
- Refined location estimation carried out in the neighborhood of the nominal target location

$$\xi_0 = (x_0, y_0).$$

- Requires phase synchronization of distributed sensors



MIMO radar signal model.

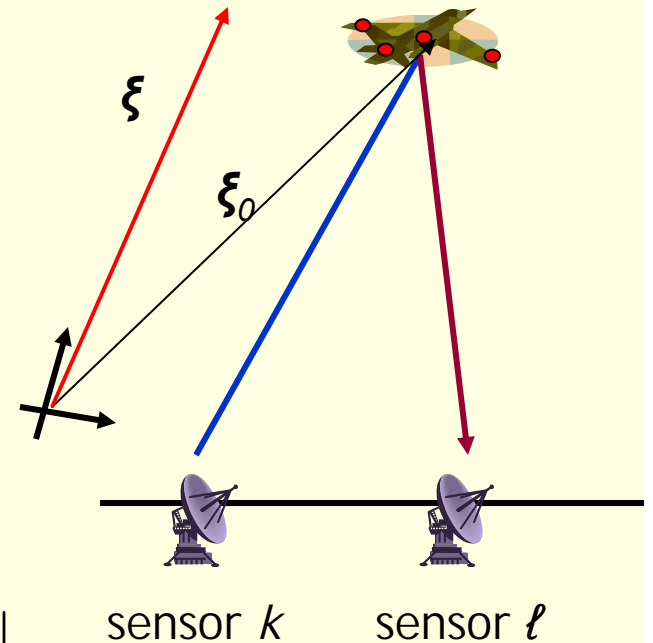
Coherent MIMO Radar Processing

[Lehmann, Haimovich...2006]

- For a hypothetical target at position vector ξ , coherent processing consists of:
 - Matched filtering with $s_k^*(t - \tau_{\ell k}(\xi))$
 - Compensate phase with $\omega_c \tau_{\ell k}(\xi)$
- The coherent MIMO radar metric is given by

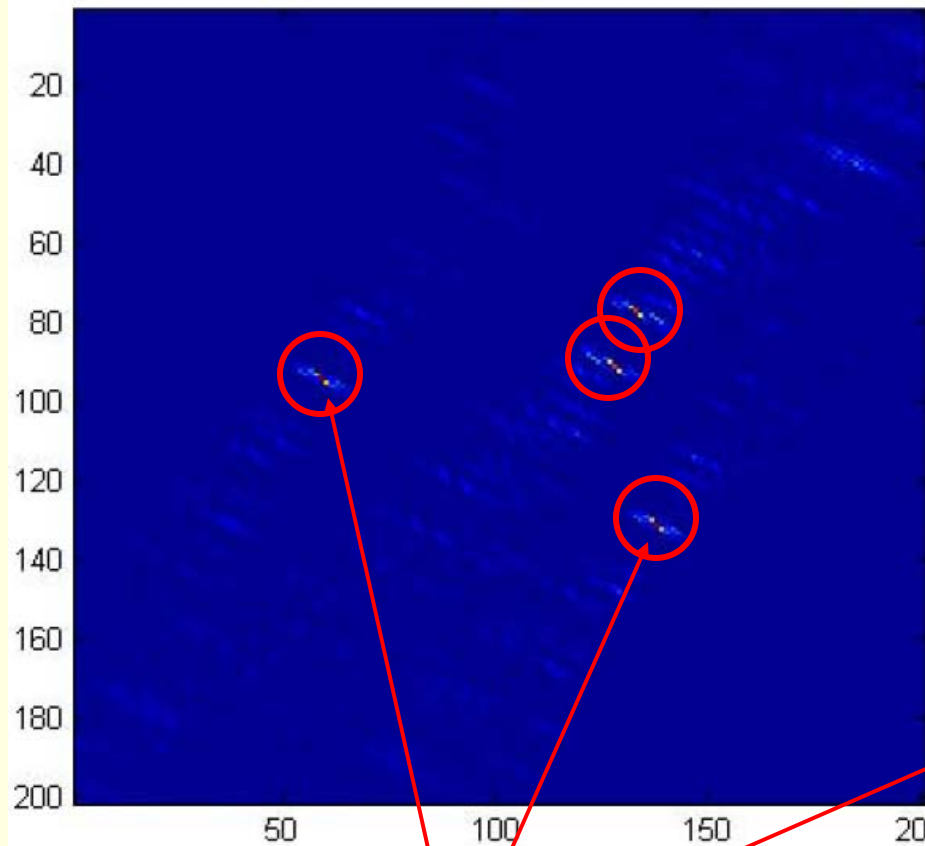
$$f(\xi) = \left| \sum_{\ell=1}^N \sum_{k=1}^M e^{j\omega_c \tau_{\ell k}(\xi)} \int r_{\ell}(t) s_k^*(t - \tau_{\ell k}(\xi)) dt \right|$$

