Week 7 Chapter 4 and 6

4-76. a) $P(X \le 0) = \int_0^0 \lambda e^{-\lambda x} dx = 0$

b) $P(X \ge 2) = \int_2^\infty 2e^{-2x} dx = -e^{-2x}\Big|_2^\infty = e^{-4} = 0.0183$

c) $P(X \le 1) = \int_0^1 2e^{-2x} dx = -e^{-2x}\Big|_0^1 = 1 - e^{-2} = 0.8647$

d) $P(1 < X < 2) = \int_1^2 2e^{-2x} dx = -e^{-2x}\Big|_1^2 = e^{-2} - e^{-4} = 0.1170$

e) $P(X \le x) = \int_0^x 2e^{-2t} dt = -e^{-2t}\Big|_0^x = 1 - e^{-2x} = 0.05$ and x = 0.0256

4-84. Let X denote the time until a message is received. Then, X is an exponential random variable and $\lambda = 1/E(X) = 1/2$.

a) P(X > 2) = $\int_2^\infty \frac{1}{2} e^{-x/2} dx = -e^{-x/2}\Big|_2^\infty = e^{-1} = 0.3679$

b) The same as part a.

c) E(X) = 2 hours.

4-90. Let Y denote the number of arrivals in one hour. If the time between arrivals is exponential, then the count of arrivals is a Poisson random variable and $\lambda = 1$ arrival per hour.

a) P(Y > 3) = $1 - P(Y \le 3) = 1 - \left[\frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} + \frac{e^{-1}1^2}{2!} + \frac{e^{-1}1^3}{3!}\right] = 0.01899$

b) From part a), P(Y > 3) = 0.01899. Let W denote the number of one-hour intervals out of 30 that contain more than 3 arrivals. By the memoryless property of a Poisson process, W is a binomial random variable with n = 30 and p = 0.01899.

P(W = 0) = $\binom{30}{0} 0.01899^0 (1 - 0.01899)^{30} = 0.5626$

c) Let X denote the time between arrivals. Then, X is an exponential random variable with $\lambda = 1$ arrivals per hour. P(X > x) = 0.1 and $P(X > x) = \int_x^\infty 1e^{-1t} dt = -e^{-1t}\Big|_x^\infty = e^{-1x} = 0.1$.

Therefore, x = 2.3 hours.

4-130 The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with 0.00004.

a) $P(X > 20{,}000) = \int_{20000}^\infty 0.00004 e^{-0.00004x} dx = -e^{-0.00004x}\Big|_{20000}^\infty = e^{-0.8} = 0.4493$

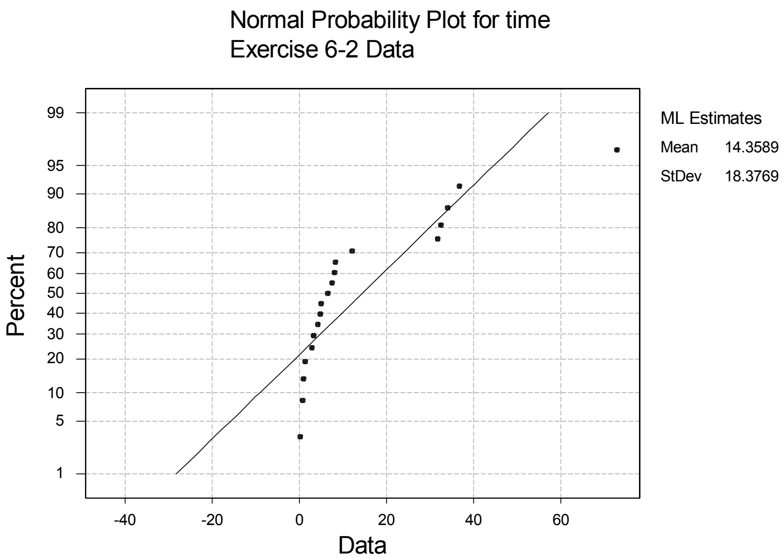
b) $P(X < 30{,}000) = \int_{30000}^{\infty} 0.00004 e^{-.0.00004x}\,dx = -e^{-0.00004x}\Big|_{0}^{30000} = 1 - e^{-1.2} = 0.6988$

