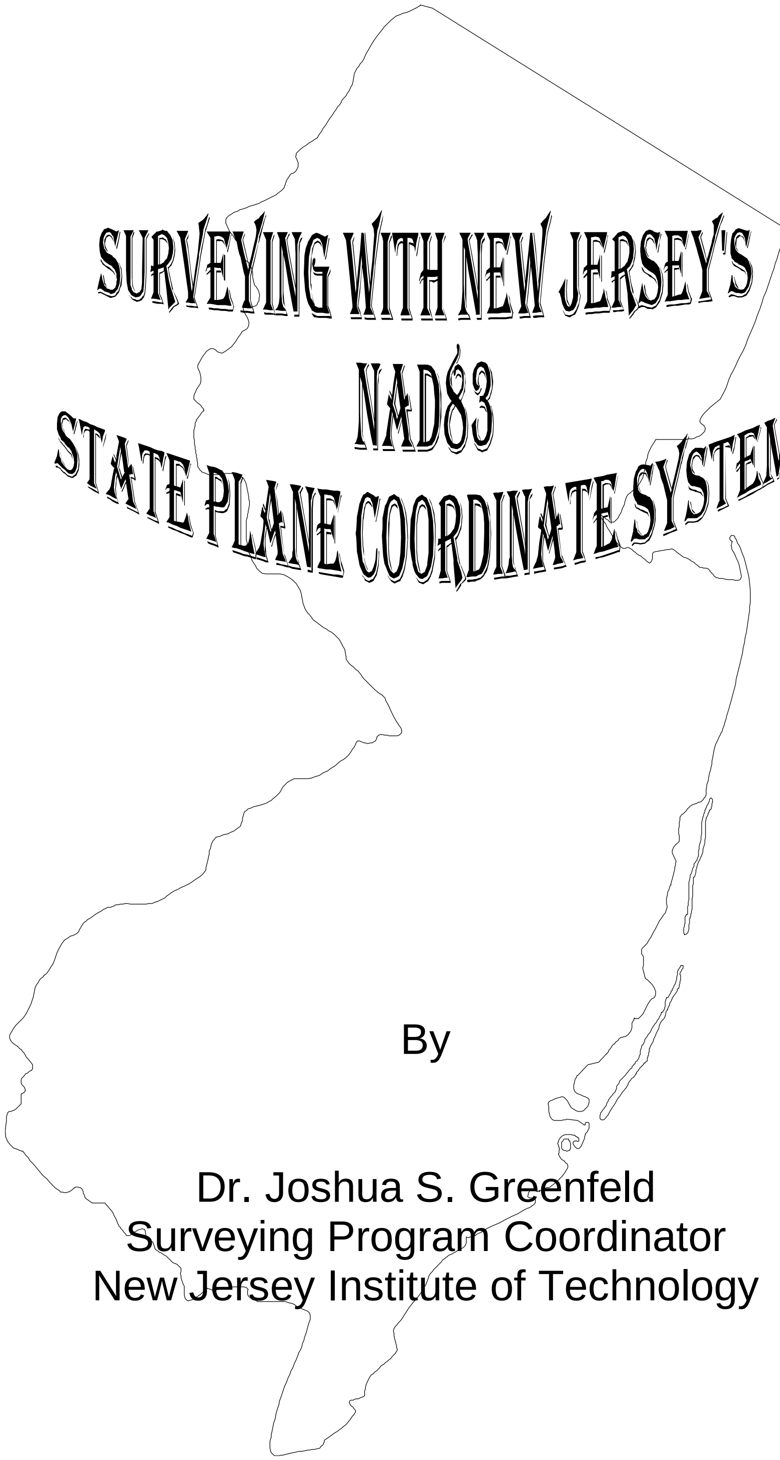
SURVEYING WITH NEW JERSEY'S NAD83 STATE PLANE COORDINATE SYSTEM



By

Dr. Joshua S. Greenfeld
Surveying Program Coordinator
New Jersey Institute of Technology

TABLE OF CONTENTS

List of Figures

List of Tables

List of Forms

PREFACE

The purpose of this Manual is to provide the Professional Land Surveyor in the state of New Jersey with all the necessary information to enable him/her to work with the State Plane Coordinate System (SPCS). It is not a comprehensive text on NAD83 nor is it a text on map projections. It contains a general discussion on why we should work with SPCS as well as specific step by step procedures on how to work with it. This Manual is complete in the sense that it provides all the necessary computational formulas and examples. The examples and some of the tables are specific for New Jersey or states with Transverse Mercator (TM) projections.

The parameters of New Jersey's State Plane Coordinate System are:

	Zone Code (NGS)	2900
	Projection	TM
	Central Meridian (CM)	74° 30'
	Scale Factor at CM	1:10,000
Grid Origin:	Latitude φ_o	38° 50'
	Longitude λ_o	74° 30'
	Eastings Shift E_o	150,000
	Northings Shift N_o	0

The definitions, the significance and the usage of these parameters will be discussed in this manual.

The first section presents a short discussion on why we need to work with SPCS. In light of the recent accelerated activities in GIS/LIS, and recognizing that SPCS provides a common reference to all surveying and mapping activities, it is not too difficult to make a case for working with State Plane Coordinates (SPC).

The second section provides a very brief discussion on the relationship between the surveying medium (the topography) and the preferred mapping medium (a flat surface with Cartesian coordinates). The key to making the connection between the two is understanding some aspects of Geodesy.

The third section includes all the formulas and corrections that may be necessary when working with SPCS. Each formula is explained in terms of its purpose and its usage. The explanation of the purpose of a formula is the description of the specific correction that is applied by it. The explanation of the usage of the formula is an outline what data is needed for the computation and when is it necessary to make the computation. Some corrections could be neglected if they are smaller than our ability to carry out the measurements.

The fourth section provides a detailed example on how to compute and balance a traverse when working with SPC. The section contains a step by step computation procedure annotated with detailed explanations. The purpose of this section is to serve as a practical guide for working with SPCS.

The last section provides tables and forms to enable the user to perform all the necessary computations to convert between Geodetic and State Plane coordinates, and to compute the meridian convergence. The section also includes a table for interpolating the grid scale factor as a function of Eastings. Using these tables is not mandatory for working with SPCS. The conversions and the meridian convergence computations as well as the determination of the grid scale factor can be done with computer software available from the National Geodetic Survey (NGS). It is provided here for convenience, for easy access and to make this manual self-contained.

For additional reading on the NAD83 State Plane Coordinate System the reader is referred to James E. Stem's publication "State Plane Coordinate System of 1983", NOS NGS 5, available from NGS (301) 713-3242.

1. WHY STATE PLANE COORDINATE SYSTEM?

State Plane Coordinate Systems (SPCS) have certain characteristics that make them very useful for surveyors, mappers, engineers and GIS/LIS specialists. Although working with SPCS requires some extra computational effort, the benefits of having all the data in one consistent coordinate system makes the extra effort worthwhile. In the era of GIS/LIS it should not be viewed as an option but be considered mandatory for everyone to work on a well defined coordinate system.

Some of the benefits for surveyors to work with SPCS are:

- All surveys correlate to a single reference framework. This means that all surveys, old and new, can be combined seamlessly to a consistent and contiguous mapping project. Points from old and new surveys can be used without the need to re-compute the old measurements. Surveyors having numerous projects in a certain area could, theoretically, “cut and paste” different projects to produce a map without compromising the accuracy of the new product.

- Large projects can be surveyed in parallel as independent sections. Although during the time of the execution of the project the different sections are not yet connected physically, they are connected computationally because they all share a common reference framework. As the work progresses, all sections will be connected and the accuracy of the entire project will be maintained throughout.

- Data sharing among surveyors is simplified if everyone is working on the same reference system. This means that surveys can be re-packaged and sold for additional profit. Data is a precious commodity in the GIS/LIS world. Surveyors have an abundance of spatial information; if it is in a useable form (such as SPCS) it has a market value.

- No point can be considered lost because it can be recovered by its coordinates. If you have the State Plane Coordinates of a point, you can use, for example, GPS to recover it. There is no need to recover points using ties that may have been also been destroyed. (Unless there is a legal issue involved.)

- Using SPCS, the earth can be viewed mathematically as a plane. This means that plane geometry and trigonometry mathematics can be used in our computations. One needs only to apply a small, well defined, correction to compensate for the plane approximation. This manual explains what corrections have to be made and provides an example of how to apply them.

- Working with SPCS provides an extra external computation check for our surveys. Loop closures such as a closed traverse check only the inner consistency of the survey. If, for example, there is a systematic scale error in the traverse, it will not be detected by summing up the latitudes and the departures. Only if we tie the traverse to two or more points with given State Plane Coordinate values can this error be realized and corrected. A similar argument holds for the orientation of the traverse. To maintain proper orientation of a traverse we need to tie it to at least two control points with State Plane Coordinates.

- In many states such as New Jersey there is a SPCS law. (See Appendix A)

2. BACKGROUND INFORMATION

In order to understand the computations and the corrections that have to be applied when working with SPCS it is prudent to provide some background on the theory of projecting measurements onto a map. It is important to understand the relationship between the surveying medium (the topography) and the preferred mapping medium (a flat surface with Cartesian coordinates). Field measurements are performed on the “real world” and then presented on a flat surface as a set of plane coordinates. The question is how do we get there (plane coordinates) from here (topography)? In this section we also explore what NAD83 is and why was it necessary to define it?