- Near-field operation: the target is at a different angle with respect to each antenna.

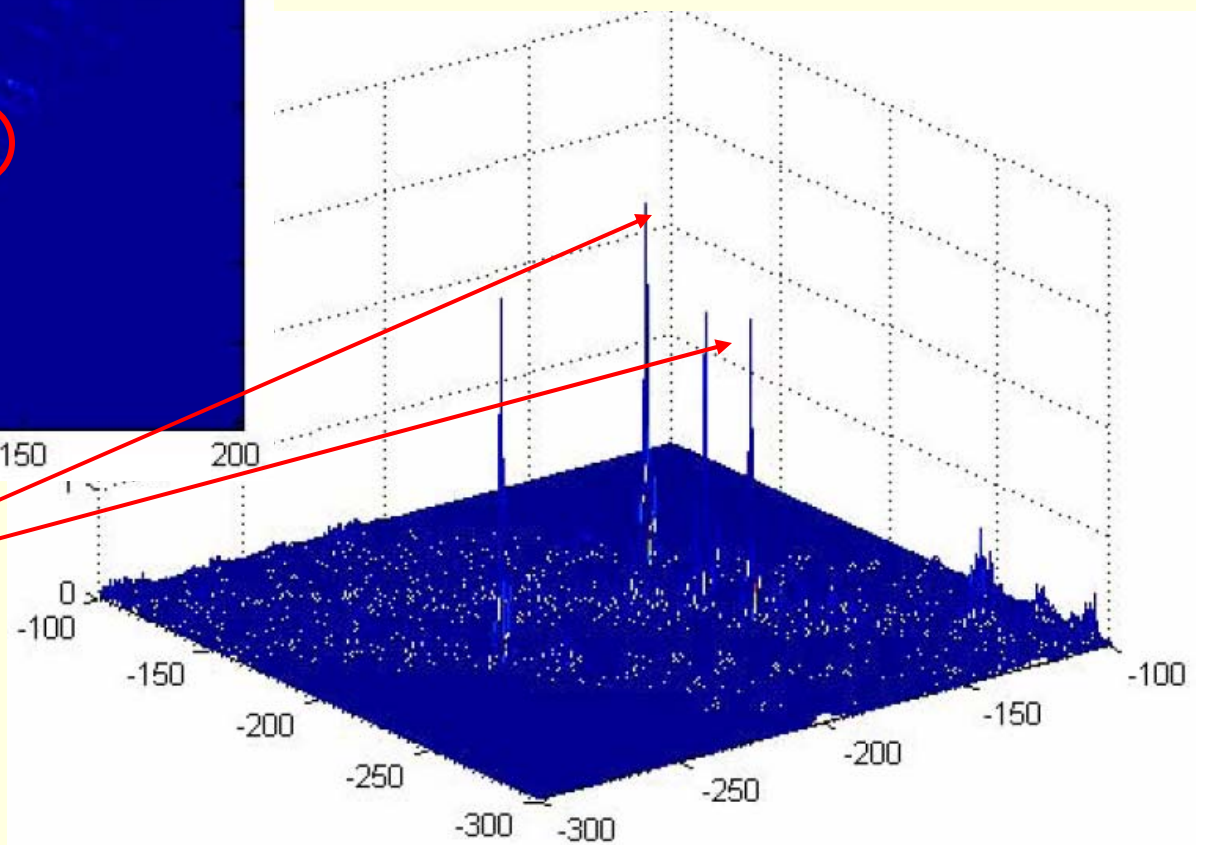


Example

Beamforming example
8 transmit x 8 receive



Targets



High Resolution Target Localization

- Target localization by coherent MIMO radar exhibits high resolution, but is subject to spurious sidelobes.
- Problems that need to be studied:
 - Localization accuracy
 - Sidelobes Statistics
 - Features of MIMO radar
- Extending Steinberg's work on random arrays to:
 - Near-field arrays
 - MIMO radar

