Target Localization Techniques and Tools for MIMO Radar

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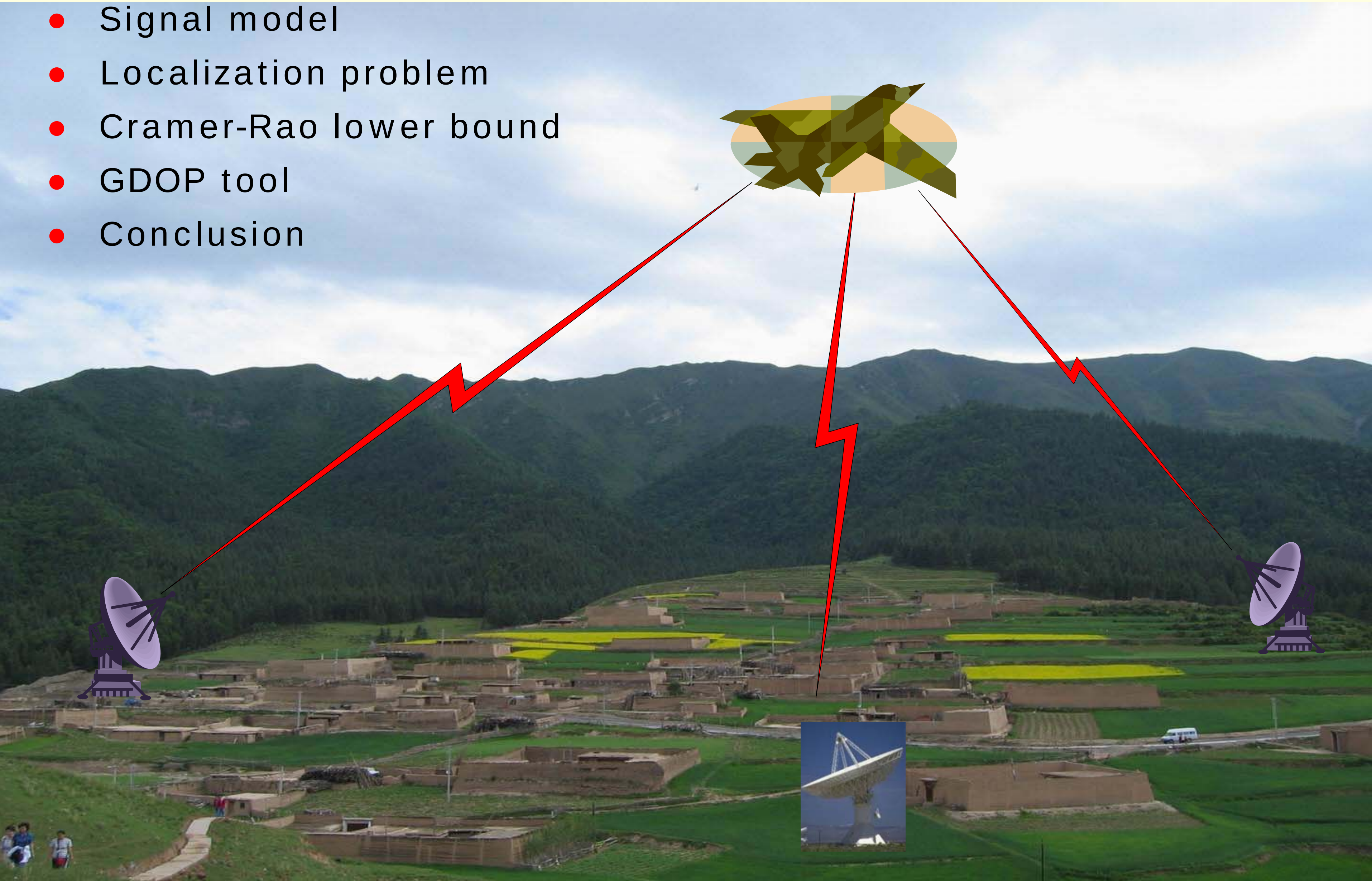
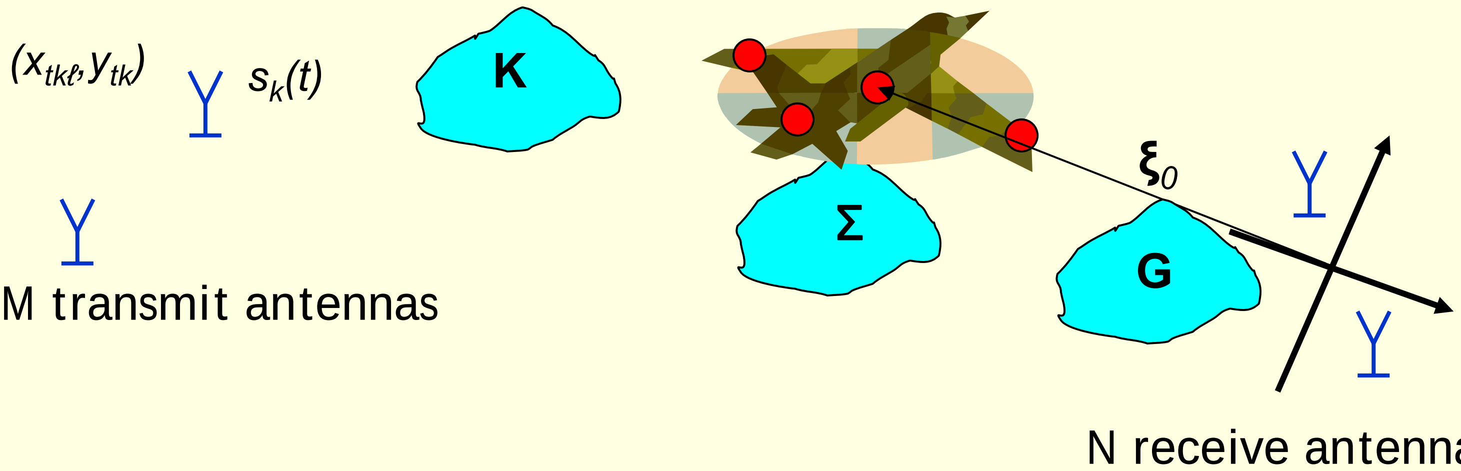
RadarCon 2008