2.1 Geodesy

To better understand the relationship between the actual curved surface of the earth and its projection onto a plane, we need to understand some of the basic concepts of geodesy. Geodesy, from the Greek, literally means dividing the earth. It is the science that determines the figure of the earth and the inter-relationship of selected points on (or near) it's surface. The inter-relationship among the selected points is based on well defined three-dimensional (actually, four including time) coordinate systems and the mathematical models that relate these coordinate systems to the actual location of points on (or near) the surface of the earth. One of the principal objectives of Geodesy is to provide an accurate framework for the control of national topographical mapping and for precise surveys. Other branches of Geodesy deal with satellite and astronomical positioning, gravity and height systems, tides, earth rotation, crustal movement and the deflection of the plumb line.

2.2 Position

A geodetic position is defined by a set of coordinates such as latitude, longitude and elevation. Coordinates, by definition, are a set of values that relate a point to a mathematically defined surface. Thus, by assigning coordinates to a point one assumes that the surface of the earth can be defined mathematically. This assumption is anything but the truth. The oceans are reasonably uniform, but the surface or topography of the land masses show large vertical variations between mountains and valleys which make it impossible to approximate the shape over a large area with any reasonably simple mathematical model. Since the earth is in fact flattened slightly at the poles and bulges somewhat at the equator, the geometrical figure used in geodesy to most nearly approximate the shape of the earth is an ellipsoid of revolution, which is a 3-D shape formed by spinning an ellipse about it's vertical axis.

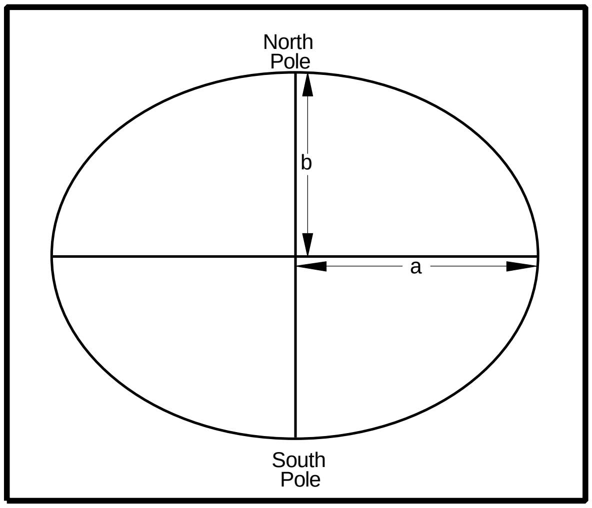


Figure 1. The ellipsoid.

In order to determine the third coordinate, elevation, we need to define another representation of the earth, namely, an equipotential surface. The main characteristics of an equipotential surface is that water will stand still on it. Water will run off from a "higher" equipotential surface to a "lower" one. The "higher" and "lower" can be expressed in terms elevation. Thus, the height of a point is determined according to the equipotential surface on which it is situated. The equipotential surface that is assigned an elevation of zero is called the Geoid.

In summary, we distinguish between three different surfaces (see figure 2);

1. The topography - the physical surface of the earth
2. The Geoid - the level (equipotential) surface at mean sea level. In simple terms a level surface is a surface on which water will stand still. The Geoid is a level surface at elevation zero.
3. The ellipsoid - the mathematical surface which approximates the shape and size of the earth, and is used as a reference frame for position computations.

From the preceding discussion, it becomes clear that in order to define the geodetic position of a point located on the topography, it must be first reduced mathematically onto the ellipsoid and the Geoid. Only on these surfaces can a point have coordinates.

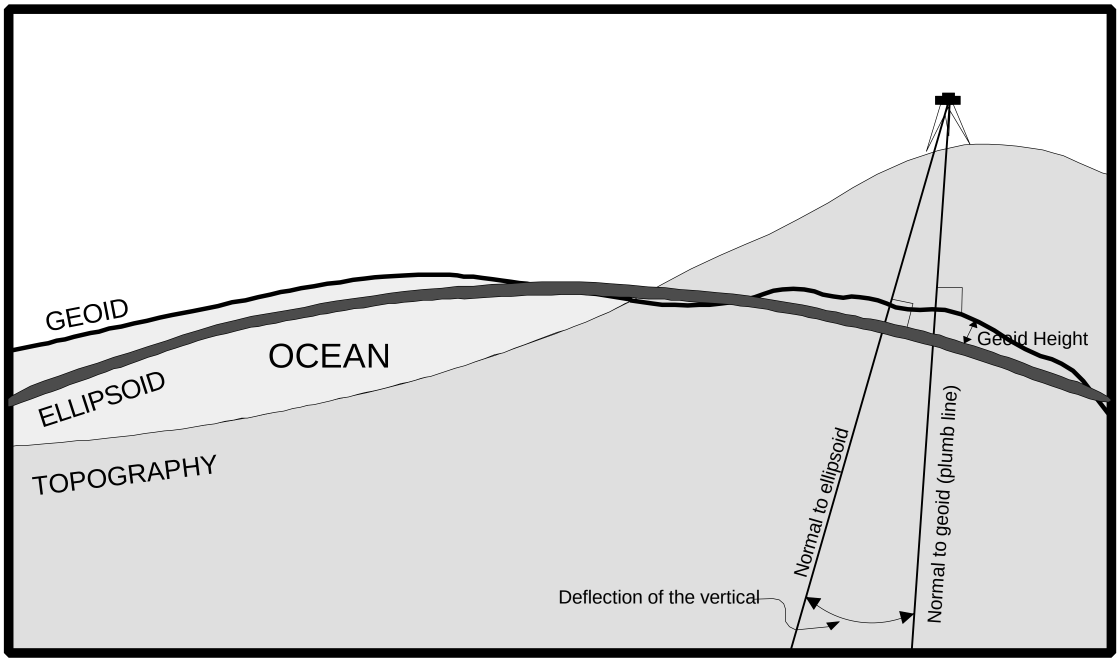


Figure 2. Surfaces used in geodesy to represent the earth.

2.3 Geodetic Horizontal Datum

A datum is defined as any numerical or geometrical quantity or set of such quantities which serve as a reference or base for other quantities. In geodetic practice two types of datums are used: horizontal and vertical. (vertical datums will not be discussed here as they are not part of the NAD83 State Plane Coordinate System).

A Horizontal datum is a surface of constant values which forms the basis for the computations of horizontal control surveys in which the curvature of the earth is considered. A reference ellipsoid is used for a horizontal datum. Five parameters are required to define a horizontal datum: two to specify the dimensions of the ellipsoid, two to specify the location of an initial point (origin), and one to specify the orientation (i.e. north) of the coordinate system. Countries, or groups of countries choose different reference ellipsoids to minimize the deviations between the topography and the ellipsoid in their region (best fit ellipsoid for their region.). These individual geodetic datums are often referred to as local datums. Local datums deal only with part of the earth's surface and are only loosely related to the earth's centermass, the geocenter.

2.4 The North American Datum Of 1983 (NAD83)

In 1986 the National Geodetic Survey (NGS) completed a project for the redefinition and adjustment of the existing horizontal reference system. The North American Datum of 1983 (NAD83) represents the single most accurate, continent-wide and comprehensive geodetic survey datum in the history of the United States. It supersedes the North American Datum of 1927 (NAD 27). In recent years, the introduction of highly accurate electronic measurement systems, and the advent of satellite tracking systems such as Doppler (an earlier satellite positioning system) and GPS unveiled many weaknesses in NAD 27. Discrepancies between existing control and newly established surveys necessitated the establishment of an entirely new datum, rather than fixing NAD 27. The new datum, NAD 83, is earth centered and relies on an ellipsoid (and other constants) of the Geodetic Reference System of 1980 (GRS 80). The primary advantage of GRS 80 is that it facilitates the computation of correct geometric relationships on a global as well as a continental scale.

2.5 Plane coordinates

So far we have discussed the definitions concerning positioning a point on an ellipsoid in terms of Latitude (φ) and Longitude (λ). Curvilinear coordinates such as φ and λ, are not the most convenient or user-friendly to work with. Computing distances and angles between points using φ and λ can be very lengthy and cumbersome. Surveyors and engineers would rather work with plane Cartesian (x, y) coordinates which facilitates simple and straight forward computation methods. In plane coordinates one makes the assumption that the points (which are being surveyed or mapped) are located on a flat horizontal plane. This assumption is, of course, not valid because, as stated earlier, points are actually located on a curved surface (topography or reduced to an ellipsoid) not on a plane.

	Plane	**Ellipsoid**
North-South Direction	Straight up or parallel to the direction of the North Arrow	Slanted (not uniformly) towards the North Pole. All lines pointing to North, converge at the North Pole.
Distances	Straight lines (C-D, Fig. 3)	Curved lines (C-S’-D, Fig. 3)
Sum of angles in a quadrilateral	360°	360° + spherical excess
Even coordinate differences correspond to:	Even (same length) distances.	Uneven distances, i.e. the length of an arc of 2° of longitude near the pole is much shorter than 2° of arc near the equator.

Table 2.1. Some differences between working on a plane and on an ellipsoid.

Figure 3 illustrates some of the differences between coordinates, distances and angles on an ellipsoid vs. those on a plane.