Cramer-Rao Lower Bound

- The Cramer-Rao lower bound can be expressed [Godrich, Haimovich, Blum...2008]:

$$\sigma_{CRLB} = \frac{c}{\sqrt{8\pi^2 \text{SNR} (f_c^2 + \beta^2)}} \cdot g$$
$$\approx \frac{c}{\sqrt{8\pi^2 \text{SNR} f_c}} \cdot g$$

- The geometric dilution of position (GDOP) term g incorporates the effect of sensor locations on the localization error
- It can be shown that with M transmit and N receive antennas, the minimum $\text{GDOP} = \sqrt{2 / MN}$
- The term β is the effective bandwidth
- Localization error is approximately proportional to $1 / f_c$
- The effective bandwidth β has little impact

Sidelobes

- Target detection test statistics is sum of complex exponentials
- Signal received from target at location ξ_0

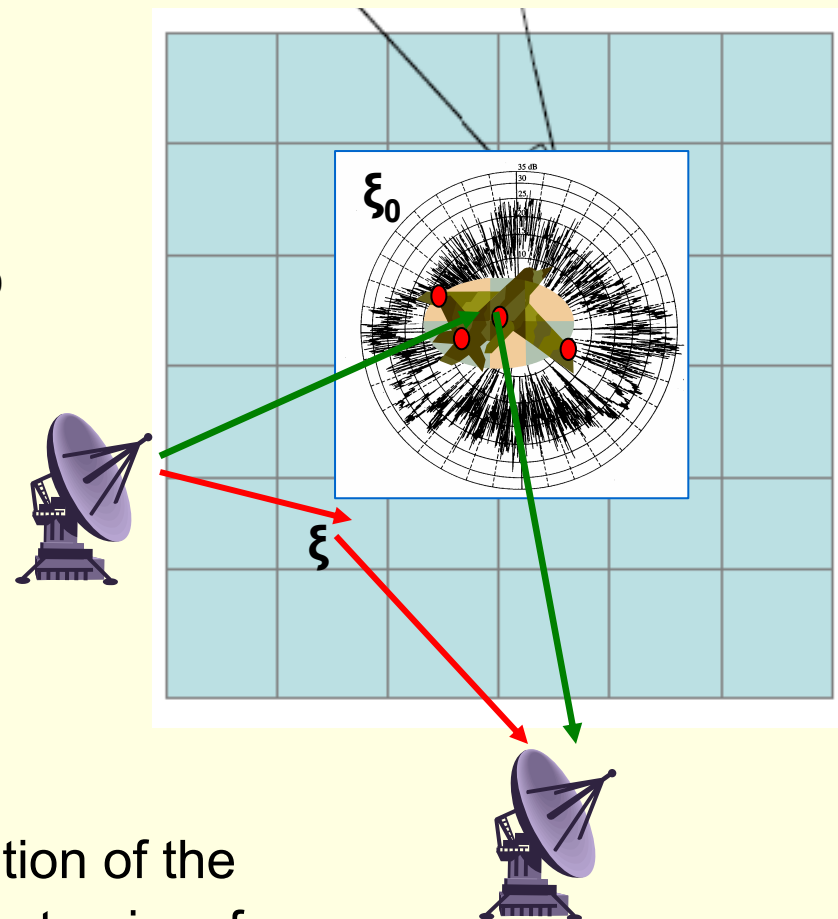
$$e^{-j\omega_c \tau_{\ell k}(\xi_0)}$$

- Test for target at ξ by applying $e^{j\omega_c \tau_{\ell k}(\xi)}$

$e^{-j\omega_c [\tau_{\ell k}(\xi_0) - \tau_{\ell k}(\xi)]}$ has spurious peaks

- Sidelobe reduction concept:

- Spurious peaks depend on the location of the sensors through the term $\tau_{\ell k}$. Different pairs of sensors yield different sidelobes.