Overview

- Signal model
- Localization problem
- Cramer-Rao lower bound
- GDOP tool
- Conclusion

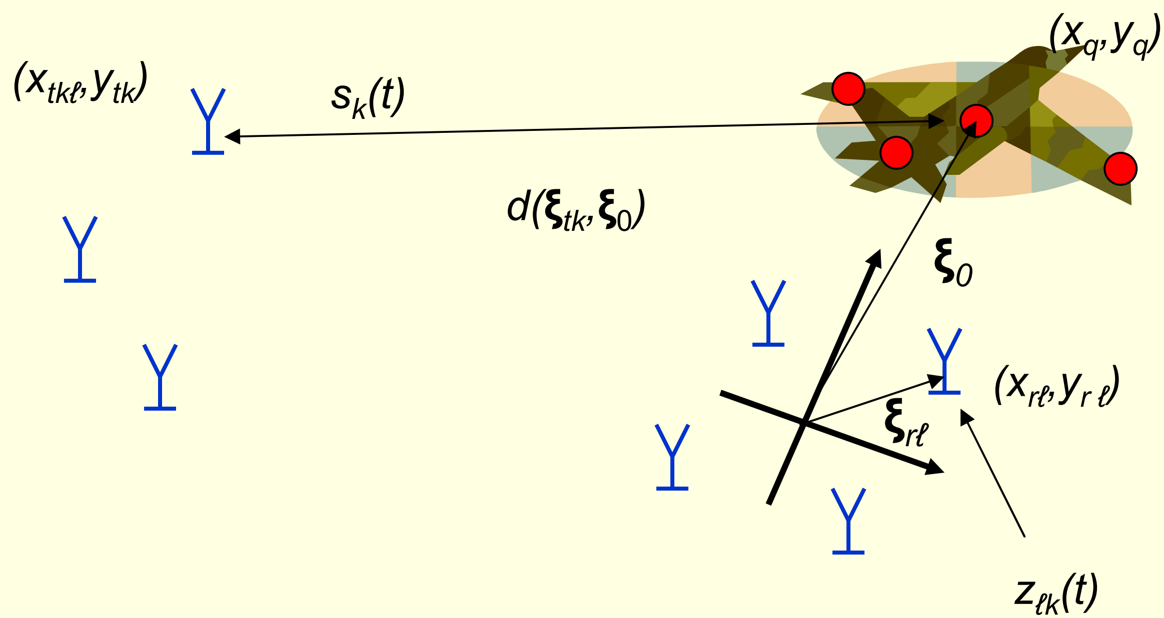


MIMO Radar Channel



- Let $h_{\ell k}(\boldsymbol{\xi}_0)$ represent the complex channel gain from transmit antenna k to the target with an RCS center of gravity located at position vector $\boldsymbol{\xi}_0$, to receive antenna ℓ
- Path gains from transmit antenna k to receive antenna ℓ, $h_{\ell k}$ are organized in a matrix $\mathbf{H} = [h_{\ell k}]$.