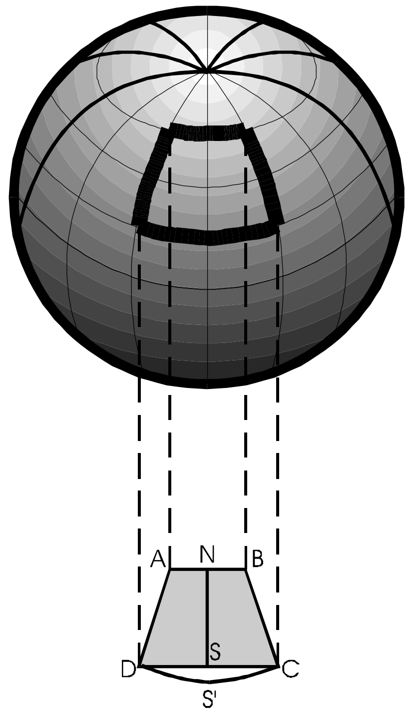


Figure 3. Projecting part of an ellipsoid onto a plane.

To enable us to work with the conveniency of plane coordinates and with the accuracy of geodetic coordinates we need to project a portion of the earth (or ellipsoid) onto a plane surface. The projection must be done in such away that the distortions can be accounted for, and corrected for. The two major corrections are the meridian convergence (γ) and the scale factor. The meridian convergence correction compensates for the skewness (or non-parallel) of the north direction lines towards the north pole. The scale factor correction (called Grid Scale Factor or GSF) compensates for the difference between a straight line on the projection plane and its corresponding curved line on the ellipsoid. It should be pointed out here that there is another scale factor that has to be applied to measured distances, the Elevation (or "sea level") Scale Factor (ESF). The elevation scale factor is applied when reducing a measured distance from the topography (the ground) to the ellipsoid. These scale factor corrections are illustrated in Figures 4 and 5.

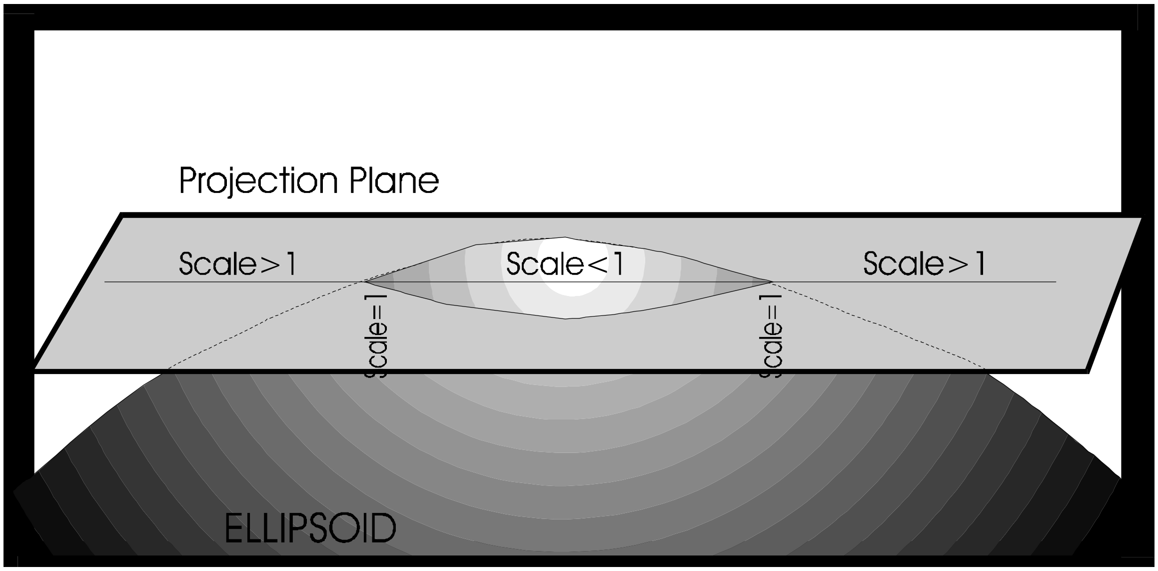


Figure 4. The Grid Scale Factor correction for projecting a curved line onto a plane

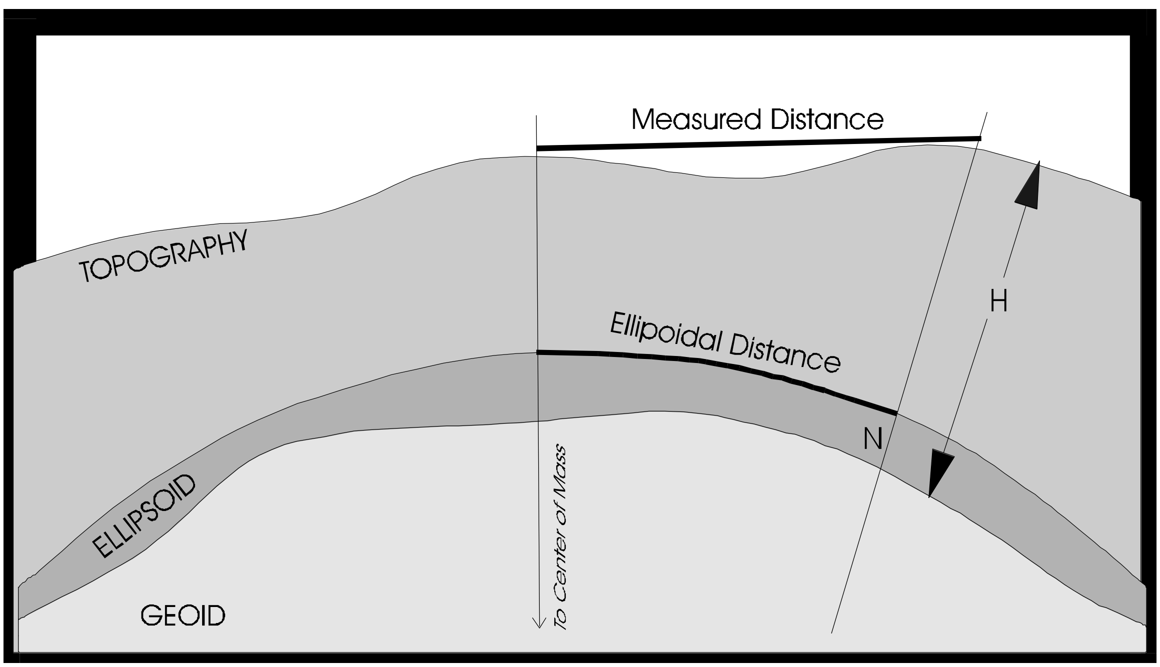


Figure 5. The Elevation Scale Factor to correct for reducing (lowering) the measured distance from the topography to the ellipsoid.

In the next section we introduce the formulas for the corrections that have to be applied to our field measurements.

3. USEFUL FORMULAS FOR COMPUTATIONS ON PLANE COORDINATE SYSTEMS

In this section we present some useful formulas which are being used in conjunction with SPCS. Not all of these formulas have to be used every time. A short explanation is provided on the purpose of each formula and on when to use it. A practical example on how to use these formulas will follow in the next section.

3.1 GRID SCALE FACTOR K_{12} FOR A LINE FROM POINT 1 TO POINT 2.

Purpose: To correct for scale distortion due to the projection of the ellipsoid onto a plane.

Formula:

$$K_{12} = \frac{K_1 + 4K_m + K_2}{6}$$

Where:

K_{12}	-	Grid Scale factor of a line between points 1 and 2.
K_1	-	Grid scale factor at point 1
K_2	-	Grid scale factor at point 2
K_m	-	Grid scale factor at the line's mid- point.

Usage: A reasonable approximation for the above formula is to compute a simple average of K_1 and K_2 . A further approximation is to compute a single K value for the entire line or for the entire survey area. This will be demonstrated later in our traverse example.

For New Jersey the value of K can be interpolated from Table 5.1 or computed from:

$$K = 0.9999 + (E - E_o)^2 \cdot 1.23 \cdot 10^{-14}$$

3.2 REDUCING MEASURED HORIZONTAL DISTANCE TO GRID DISTANCE.

Purpose: To reduce a measured horizontal distance to the projection plane. As mentioned earlier, field measurements are carried out on the physical surface of the earth, while office computation are performed on the projection of the earth onto a plane. This reduction is in essence the bridge between field measurements and the computations on the state plane coordinate system.

Formula:

$$S = D \times \left(\frac{R}{R + H + N}\right) \times K_{12}$$

Where:

S	-	Grid Distance
D	-	Horizontal (Measured) Distance
H	-	Mean Elevation (Above Mean Sea Level)
N	-	Mean Geoid Height (About -32m in NJ)
R	-	Mean Radius of the Earth (About 6,372,000m)
K_{12}	-	Grid Scale factor of the Line.

Usage: Obtain the elevations of the terminal points of the line, the Geoid height of the region and compute.

3.3 Relationship Between Geodetic and Grid Azimuths

Purpose: To account for the convergence of the north direction towards the pole vs. parallel north direction on a plane

Formula:

$$AZ_{Grid} = AZ_{Geodetic} - \gamma + (t-T)$$

Where:

AZ_{Grid}	-	Grid Azimuth
$AZ_{Geodetic}$	-	Geodetic Azimuth
γ	-	Meridian Convergence
(t-T)	-	Arc-to-chord correction (explained in section 3.4)

Usage: This formula (except for the t-T correction) has to be used only if we are given with Geodetic Azimuth and want to perform our computations on plane coordinates. In other words, if we have plane coordinates and compute the Azimuth from them, we do not need to apply the meridian convergence γ. Grid Azimuth can be computed from an inverse between two points with plane coordinates. Geodetic Azimuth is usually provided by NGS or from GPS measurements. They can also be computed from Astronomical observations corrected for the deflection of the vertical (called Laplace Correcetion).