Sidelobes Statistics

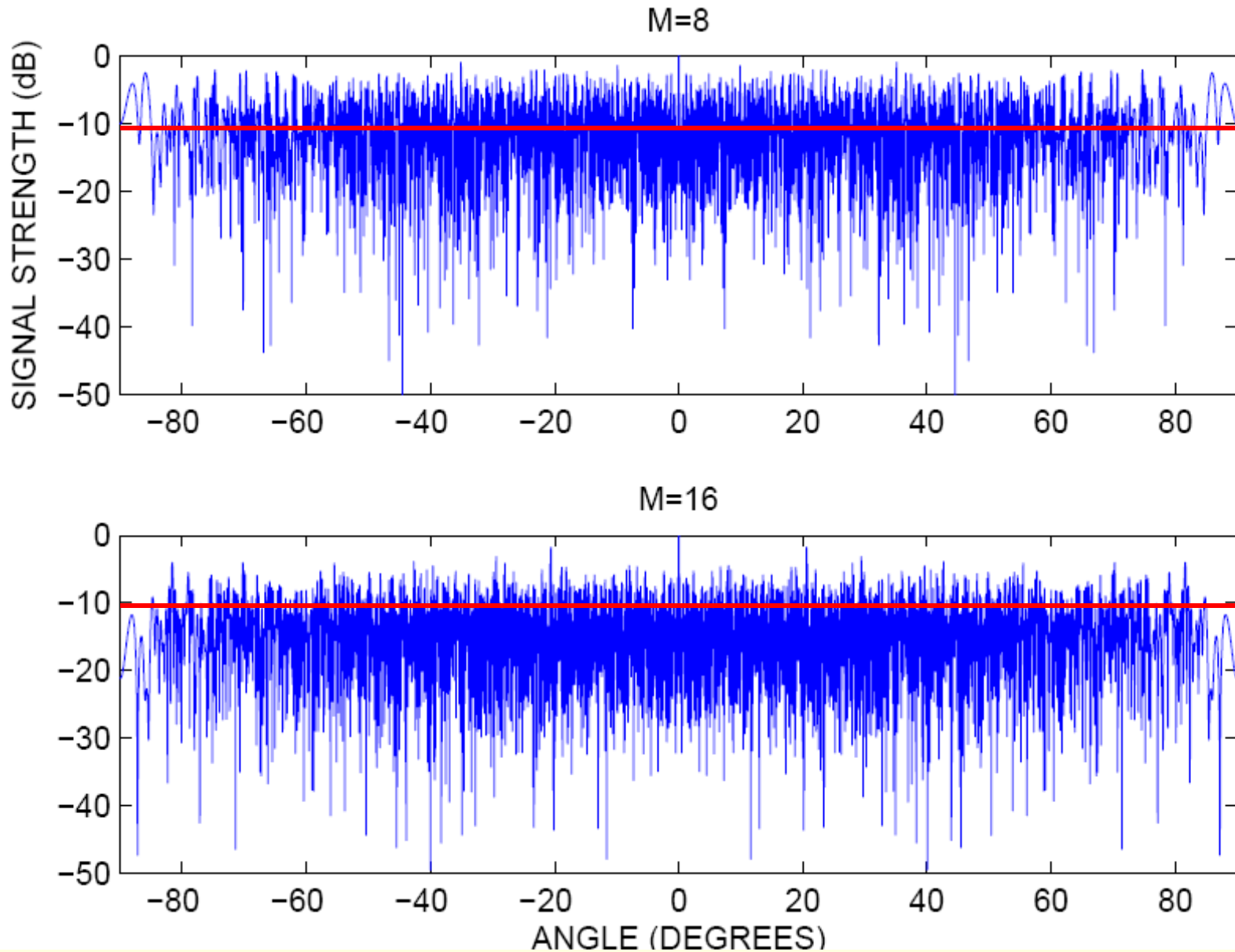
- Localization ambiguities in coherent MIMO radar with distributed antennas [Haleem, Haimovich 2008]

- The localization test statistic

$$f(\boldsymbol{\xi}) = \left| \sum_{\ell=1}^N \sum_{k=1}^M e^{j\omega_c \tau_{\ell k}(\boldsymbol{\xi})} \int r_{\ell}(t) s_k^*(t - \tau_{\ell k}(\boldsymbol{\xi})) dt \right|$$