Signal Model



- Lowpass equivalent of the signal observed at sensor ℓ due to to transmission from sensor k and reflection from scatterer at $\boldsymbol{\xi}_0$

$$z_{\ell k}(t) = \zeta_0 s_k\left(t - \tau_{tk}(\boldsymbol{\xi}_0) - \tau_{r\ell}(\boldsymbol{\xi}_0)\right) e^{-j\omega_c\left[\tau_{tk}(\boldsymbol{\xi}_0) + \tau_{r\ell}(\boldsymbol{\xi}_0)\right]}$$

where the propagation time delay $\tau_{tk}(\boldsymbol{\xi}_0) = \dfrac{d(\boldsymbol{\xi}_{tk}, \boldsymbol{\xi}_0)}{c}$

the distance $d(\boldsymbol{\xi}_{tk}, \boldsymbol{\xi}_0) = \sqrt{(x_{tk} - x_0)^2 + (y_{tk} - y_0)^2}$

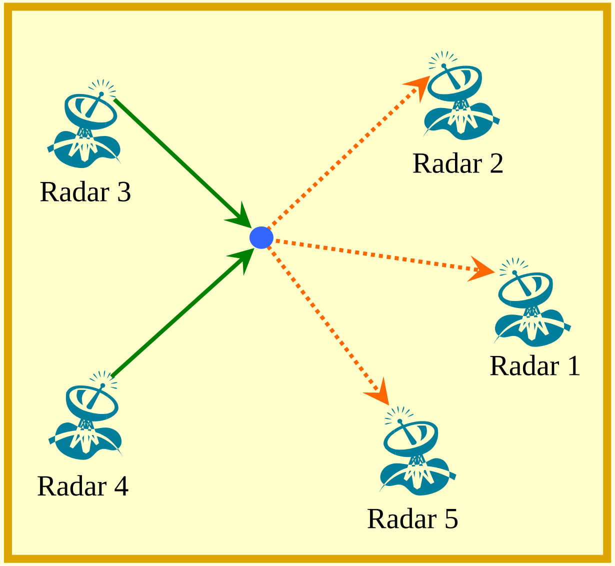
Non-coherent MIMO Radar

- Neyman-Pearson detector for target at coordinates $\boldsymbol{\xi} = (x, y)$:
  - at each sensor, form the correlation

$$\left[\mathbf{y}(\boldsymbol{\xi})\right]_{\ell k} = \int r_\ell(t)\, s_k\left(t - \tau_{\ell k}(\boldsymbol{\xi})\right) dt$$

  - average over paths
  - set threshold γ according to tolerated FA
  - compute

$$\left\|\mathbf{y}(\boldsymbol{\xi})\right\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

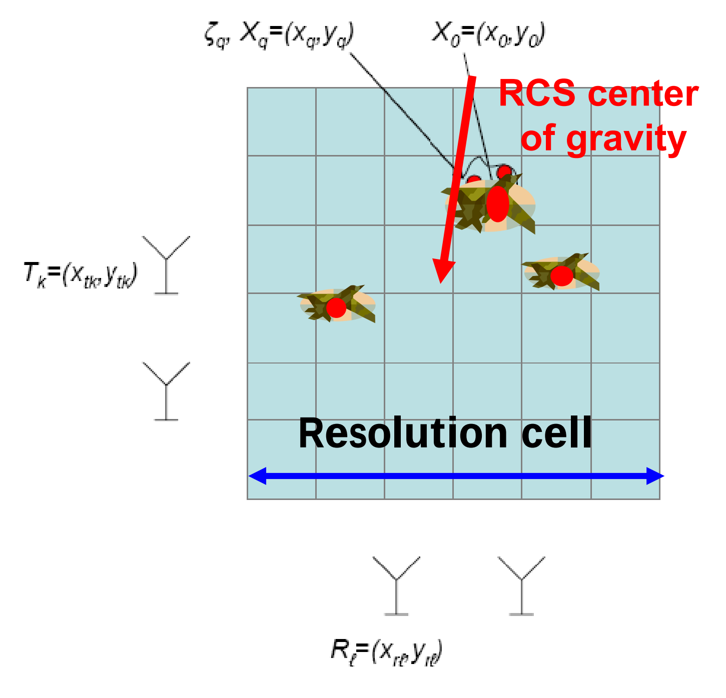


- Processing based only on time delay measurements
- Resolution cell set by bandwidth of transmitted waveform
- Since "channel" is not known, orthogonal waveforms are needed to separate signals at the receiver
- Diversity paths combined non-coherently