3.4 Arc-to-Chord Correction (t-T) for line 1-2.

Purpose: The (t-T) correction is due to the fact that the measured direction between two points is actually a curved line, on the surface on a body such as an ellipsoid, that passes through these points. When projected onto a plane, the geodetic direction looks like an arc not a straight line (see Figure 7.) The angle that we compute from field notes is defined by the difference between two measured directions. Thus, the computed angle differs from the plane angle that we have to use when working with the State Plane Coordinate System. This difference is expressed by (t-T).

Formula:

$$(t\text{-}T)'' = 25.4 \times \Delta N \times \Delta E \times 10^{-10}$$

Where:

$$\Delta N = N_2 - N_1$$

$$\Delta E = \frac{E_2 - E_1}{2} - E_0$$

(t-T)" - Arc-to-chord correction in seconds of arc.
N_1 - Northing of point 1
N_2 - Northing of point 2
E_1 - Easting of point 1
E_2 - Easting of point 2

Usage: The size of the correction is rather small and can be neglected for most ordinary work (not high accuracy). The sign of the correction is dependent on the direction of the line with respect to the north (Azimuth dependent). Table 3.1 presents the size (magnitude) of the (t-T) correction and figure 6, describes the sign convention for (t-T), as it applied for the New Jersey State Plane Coordinate System. One should note that the following example does not apply to States with Lambert Conformal Conic projection (e. g. Pennsylvania) or states that their central meridian is not 150,000.

3.4.1 Size of the correction in seconds of arc ("): (for New Jersey where the Central Meridiad E_o = 150,000) is:

	Average Easting of the line						
ΔN	0	50,000	100,000	150,000	200,000	250,000	300,000
2 km	0.8"	0.5"	0.3"	0"	0.3"	0.5"	0.8"
5 km	1.9"	1.3"	0.6"	0"	0.6"	1.3"	1.9"
10 km	3.8"	2.5"	1.3"	0"	1.3"	2.5"	3.8"
20 km	7.6"	5.1"	2.5"	0"	2.5"	5.1"	7.6"

3.4.2 Sign of the correction: (for States using Transverse Mercator projection)

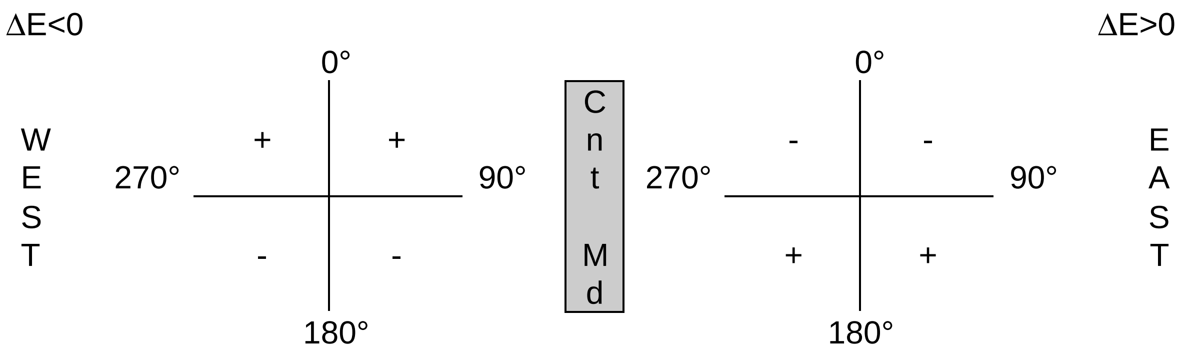


Figure 6. The sign of (t-T) as a function of the Azimuth and location of the point with respect to the central meridian.

Sign Rule: The actual direction always curves away from the central meridian.

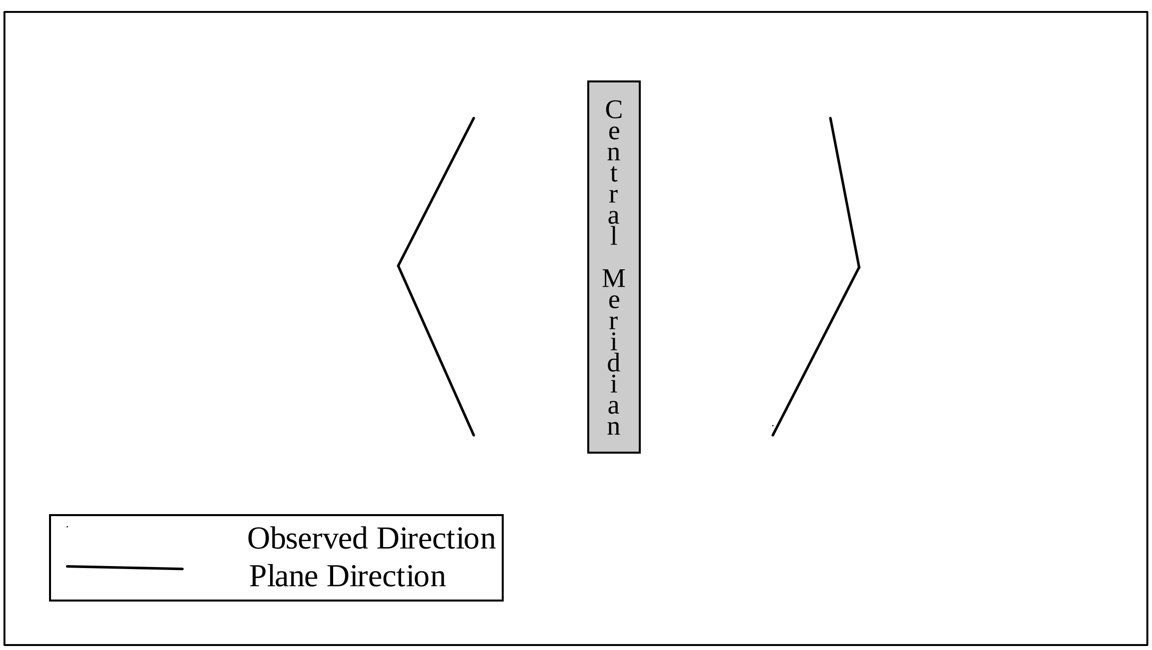


Figure 7. The difference between the observed and the plane directions between two points.

4. TRAVERSE COMPUTATION ON SPCS NAD83

In this section we outline the procedure for computing and balancing a traverse when working with a State Plane Coordinate System (SPCS). In addition, a practical example of this procedure is presented so that the reader can use it as a guide in his/her own work. The example traverse that will be solved in this section is a connecting traverse rather than a closed (loop) one for the following reasons.

Traditionally, surveyors utilize a closed loop traverse to provide them with a check on the measured angles and distances. This closure check is necessary in order to provide quality control and to ensure the consistency of the survey. Most closed traverses use an assumed coordinate system and an arbitrary initial bearing. This type of closed traverses lacks the mechanism for relating the current survey to others that may have been performed in the area. As long as each surveyor worked on his own project with his locally defined (assumed) coordinate system this practice was acceptable. However, when working with SPCS all surveys are implicitly related by sharing the same coordinate system. Thus, it is imperative that we have a check on the coordinates of the (initial) known point and the initial bearing. These values are not arbitrary but must be consistent with the state-wide coordinate system. The best way to achieve that extra check on the coordinates and the bearings is by designing a traverse that starts at a known control point and terminates at another known point. This type of traverse is called a connecting traverse.

The analysis that follows assumes that a connecting traverse is used. It is also assumed that the reader is familiar with computing the closure and balancing of a traverse. The main difference between computing a traverse on a local (assumed) coordinate system and State Plane Coordinate System is in what is used as observations. In a local traverse we use the observations as recorded in the field. When working with State Plane Coordinates we first have to reduce the observations from the topography to the ellipsoid and to the projection plane, and only then compute closures and balancing.

4.1 THE 10 STEPS FOR COMPUTING A TRAVERSE ON NAD 83 SPCS

Computing a traverse on NAD 83 State Plane Coordinate System may include the following steps:

1. *Obtain starting and ending coordinates and grid azimuth.*

2. *Compute preliminary Azimuth for each line.*

3. *Compute preliminary coordinates for each traverse point.*

    These three steps are essentially computing an open traverse without applying corrections for closure errors.

4. *Obtain or compute approximate elevations (mean or for each point) of the traverse.*

    Step 4 is necessary in order to reduce the distances from the topography to mean sea level. Unless high precision results are sought or there are substantial elevation differences between the various traverse points, a mean elevation of the area will be sufficient.

5. *Compute grid scale factor (mean or individual for each line).*

    Again, the choice between computing a mean value or individual scale factors for each line depends on the accuracy objectives of the project and the magnitude of elevation differences between the traverse points.

6. *Reduce horizontal distances to grid distances.*

7. *Compute (t-T) correction for each line (if necessary).*

    For most traverses, (t-T) correction is practically negligible. If the Northing component (usually called the Latitude) of a side of the traverse is less than 1 mile long, the maximum (t-T) correction will not exceed 1″ (second of arc). Thus, the correction is equal or smaller than the accuracy with which we are able to carry out our measurements. (t-T) becomes significant only for long Latitudes (larger than 10 km) and at the east/west most parts of the State. One can assume that the type of work for which (t-T) is significant will be done with GPS.