- The mean pattern of the localization test statistic is found to be independent of the number of antennas of the MIMO system.
- The variance of the sidelobes of the test statistic pattern scales down as $1/2MN$

Effect of Number of Sensors



Waveform Coding

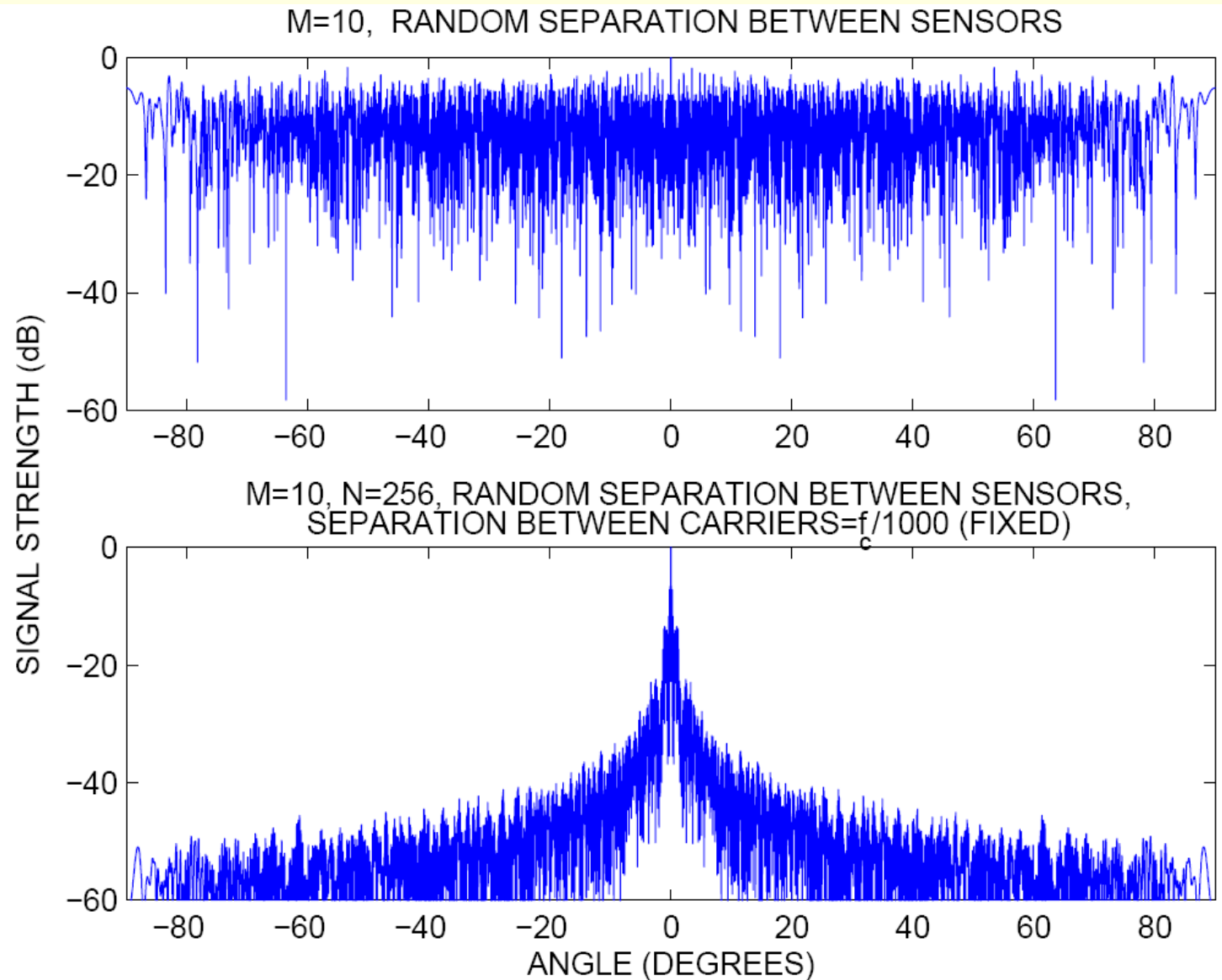
- The location of sidelobes depends on the signal frequency

$$e^{-j\omega_c[\tau_{ck}(\boldsymbol{\xi}_0) - \tau_{ck}(\boldsymbol{\xi})]}$$

- Sidelobes can be controlled with suitably signals suitably separated in frequency
- An OFDMA scheme for MIMO radar: each sensor transmits a set of subcarriers of an OFDM signal [Haleem, Haimovich 2008].