Non-coherent Localization

Applications:

- Multiple targets at long distance. Each target appears as point scatterer.
- Targets are unresolvable by radar waveform
- This model results in RCS diversity, i.e., full rank channel matrix **H**
- Channel matrix **H** is unknown, its pdf is known



Distinguishing features of non-coherent MIMO radar:

- Orthogonal waveforms
- Time delay measurements only (non-coherent)

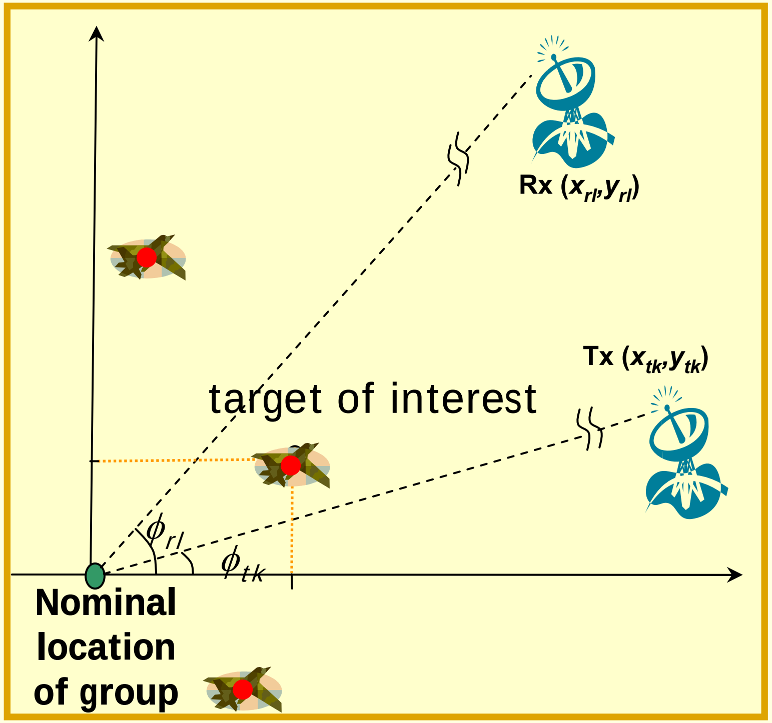
MIMO "gain"

- Illumination of full surveillance volume
- Exploit RCS diversity
- Geometric dilution of precision (GDOP) advantage of the radar system footprint

Coherent Mode

- Non-coherent mode identified a target of interest
- Now, switch to high resolution, coherent mode to investigate the target
- Goal is to obtain resolution beyond possible with the radar waveform
- Refined location estimation carried out in the neighborhood of the nominal target location

  $\xi_0 = (x_0, y_0)$.
- Requires phase synchronization of distributed sensors



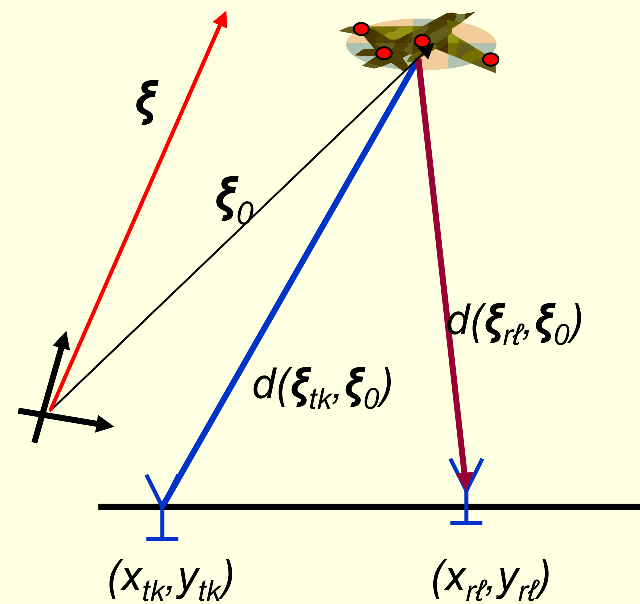
MIMO radar signal model.