8. *Apply (t-T) correction to each Azimuth (if necessary).*

9. *Balance (or adjust) the traverse.*

10. *Compute final State Plane Coordinates for the traverse points.*

    Steps 9 and 10 are the standard procedures for balancing the traverse with the Compass Rule or adjusting it with Least Squares.

4.2 AN EXAMPLE OF TRAVERSE COMPUTATION ON NEW JERSEY'S SPCS (NAD83)

The following is an example illustrating the implementation of the 10 step procedure which was described above. The example focuses on New Jersey's State Plane Coordinate System, but could be applied to any state that uses a Transverse Mercator (TM) projection.

The example traverse starts at control point B and ends at point C. Points B and C have known coordinate values in New Jersey's SPCS. In addition, an initial Azimuth is given from point B to point A and from point C to point D.

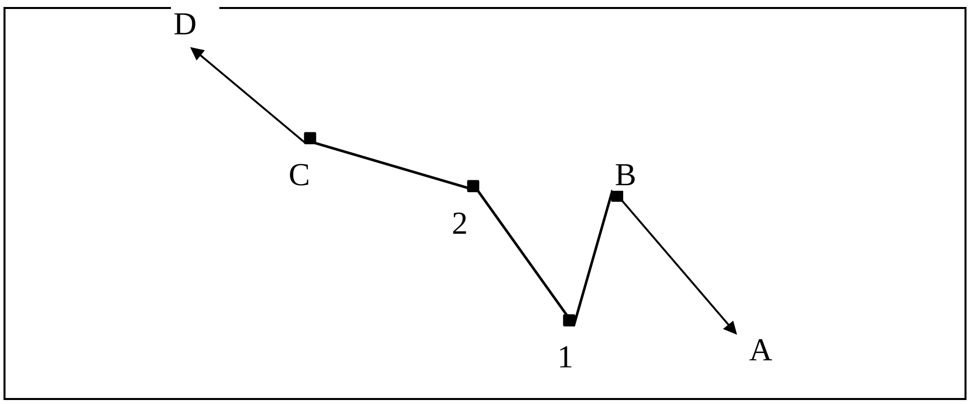


Figure 8. A sample traverse

- **Step 1**: Obtain starting and ending coordinates and grid azimuth and place the information in a table to facilitate the computations.

OBSERVATIONS | **GIVEN DATA**

Point	Angle	Distance (m)	AZIMUTH	N (m)	E (m)
A					
			341° 22' 35.0"		
B	60° 50' 37"			199694.411	130266.014
		3581.556			
1	295° 51' 32"				
		3092.569			
2	161° 26' 40"				
		2906.883			
C	185° 58' 52"			202121.962	124818.850
			325° 30' 18.6"		
D					

Also given: Average Elevation of the traverse is: 200m above MSL

- **Steps 2 and 3:** Compute preliminary Azimuth for each line and compute preliminary coordinates for each traverse points.

PRELIMINARY TRAVERSE COMPUTATION

Point	Angle	Distance	AZIMUTH	N	E
A				LAT	DEP
			341 22 35.0		
B	60 50 37			199694.411	130266.014
		3581.556	222 13 12.0	-2652.393	-2406.731
1	295 51 32			197042.018	127859.283
		3092.569	338 4 44.0	2868.973	-1154.547
2	161 26 40			199910.991	126704.736
		2906.883	319 31 24.0	2211.180	-1886.969
C	185 58 52			202122.171	124817.767
		Comp.	325 30 16.0		
D		Given	325 30 18.6	202121.962	124818.850
		Closure	0 0 2.6	-0.209	1.083

- **Step 4:** Obtain or compute approximate elevations (mean or for each point) of the traverse.

  The average elevation is given as 200m above MSL. It is assumed that there are no substantial height differences among the traverse points. Thus, the average value will be used.

- **Step 5:** Compute Grid and Elevation scale factors (mean or individual for each line)

  The Grid scale factor (GSF) is interpolated from table 5.1 based on the approximate Easting of the points. The results of the interpolation are:

Point	GSF
B	0.9999048
C	0.9999078
1	0.9999060
2	0.9999067
Mean	0.9999063

One should note that the largest error committed by using the mean GSF instead of GSF for individual points will occur on side B-1 (the longest). The magnitude of this error is about 3mm (0.01 ft). This error is smaller than our ability to measure the distance B-1 with a total station. Thus, it is justifiable to use a mean GSF for our example traverse.

The Elevation scale factor (ESF) is required in order to reduce the traverse from the topography to the ellipsoid. The computation is based on the elevation of traverse points above MSL (actually the Geoid) and the height of the ellipsoid above the Geoid (N).

Data:

$R = 6372160$

$H = 200$

$N = -32$

Using the ratio: $\frac{R}{R+H+N}$

Mean ESF is = 0.99997364

Finally, the combined scale factor = GSF×ESF = 0.99987994

- **Step 6:** Reduce horizontal distances to grid distances. The reduction of the distances is performed by multiplying each measured distance with the combined scale factor from step 5.

Line	Dist	Grid Dist
B-1	3581.56	3581.126
1-2	3092.57	3092.198
2-C	2906.88	2906.534

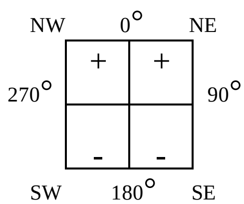
- **Step 7:** Compute (t-T) correction for each line. See section 3.4 for details on how (t-T) is computed.

Line	ΔN	ΔE	(t-T)"
B-1	-2652.393	-20937.352	0.14
1-2	2868.973	-22717.991	-0.17
2-C	2211.180	-24238.749	-0.14

- **Step 8:** Apply (t-T) correction to each Azimuth (if necessary).

  It can be seen from step 7 that the magnitude of this correction is negligible for most projects. Only very precise traverses performed with first order theodolites may be subject to such a small correction. Nevertheless, we will show how to apply the correction for reference purposes only.

  Section 3.4 contains information on how to determine the sign of the correction (the sign is Azimuth dependent). From the sign diagram we can see that we are on the west side (left) of the central meridian. This is because the central meridian in New Jersey has an Easting value of 150,000. and our traverse points have smaller Eastings (between 124,000. and 131,000.) Thus the sign diagram that should be used is as follows:



From the preliminary traverse computation (steps 2-3) we have the following information:

Point	Backsight AZ	Quad./Sign	Foresight AZ	Quad./Sign
B	161 22 35.0	SE / -	222 13 12.0	SW / -
1	42 13 12.0	NE / +	338 4 44.0	NW / +
2	158 4 44.0	SE / -	319 31 24.0	NW / +
C	139 31 24.0	SE / -	325 30 16.0	NW / +

Combining the sign information with the computed values for (t-T) we can now compute the corrections for each observed angle.

Point	Observed Angle	BS Corr	FS Corr	FS-BS Total	Corrected Angle
B	60 50 37	0.0	-0.14	-0.14	60 50 36.9
1	295 51 32	0.14	0.17	0.03	295 51 32.0
2	161 26 40	-0.17	0.14	0.31	161 26 40.3
C	185 58 52	-0.14	0.0	0.14	185 58 52.1

Note that the Backsight at point B and the foresight at point C are control points with State Plane coordinates. Consequently, the Azimuths to these points are Grid Azimuths and thus already corrected for (t-T).

- **Step 9:** Balance (or adjust) the traverse. In this example we use the Compass Rule.

Point	Angle	Distance	Balanced Azimuth	N LAT	E DEP
A					
			341 22 35.0		
B	60 50 36.9		+0.6	199694.411	130266.014
		3581.126	222 13 12.5	-2652.069	-2406.448
1	295 51 32.0		+0.5	197042.342	127859.566
		3092.198	338 4 45.0	2868.634	-1154.394
2	161 26 40.3		+0.6	199910.975	126705.172
		2906.534	319 31 25.9	2210.932	-1886.722
C	185 58 52.1		+0.6	202121.907	124818.449
	Closure 2.3”	Comput.	325 30 18.6		
D		Given	325 30 18.6	202121.962	124818.850
		Closure	0 0 0	0.055	0.401

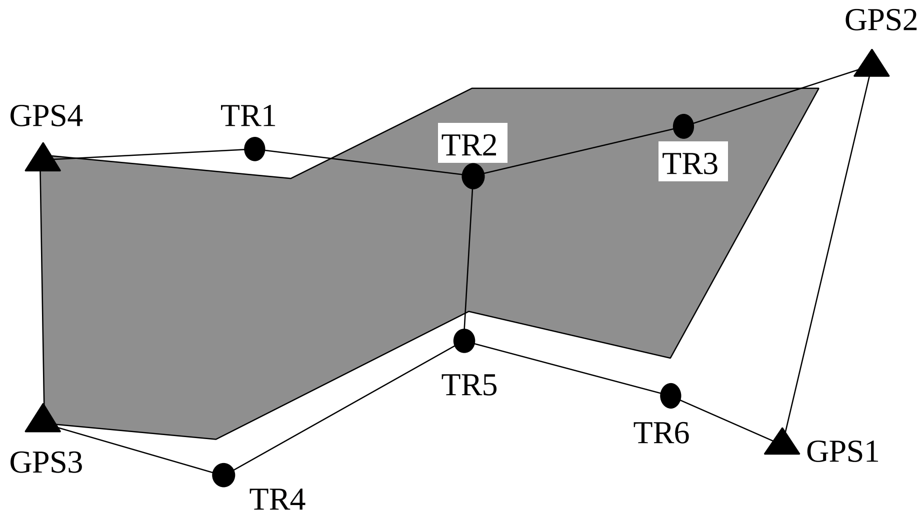
Relative precision = 1: 23,698

- **Step 10:** Compute final State Plane Coordinates for the traverse points.