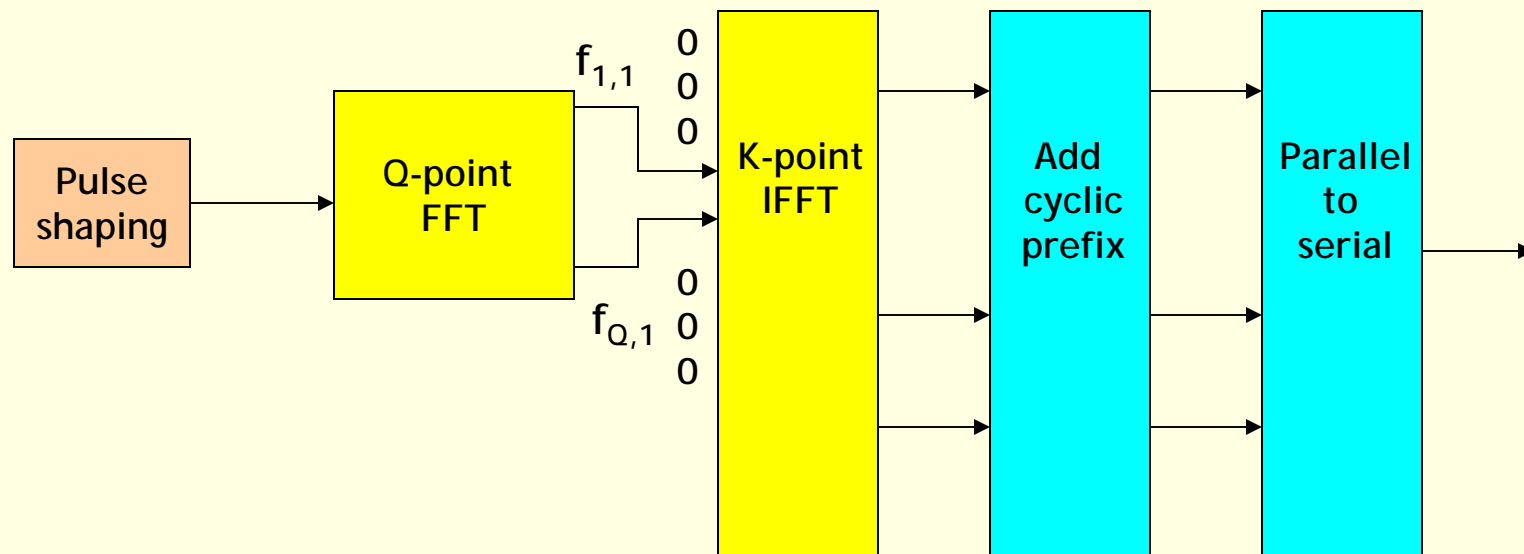
$$f(\boldsymbol{\xi}) = \left| \sum_{n=1}^N \sum_{m=1}^M \sum_{k=1}^K e^{j2\pi(f_c + f_{k,m})\tau_{nm}(\boldsymbol{\xi})} \int r_n(t) s_{k,m}^*(t - \tau_{ck}(\boldsymbol{\xi})) dt \right|$$

OFDMA Coding



SC-FDMA Implementation

- Single carrier – frequency division multiple access (SC-FDMA) is a multiple access method similar to OFDM.
- Proposed for the uplink of 3GPP-LTE
- Seems to be well-suited to MIMO radar



One of the M transmitters of the MIMO radar

Concluding Remarks

- Non-coherent MIMO radar seeks to exploit target RCS diversity to improve detection and estimation performance.
- Coherent MIMO radar supports high resolution, albeit ambiguous target localization.
- Ambiguities can be controlled through the number of sensors.
- Localization with coherent MIMO radar exhibits an error that scales with carrier wavelength
- The use of multiple sensors improves localization accuracy by a factor as low as $\sqrt{2 / MN}$
- The transmission of multiple frequency can mitigate the sidelobe problem in MIMO radar with distributed antennas.