Coherent MIMO Radar Processing

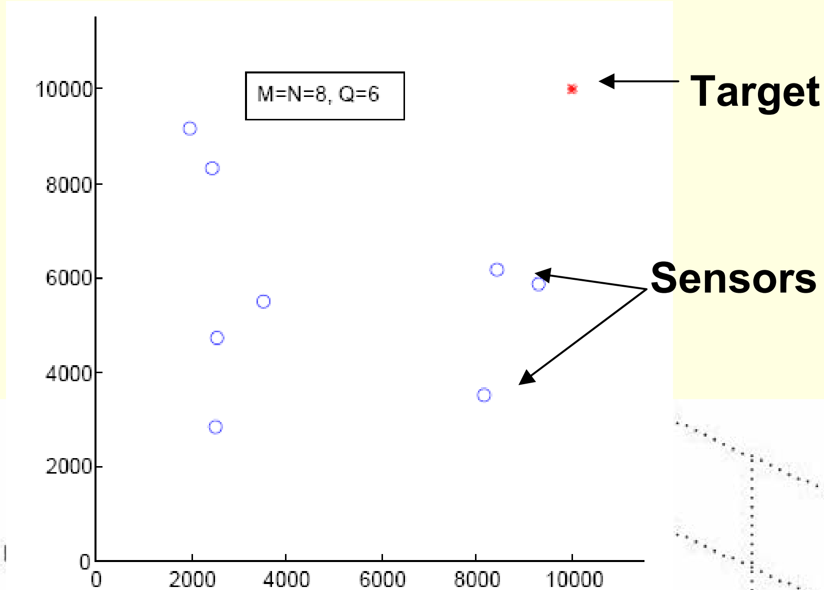
- For a hypothetical target at position vector $\boldsymbol{\xi}$, coherent processing consists of:
  - Matched filtering with $s_k^*\left(t-\tau_{\ell k}\left(\boldsymbol{\xi}\right)\right)$
  - Compensate phase with $\omega_c \tau_{\ell k}\left(\boldsymbol{\xi}\right)$
- The coherent MIMO radar metric is given by

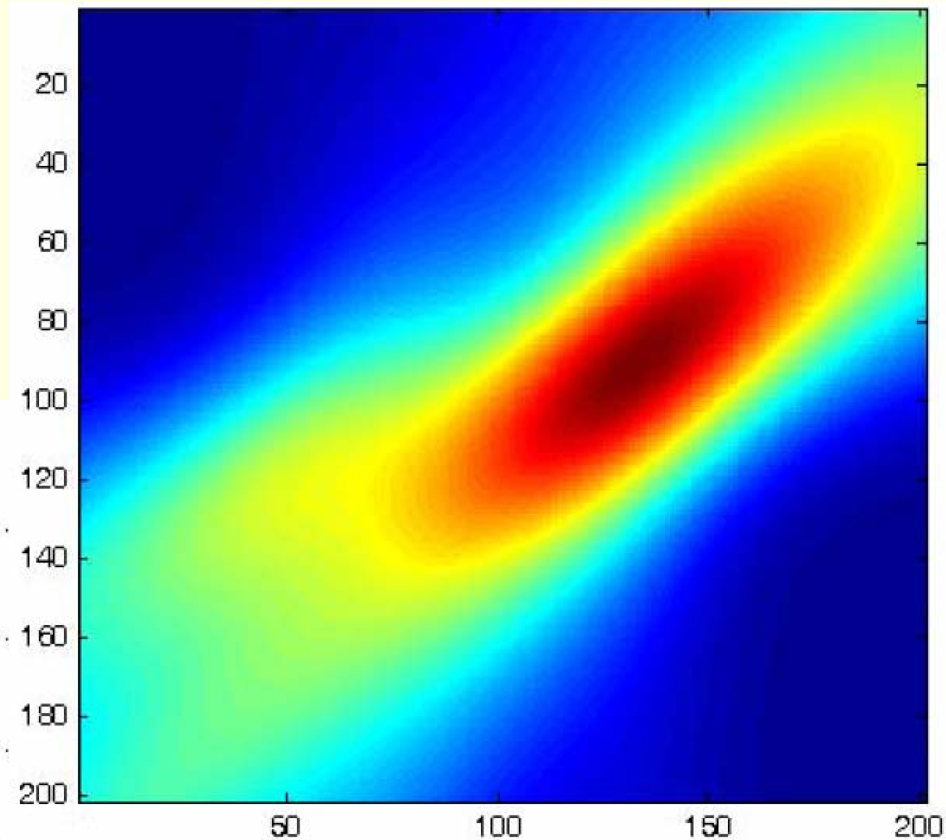
$$f\left(\boldsymbol{\xi}\right)=\left|\sum_{\ell=1}^{N}\sum_{k=1}^{M} e^{j\omega_c \tau_{\ell k}\left(\boldsymbol{\xi}\right)}\int r_\ell\left(t\right)s_k^*\left(t-\tau_{\ell k}\left(\boldsymbol{\xi}\right)\right)dt\right|$$