Point	N	E
B	199694.411	130266.014
1	197042.362	127859.716
2	199911.014	126705.451
C	202121.962	124818.850

4.3 AN EXAMPLE (PROBLEM) WITH GPS DATA.

A large tract of land has to be surveyed and delivered with State Plane Coordinates. The monuments of the two west corners of the tract were recovered and documents revealed that the Azimuth of the line is 29° 53' (determined from "sun shots"). In order to establish base lines for your traverses and to tie the survey into SPC, four GPS points were ordered from a GPS service firm (see figure below).



The following data was obtained from the GPS observations:

Point	E	N	Elev
GPS 1	106245.2422	131565.4149	89.45
GPS 2	106801.2074	132412.7078	67.91
GPS 3	101045.0197	133988.5688	101.43
GPS 4	101672.8785	135065.1054	117.18

You decided to check if the GPS data is correct by performing the following computations:

1. Compute the Bearing of the line from GPS3 to GPS4 and compare it with the given Azimuth. You discover that the computed Bearing is: 30° 15' 06" while the given Azimuth is 29° 53'. What is wrong? (Answer: compute meridian convergence)

2. Now that you found a problem you decided to measure the distance between GPS3 and GPS4 with your EDM. The measured distance was 4088.356. The computed distance from GPS data is 1246.250. problem ?

You remember that NAD83 coordinates are in Meters, so you convert the measured distance to Meters. Now the computed distance is 1246.146. Why is there a 0.1m (0.3') difference?

You realize that the discrepancy could be because of the Grid Scale Factor. Computing the corrected distance for GSF yields 1246.161. This improves the situation somewhat, but still the distances are about 1/4 ft off. How can you account for this difference? (Answer: compute "sea level" SF)

4.4 COMPUTE GROUND LEVEL DISTANCES FROM STATE PLANE COORDINATES.

In some cases it is necessary to compute actual ground level distances between points that have coordinates in SPCS. The actual distance is computed from:

$$\text{Distance}_{actual} = \frac{\text{Distance}_{grid}}{Comb.\ S.F.}$$

Thus, in order to compute the actual distance from a computed distance from coordinates we need to reverse the computation procedure of section 3.4. The reversed procedure is to compute GSF (Grid scale factor), ESF (elevation scale factor) and the combined scale factor as described in step 5. Subsequently, apply the correction by ***dividing*** the grid distance by the combined scale factor.

For example, the actual ground distance between point 1 and 2 in our example is computed as follows:

Grid Distance	=	3092.166m	(computed from coordinates)
Combined scale factor	=	0.99987994	
Actual ground distance	=	3092.538m	

5. COMPUTATION TABLES FOR NEW JERSEY'S STATE PLANE COORDINATE SYSTEM OF NAD83

The state plane coordinate system of New Jersey is based on the Transverse Mercator (TM) projection using a reduced scale for the central meridian. This section contains four sets of tables which enables the user to perform all the necessary computations associated with State Plane Coordinate Systems. The first table is to interpolate the Grid Scale Factor K as a function of eastings. No additional computations are necessary accept for the interpolation. The second set of tables ('A' coefficients) are used to convert Geodetic positions (φ,λ) to plane coordinates (N,E). The third set of tables ('B' coefficients) are used to compute the inverse conversion, namely, converting (N,E) to (φ,λ). The forth set of tables are used to compute the meridian convergence γ ('D' coefficients). Special computation forms and sample computations are also included in this manual in order to make it easier for the user to follow the computation process.

All computations can be carried out on a simple scientific calculator which has a display of at least 10 digits. Since virtually all of these calculators have a function to convert angles from Degrees, Min., Sec. to Decimal Degrees, angular input and output (φ,λ and γ) are in Decimal Degrees. The units for (N,E) are in meters.

The accuracies of the computed Northings and Eastings from the tables are limited to ±0.001m or ±1mm. The accuracies of the computed Latitude and Longitude from the tables are limited to ±0.0001" (second of arc). To obtain more accurate results a second order correction must be subtracted from the computed coordinates. Tables for these second order corrections are included in this manual.

When do you need to use these tables? If you use a computer software to make these computations then you don't have to use these tables. Otherwise, follow these rules:

- All measured distances MUST be corrected for Grid Scale Factor. So table 5.1 will be used whenever you measure a distance.
- Tables for the direct problem (converting Geodetic to SP coordinates) have to be used ONLY if your control points are given in terms of (φ,λ). Otherwise, you do not have to use these tables.
- Tables for the inverse problem (converting SP to Geodetic coordinates) have to be used ONLY if you have to report your results in terms of (φ,λ). Otherwise, you do not have to use these tables
- Tables for Meridian Convergence have to be used ONLY if you have to compute a Geodetic Azimuth from Grid Azimuth or visa versa. Otherwise, you do not have to use these tables.

Table 5.1 Scale Factor (k) as a Function of Easting

E	k	E	k	E	k	E	k	E	k	E	k
0	1.0001769	50000	1.0000231	100000	0.9999308	150000	0.9999000	200000	0.9999308	250000	1.0000231
1000	1.0001732	51000	1.0000206	101000	0.9999295	151000	0.9999000	201000	0.9999320	251000	1.0000255
2000	1.0001696	52000	1.0000182	102000	0.9999284	152000	0.9999000	202000	0.9999333	252000	1.0000280
3000	1.0001659	53000	1.0000158	103000	0.9999272	153000	0.9999001	203000	0.9999346	253000	1.0000306
4000	1.0001623	54000	1.0000134	104000	0.9999260	154000	0.9999002	204000	0.9999359	254000	1.0000331
5000	1.0001587	55000	1.0000111	105000	0.9999249	155000	0.9999003	205000	0.9999372	255000	1.0000357
6000	1.0001552	56000	1.0000087	106000	0.9999238	156000	0.9999004	206000	0.9999386	256000	1.0000383
7000	1.0001516	57000	1.0000064	107000	0.9999228	157000	0.9999006	207000	0.9999400	257000	1.0000409
8000	1.0001481	58000	1.0000042	108000	0.9999217	158000	0.9999008	208000	0.9999414	258000	1.0000435
9000	1.0001447	59000	1.0000019	109000	0.9999207	159000	0.9999010	209000	0.9999428	259000	1.0000462
10000	1.0001412	60000	0.9999997	110000	0.9999197	160000	0.9999012	210000	0.9999443	260000	1.0000489
11000	1.0001378	61000	0.9999975	111000	0.9999187	161000	0.9999015	211000	0.9999458	261000	1.0000516
12000	1.0001344	62000	0.9999953	112000	0.9999178	162000	0.9999018	212000	0.9999473	262000	1.0000544
13000	1.0001310	63000	0.9999931	113000	0.9999168	163000	0.9999021	213000	0.9999488	263000	1.0000571
14000	1.0001276	64000	0.9999910	114000	0.9999159	164000	0.9999024	214000	0.9999504	264000	1.0000599
15000	1.0001243	65000	0.9999889	115000	0.9999151	165000	0.9999028	215000	0.9999520	265000	1.0000627
16000	1.0001210	66000	0.9999868	116000	0.9999142	166000	0.9999032	216000	0.9999536	266000	1.0000656
17000	1.0001177	67000	0.9999848	117000	0.9999134	167000	0.9999036	217000	0.9999552	267000	1.0000685
18000	1.0001144	68000	0.9999827	118000	0.9999126	168000	0.9999040	218000	0.9999569	268000	1.0000713
19000	1.0001112	69000	0.9999807	119000	0.9999118	169000	0.9999044	219000	0.9999586	269000	1.0000743
20000	1.0001080	70000	0.9999788	120000	0.9999111	170000	0.9999049	220000	0.9999603	270000	1.0000772
21000	1.0001048	71000	0.9999768	121000	0.9999103	171000	0.9999054	221000	0.9999620	271000	1.0000802
22000	1.0001016	72000	0.9999749	122000	0.9999096	172000	0.9999060	222000	0.9999638	272000	1.0000832
23000	1.0000985	73000	0.9999730	123000	0.9999090	173000	0.9999065	223000	0.9999656	273000	1.0000862
24000	1.0000954	74000	0.9999711	124000	0.9999083	174000	0.9999071	224000	0.9999674	274000	1.0000892
25000	1.0000923	75000	0.9999692	125000	0.9999077	175000	0.9999077	225000	0.9999692	275000	1.0000923
26000	1.0000892	76000	0.9999674	126000	0.9999071	176000	0.9999083	226000	0.9999711	276000	1.0000954
27000	1.0000862	77000	0.9999656	127000	0.9999065	177000	0.9999090	227000	0.9999730	277000	1.0000985
28000	1.0000832	78000	0.9999638	128000	0.9999060	178000	0.9999096	228000	0.9999749	278000	1.0001016
29000	1.0000802	79000	0.9999620	129000	0.9999054	179000	0.9999103	229000	0.9999768	279000	1.0001048
30000	1.0000772	80000	0.9999603	130000	0.9999049	180000	0.9999111	230000	0.9999788	280000	1.0001080
31000	1.0000743	81000	0.9999586	131000	0.9999044	181000	0.9999118	231000	0.9999807	281000	1.0001112
32000	1.0000713	82000	0.9999569	132000	0.9999040	182000	0.9999126	232000	0.9999827	282000	1.0001144
33000	1.0000685	83000	0.9999552	133000	0.9999036	183000	0.9999134	233000	0.9999848	283000	1.0001177
34000	1.0000656	84000	0.9999536	134000	0.9999032	184000	0.9999142	234000	0.9999868	284000	1.0001210
35000	1.0000627	85000	0.9999520	135000	0.9999028	185000	0.9999151	235000	0.9999889	285000	1.0001243
36000	1.0000599	86000	0.9999504	136000	0.9999024	186000	0.9999159	236000	0.9999910	286000	1.0001276
37000	1.0000571	87000	0.9999488	137000	0.9999021	187000	0.9999168	237000	0.9999931	287000	1.0001310
38000	1.0000544	88000	0.9999473	138000	0.9999018	188000	0.9999178	238000	0.9999953	288000	1.0001344
39000	1.0000516	89000	0.9999458	139000	0.9999015	189000	0.9999187	239000	0.9999975	289000	1.0001378
40000	1.0000489	90000	0.9999443	140000	0.9999012	190000	0.9999197	240000	0.9999997	290000	1.0001412
41000	1.0000462	91000	0.9999428	141000	0.9999010	191000	0.9999207	241000	1.0000019	291000	1.0001447
42000	1.0000435	92000	0.9999414	142000	0.9999008	192000	0.9999217	242000	1.0000042	292000	1.0001481
43000	1.0000409	93000	0.9999400	143000	0.9999006	193000	0.9999228	243000	1.0000064	293000	1.0001516
44000	1.0000383	94000	0.9999386	144000	0.9999004	194000	0.9999238	244000	1.0000087	294000	1.0001552
45000	1.0000357	95000	0.9999372	145000	0.9999003	195000	0.9999249	245000	1.0000111	295000	1.0001587
46000	1.0000331	96000	0.9999359	146000	0.9999002	196000	0.9999260	246000	1.0000134	296000	1.0001623
47000	1.0000306	97000	0.9999346	147000	0.9999001	197000	0.9999272	247000	1.0000158	297000	1.0001659
48000	1.0000280	98000	0.9999333	148000	0.9999000	198000	0.9999284	248000	1.0000182	298000	1.0001696
49000	1.0000255	99000	0.9999320	149000	0.9999000	199000	0.9999295	249000	1.0000206	299000	1.0001732
50000	1.0000231	100000	0.9999308	150000	0.9999000	200000	0.9999308	250000	1.0000231	300000	1.0001769