c)

$$P(20{,}000 < X < 30{,}000) = \int_{20000}^{30000} 0.00004 e^{-.0.00004x}\,dx$$

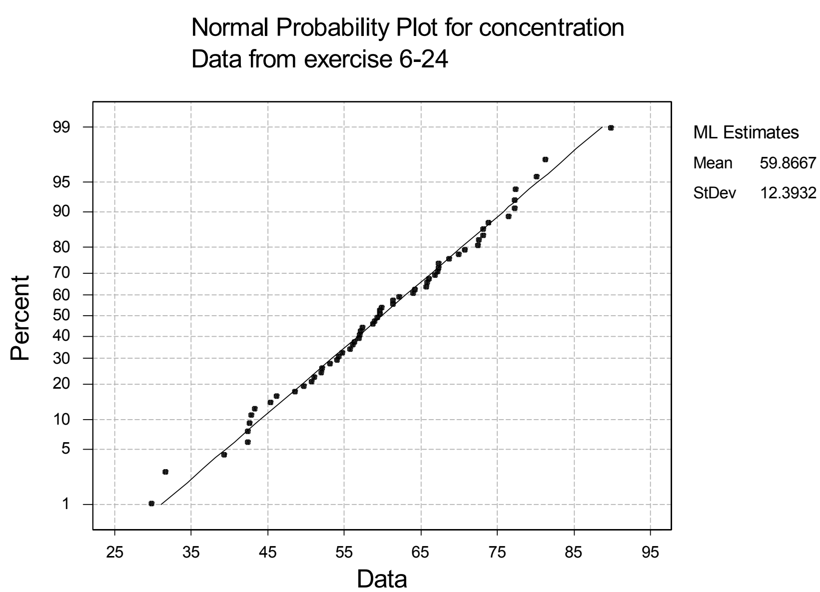
$$= -e^{-0.00004x}\Big|_{20000}^{30000} = e^{-0.8} - e^{-1.2} = 0.1481$$

6-66.



It appears that the data do not come from a normal distribution. Very few of the data points fall on the line.

6-72.

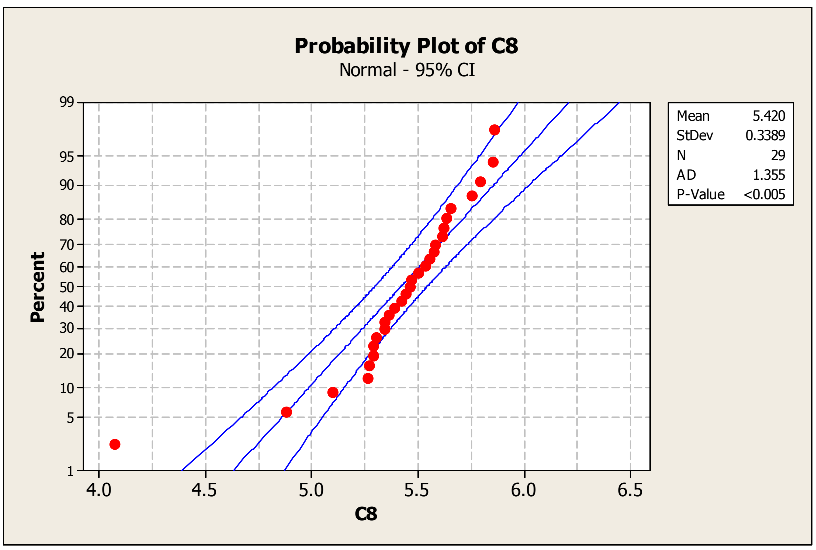


The data appear to be normally distributed. Nearly all of the data points fall very close to, or on the line.

6-94. a) Descriptive Statistics

Variable	N	N*	Mean	SE Mean	StDev	Variance
Density	29	0	5.4197	0.0629	0.3389	0.1148

Variable	Minimum	Q1	Median	Q3	Maximum
Density	4.0700	5.2950	5.4600	5.6150	5.8600



b) There does appear to be a low outlier in the data.

c) Due to the very low data point at 4.07, the mean may be lower than it should be. Therefore, the median would be a better estimate of the density of the earth. The median is not affected by a few outliers.