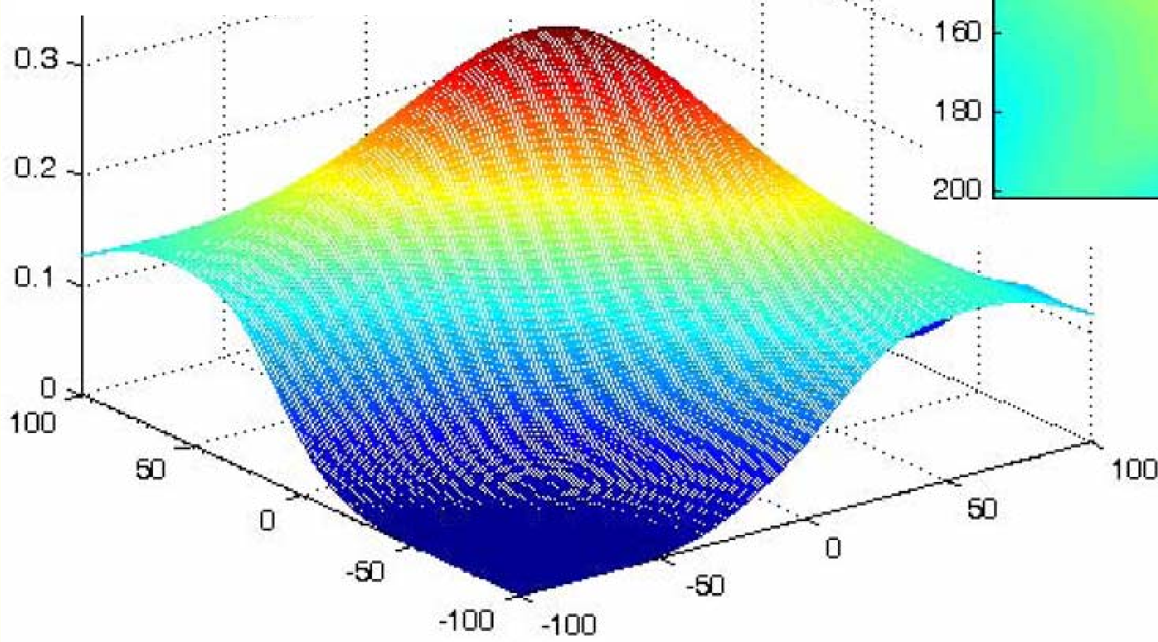
- Near-field operation: the target is at a different angle with respect to each antenna.