Form 1. Direct Problem.

STATE PLANE COORDINATES FROM GEODETIC $(\varphi, \lambda) \longrightarrow (N, E)$

$L = \lambda^d - \lambda_0$

$$N = S' + A_2 \times L^2 + A_4 \times L^4 - {}^{*}\text{2nd order correction}$$

$$E = E_0 + A_1 \times L + A_3 \times L^3 - {}^{*}\text{2nd order correction}$$

Where:

λ^d	-	Geodetic Longitude in D°.ddddd (Decimal Degrees)
λ_0	-	74°.5 (NJ's Geodetic Longitude of Central Meridian)
E_0	-	150,000m (NJ's Grid Easting Origin Shift)
S', A_2, A_4, A_1, A_3,	-	Tabulated coefficients as a function of φ
φ	-	Geodetic Latitude in D° M' S"
N,E	-	Grid Northing and Easting (m)

*2nd order corrections (table 5.5) and are necessary only for accuracies better than ±0.001mm or ±0.0001".

~~~~~~

COMPUTATION FORM

Station: ____

Given:

φ = ° ′ ″	λ = ° ′ ″	$L=\lambda^d-74.5$ =

λ^d -Longitude in Decimal Degrees

Compute:

	S'	A_2	A_4
φ°′			
$\Delta \times \varphi$ ″			
Sum			
+		$A_2 \times L^2$	
+		$A_4 \times L^4$	
N=			

	A_1	A_3
	150000.000	NJ const.
+		$A_1 \times L$
+		$A_3 \times L^3$
E=		m

For better than ±1mm accuracy subtract the respective 2nd order correction from N and E
~~~~~~

COMPUTING STATE PLANE COORDINATES FROM GEODETIC

Station: Example

Given:

φ = 38° 52′ 34.53761 ″	λ = 74 ° 54′ 28.12345 ″	L=λ^d-74.5 =0.407812069

λ^d -Longitude in Decimal Degrees

Compute:

	S'	A_2	A_4
φ°′	3700.0506	475.21283	0.03208
Δ×φ ″	1064.9288	0.03481	
Sum	4764.9794	475.24764	0.03208
+	79.0388	$A_2 \times L^2$	
+	0.0009	$A_4 \times L^4$	
N=	4844.019		

	A_1	A_3
	-86780.0105	-0.94697
	11.66533	0.00157
	-86768.40572	-0.94540
	150000.000	NJ const.
+	-35385.2031	$A_1 \times L$
+	-0.0641	$A_3 \times L^3$
E=	114614.733	m

For better than ±1mm accuracy subtract the respective 2nd order correction from N and E

TABLE 5.2 COMPUTING STATE PLANE COORDINATES FROM GEODETIC

φ	S'	A_2	A_4	A_1	A_3
38°50'	0.0000	475.091 38	0.032 12	-86 820.579 81	-0.952 45
Δ	30.8337 12	0.001 0133	-0.000 0003	0.337 5115	0.000 0457
38°51'	1 850.0227	475.152 18	0.032 10	-86 800.329 12	-0.949 71
Δ	30.8337 98	0.001 0108	-0.000 0003	0.337 6345	0.000 0457
38°52'	3 700.0506	475.212 83	0.032 08	-86 780.071 05	-0.946 97
Δ	30.8338 88	0.001 0080	-0.000 0002	0.337 7573	0.000 0455
38°53'	5 550.0839	475.273 31	0.032 07	-86 759.805 61	-0.944 24
Δ	30.8339 77	0.001 0053	-0.000 0003	0.337 8802	0.000 0457
38°54'	7 400.1225	475.333 63	0.032 05	-86 739.532 80	-0.941 50
Δ	30.8340 63	0.001 0028	-0.000 0003	0.338 0030	0.000 0455
38°55'	9 250.1663	475.393 80	0.032 03	-86 719.252 62	-0.938 77
Δ	30.8341 53	0.001 0000	-0.000 0002	0.338 1257	0.000 0455
38°56'	11 100.2155	475.453 80	0.032 02	-86 698.965 08	-0.936 04
Δ	30.8342 40	0.000 9973	-0.000 0003	0.338 2485	0.000 0457
38°57'	12 950.2699	475.513 64	0.032 00	-86 678.670 17	-0.933 30
Δ	30.8343 30	0.000 9947	-0.000 0003	0.338 3712	0.000 0455
38°58'	14 800.3297	475.573 32	0.031 98	-86 658.367 90	-0.930 57
Δ	30.8344 17	0.000 9920	-0.000 0002	0.338 4940	0.000 0455
38°59'	16 650.3947	475.632 84	0.031 97	-86 638.058 26	-0.927 84
Δ	30.8345 05	0.000 9893	-0.000 0003	0.338 6165	0.000 0455

φ	S'	A_2	A_4	A_1	A_3
39°00'	18 500.4650	475.692 20	0.031 95	-86 617.741 27	-0.925 11
Δ	30.8345 95	0.000 9867	-0.000 0003	0.338 7392	0.000 0455
39°01'	20 350.5407	475.751 40	0.031 93	-86 597.416 92	-0.922 38
Δ	30.8346 82	0.000 9840	-0.000 0002	0.338 8617	0.000 0455
39°02'	22 200.6216	475.810 44	0.031 92	-86 577.085 22	-0.919 65
Δ	30.8347 72	0.000 9812	-0.000 0003	0.338 9843	0.000 0453
39°03'	24 050.7079	475.869 31	0.031 90	-86 556.746 16	-0.916 93
Δ	30.8348 58	0.000 9787	-0.000 0003	0.339 1068	0.000 0455
39°04'	25 900.7994	475.928 03	0.031 88	-86 536.399 75	-0.914 20
Δ	30.8349 48	0.000 9758	-0.000 0002	0.339 2295	0.000 0455
39°05'	27 750.8963	475.986 58	0.031 87	-86 516.045 98	-0.911 47
Δ	30.8350 35	0.000 9733	-0.000 0003	0.339 3518	0.000 0453
39°06'	29 600.9984	476.044 98	0.031 85	-86 495.684 87	-0.908 75
Δ	30.8351 25	0.000 9705	-0.000 0003	0.339 4743	0.000 0455
39°07'	31 451.1059	476.103 21	0.031 83	-86 475.316 41	-0.906 02
Δ	30.8352 12	0.000 9678	-0.000 0003	0.339 5968	0.000 0453
39°08'	33 301.2186	476.161 28	0.031 81	-86 454.940 60	-0.903 30
Δ	30.8353 02	0.000 9652	-0.000 0002	0.339 7192	0.000 0453
39°09'	35 151.3367	476.219 19	0.031 80	-86 434.557 45	-0.900 58
Δ	30.8353 90	0.000 9625	-0.000 0003	0.339 8415	0.000 0455
39°10'	37 001.4601	476.276 94	0.031 78	-86 414.166 96	-0.897 85
Δ	30.8354 78	0.000 9598	-0.000 0003	0.339 9640	0.000 0453
39°11'	38 851.5888	476.334 53	0.031 76	-86 393.769 12	-0.895 13
Δ	30.8355 65	0.000 9572	-0.000 0002	0.340 0862	0.000 0453
39°12'	40 701.7227	476.391 96	0.031 75	-86 373.363 95	-0.892 41
Δ	30.8356 55	0.000 9543	-0.000 0003	0.340 2085	0.000 0453
39°13'	42 551.8620	476.449 22	0.031 73	-86 352.951 44	-0.889 69
Δ	30.8357 43	0.000 9518	-0.000 0003	0.340 3308	0.000 0453
39°14'	44 402.0066	476.506 33	0.031 71	-86 332.531 59	-0.886 97
Δ	30.8358 33	0.000 9490	-0.000 0003	0.340 4530	0.000 0452
39°15'	46 252.1566	476.563 27	0.031 69	-86 312.104 41	-0.884 26
Δ	30.8359 20	0.000 9465	-0.000 0002	0.340 5753	0.000 0453
39°16'	48 102.3118	476.620 06	0.031 68	-86 291.669 89	-0.881 54
Δ	30.8360 08	0.000 9437	-0.000 0003	0.340 6973	0.000 0453
39°17'	49 952.4723	476.676 68	0.031 66	-86 271.228 05	-0.878 82
Δ	30.8360 98	0.000 9410	-0.000 0003	0.340 8197	0.000 0452
39°18'	51 802.6382	476.733 14	0.031 64	-86 250.778 87	-0.876 11
Δ	30.8361 85	0.000 9383	-0.000 0002	0.340 9417	0.000 0453
39°19'	53 652.8093	476.789 44	0.031 63	-86 230.322 37	-0.873 39
Δ	30.8362 75	0.000 9357	-0.000 0003	0.341 0638	0.000 0452
39°20'	55 502.9858	476.845 58	0.031 61	-86 209.858 54	-0.870 68
Δ	30.8363 63	0.000 9328	-0.000 0003	0.341 1858	0.000 0452
39°21'	57 353.1676	476.901 55	0.031 59	-86 189.387 39	-0.867 97
Δ	30.8364 52	0.000 9303	-0.000 0003	0.341 3078	0.000 0453