Example



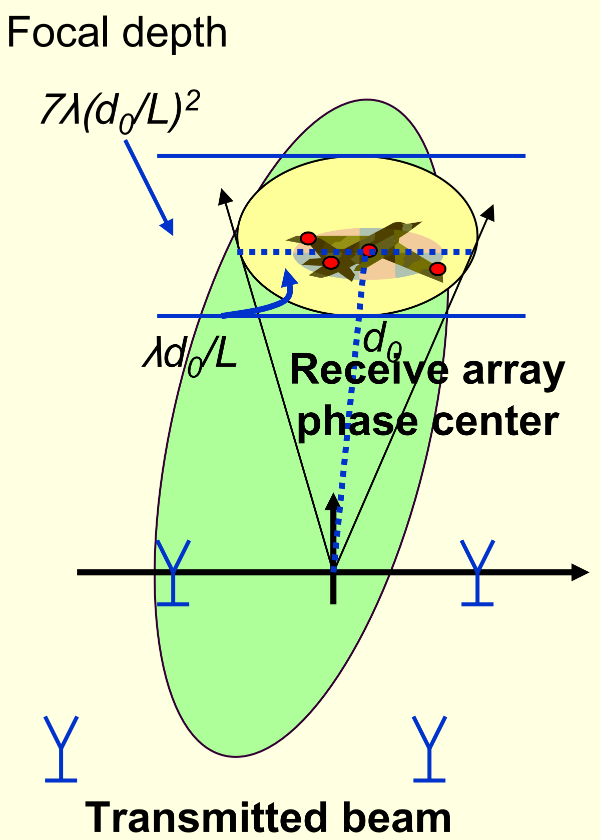




Non-coherent processing
8 transmit x 8 receive
Target consists of 6 scatterers

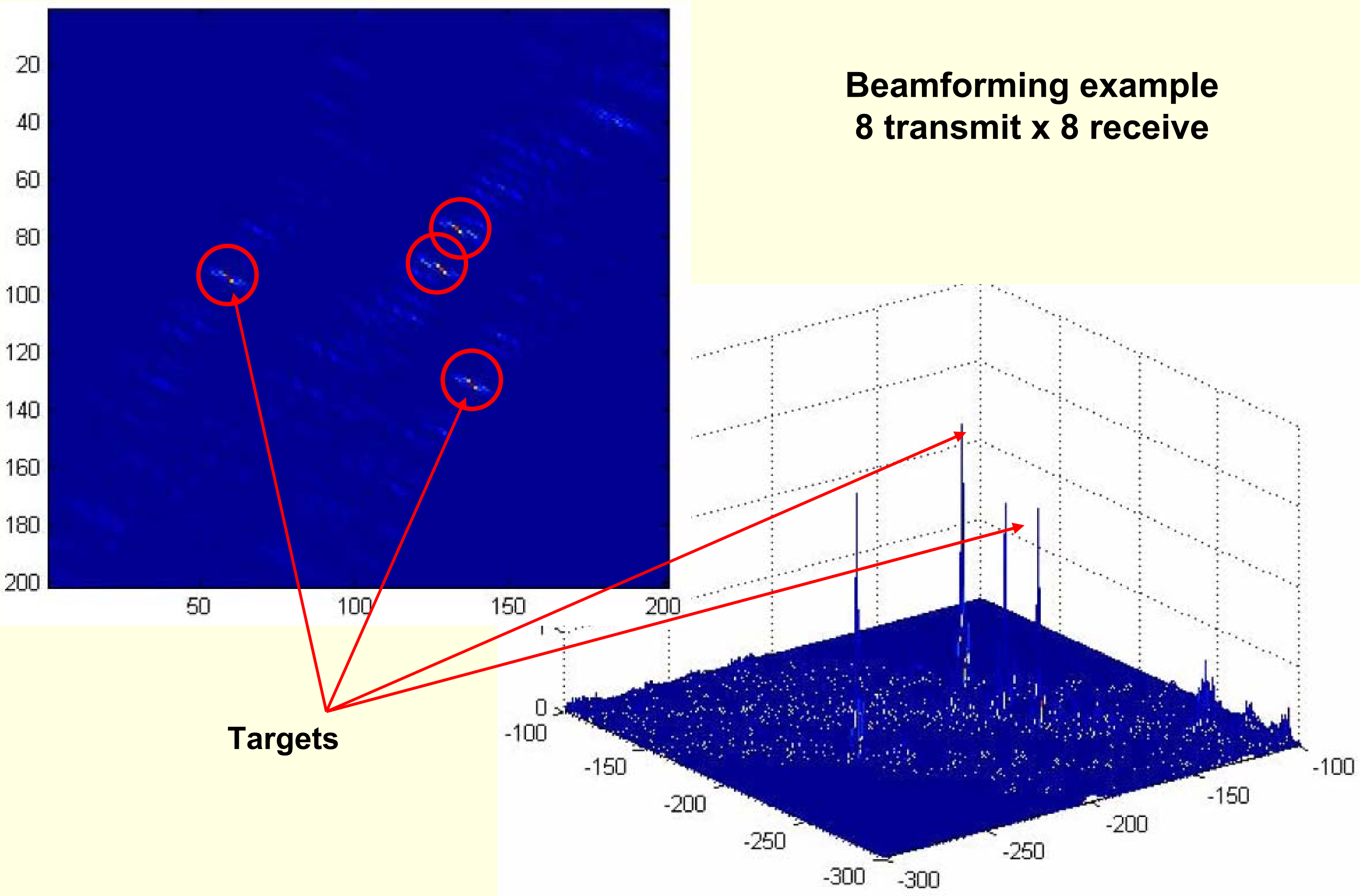
SingleTransmitted Waveform

- All sensors transmit the same waveform with phase shifts.
- The resulting beam is focused:
  - In a certain direction in the far field
  - On a spot in the near field
- The theory of random, thinned arrays predicts that the mean sidelobe level is 1/N of the mainlobe power response [Lo64, Steinberg76]
- With M transmit sensors and N receive sensors, the mean sidelobe level is 1/(MN). The size of spurious peaks can be reduced by increasing the number of transmit and/or receive sensors.
- A common phase difference is required between all transmitters and receivers.



Example

Beamforming example
8 transmit x 8 receive



Targets

Cramer-Rao Lower Bound

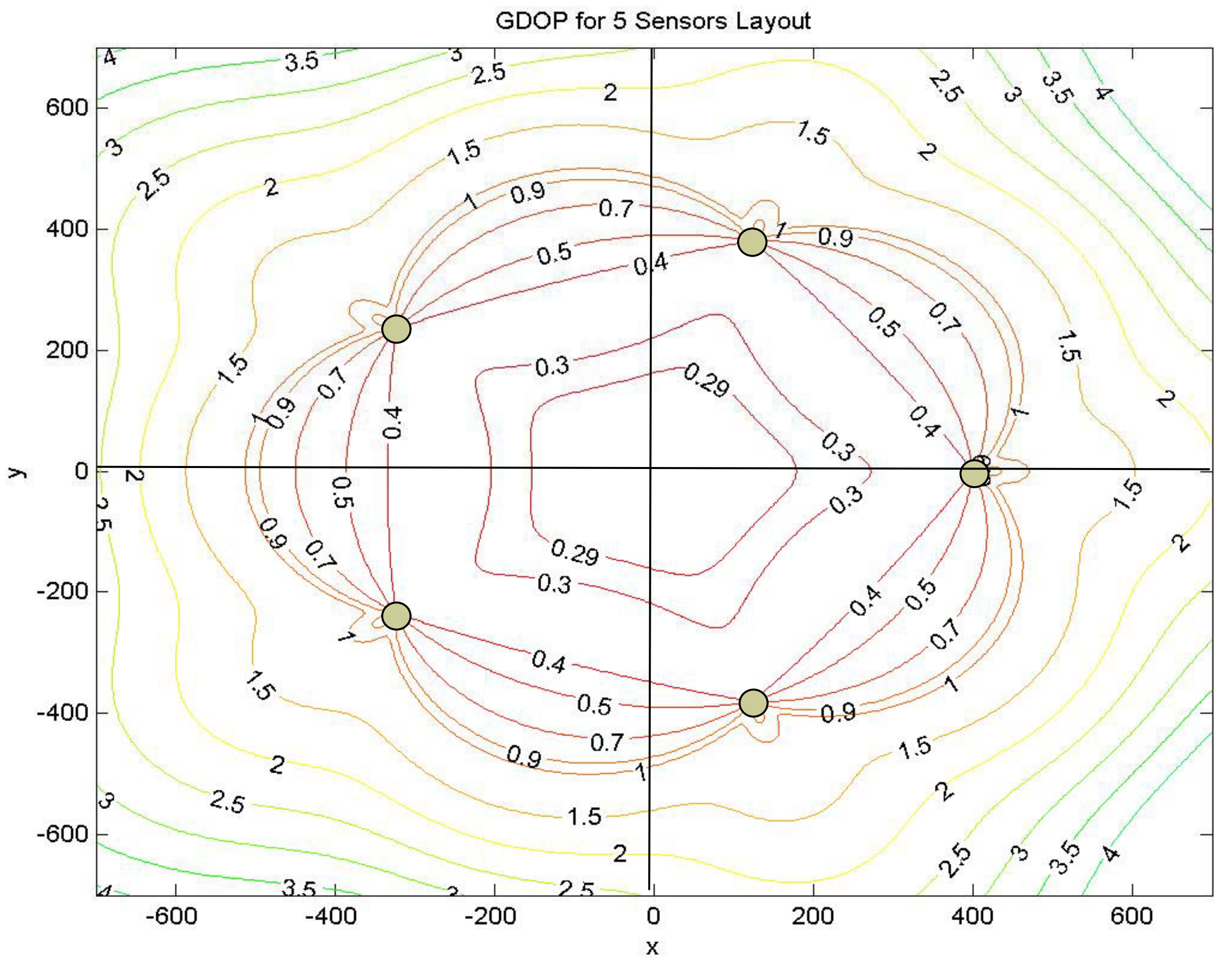
- The Cramer-Rao lower bound can be expressed:

$$\sigma_{CRLB}^2 = \frac{c^2}{8\pi^2 \,\mathrm{SNR}\left(f_c^2 + \beta^2\right)} \cdot \mathrm{GDOP}$$

- The geometric dilution of position (GDOP) term incorporates the effect of sensor locations on the localization error of the target
- It can be shown that with M transmit and N receive antennas, the minimum $GDOP = \sqrt{2/MN}$
- The term β is the effective bandwidth
- Localization error is approximately proportional to $1/f_c^2$
- The effective bandwidth β has little impact

GDOP Numerical Example

- MIMO radar 5x5
- All sensors transmit and receive



Concluding Remarks

- Non-coherent MIMO radar seeks to exploit target RCS diversity to improve detection and estimation performance.
- Coherent MIMO radar supports high resolution, albeit ambiguous target localization.
- Ambiguities can be controlled through the number of sensors.
- Localization with coherent MIMO radar exhibits an error that scales with 1/carrier frequency^2
- The use of multiple sensors improves localization accuracy by a factor as low as $GDOP = \sqrt{2/MN}$