φ	S'	A_2	A_4		A_1	A_3
39°22'	59 203.3547	476.957 37	0.031 57	-86	168.908 92	-0.865 25
Δ	30.8365 40	0.000 9275	-0.000 0002		0.341 4300	0.000 0452
39°23'	61 053.5471	477.013 02	0.031 56	-86	148.423 12	-0.862 54
Δ	30.8366 28	0.000 9250	-0.000 0003		0.341 5518	0.000 0452
39°24'	62 903.7448	477.068 52	0.031 54	-86	127.930 01	-0.859 83
Δ	30.8367 17	0.000 9222	-0.000 0003		0.341 6740	0.000 0452
39°25'	64 753.9478	477.123 85	0.031 52	-86	107.429 57	-0.857 12
Δ	30.8368 07	0.000 9195	-0.000 0003		0.341 7957	0.000 0452
39°26'	66 604.1562	477.179 02	0.031 50	-86	086.921 83	-0.854 41
Δ	30.8368 95	0.000 9168	-0.000 0002		0.341 9178	0.000 0452
39°27'	68 454.3699	477.234 03	0.031 49	-86	066.406 76	-0.851 70
Δ	30.8369 83	0.000 9142	-0.000 0003		0.342 0395	0.000 0450
39°28'	70 304.5889	477.288 88	0.031 47	-86	045.884 39	-0.849 00
Δ	30.8370 72	0.000 9113	-0.000 0003		0.342 1615	0.000 0452
39°29'	72 154.8132	477.343 56	0.031 45	-86	025.354 70	-0.846 29
Δ	30.8371 60	0.000 9088	-0.000 0002		0.342 2832	0.000 0450
39°30'	74 005.0428	477.398 09	0.031 44	-86	004.817 71	-0.843 59
Δ	30.8372 50	0.000 9060	-0.000 0003		0.342 4050	0.000 0452
39°31'	75 855.2778	477.452 45	0.031 42	-85	984.273 41	-0.840 88
Δ	30.8373 37	0.000 9035	-0.000 0003		0.342 5268	0.000 0450
39°32'	77 705.5180	477.506 66	0.031 40	-85	963.721 80	-0.838 18
Δ	30.8374 27	0.000 9007	-0.000 0003		0.342 6485	0.000 0452
39°33'	79 555.7636	477.560 70	0.031 38	-85	943.162 89	-0.835 47
Δ	30.8375 15	0.000 8980	-0.000 0002		0.342 7703	0.000 0450
39°34'	81 406.0145	477.614 58	0.031 37	-85	922.596 67	-0.832 77
Δ	30.8376 05	0.000 8952	-0.000 0003		0.342 8918	0.000 0450
39°35'	83 256.2708	477.668 29	0.031 35	-85	902.023 16	-0.830 07
Δ	30.8376 92	0.000 8927	-0.000 0003		0.343 0135	0.000 0450
39°36'	85 106.5323	477.721 85	0.031 33	-85	881.442 35	-0.827 37
Δ	30.8377 82	0.000 8900	-0.000 0003		0.343 1352	0.000 0450
39°37'	86 956.7992	477.775 25	0.031 31	-85	860.854 24	-0.824 67
Δ	30.8378 70	0.000 8872	-0.000 0002		0.343 2568	0.000 0450
39°38'	88 807.0714	477.828 48	0.031 30	-85	840.258 83	-0.821 97
Δ	30.8379 58	0.000 8845	-0.000 0003		0.343 3783	0.000 0450
39°39'	90 657.3489	477.881 55	0.031 28	-85	819.656 13	-0.819 27
Δ	30.8380 48	0.000 8818	-0.000 0003		0.343 4998	0.000 0448
39°40'	92 507.6318	477.934 46	0.031 26	-85	799.046 14	-0.816 58
Δ	30.8381 37	0.000 8792	-0.000 0003		0.343 6213	0.000 0450
39°41'	94 357.9200	477.987 21	0.031 24	-85	778.428 86	-0.813 88
Δ	30.8382 25	0.000 8765	-0.000 0003		0.343 7428	0.000 0448
39°42'	96 208.2135	478.039 80	0.031 22	-85	757.804 29	-0.811 19
Δ	30.8383 13	0.000 8737	-0.000 0002		0.343 8643	0.000 0450
39°43'	98 058.5123	478.092 22	0.031 21	-85	737.172 43	-0.808 49
Δ	30.8384 03	0.000 8710	-0.000 0003		0.343 9858	0.000 0448
39°44'	99 908.8165	478.144 48	0.031 19	-85	716.533 28	-0.805 80

φ	S’	A_2	A_4	A_1	A_3
Δ	30.8384 92	0.000 8685	-0.000 0003	0.344 1070	0.000 0448
39°45'	101 759.1260	478.196 59	0.031 17	-85 695.886 86	-0.803 11
Δ	30.8385 80	0.000 8657	-0.000 0003	0.344 2285	0.000 0450
39°46'	103 609.4408	478.248 53	0.031 15	-85 675.233 15	-0.800 41
Δ	30.8386 68	0.000 8630	-0.000 0002	0.344 3498	0.000 0448
39°47'	105 459.7609	478.300 31	0.031 14	-85 654.572 16	-0.797 72
Δ	30.8387 58	0.000 8602	-0.000 0003	0.344 4712	0.000 0448
39°48'	107 310.0864	478.351 92	0.031 12	-85 633.903 89	-0.795 03
Δ	30.8388 47	0.000 8577	-0.000 0003	0.344 5923	0.000 0448
39°49'	109 160.4172	478.403 38	0.031 10	-85 613.228 35	-0.792 34
Δ	30.8389 35	0.000 8548	-0.000 0003	0.344 7137	0.000 0447
39°50'	111 010.7533	478.454 67	0.031 08	-85 592.545 53	-0.789 66
Δ	30.8390 25	0.000 8522	-0.000 0002	0.344 8350	0.000 0448
39°51'	112 861.0948	478.505 80	0.031 07	-85 571.855 43	-0.786 97
Δ	30.8391 13	0.000 8495	-0.000 0003	0.344 9560	0.000 0448
39°52'	114 711.4416	478.556 77	0.031 05	-85 551.158 07	-0.784 28
Δ	30.8392 03	0.000 8468	-0.000 0003	0.345 0772	0.000 0447
39°53'	116 561.7938	478.607 58	0.031 03	-85 530.453 44	-0.781 60
Δ	30.8392 90	0.000 8440	-0.000 0003	0.345 1985	0.000 0448
39°54'	118 412.1512	478.658 22	0.031 01	-85 509.741 53	-0.778 91
Δ	30.8393 80	0.000 8415	-0.000 0003	0.345 3195	0.000 0447
39°55'	120 262.5140	478.708 71	0.030 99	-85 489.022 36	-0.776 23
Δ	30.8394 70	0.000 8387	-0.000 0002	0.345 4405	0.000 0448
39°56'	122 112.8822	478.759 03	0.030 98	-85 468.295 93	-0.773 54
Δ	30.8395 57	0.000 8360	-0.000 0003	0.345 5617	0.000 0447
39°57'	123 963.2556	478.809 19	0.030 96	-85 447.562 23	-0.770 86
Δ	30.8396 47	0.000 8333	-0.000 0003	0.345 6827	0.000 0447
39°58'	125 813.6344	478.859 19	0.030 94	-85 426.821 27	-0.768 18
Δ	30.8397 37	0.000 8307	-0.000 0003	0.345 8037	0.000 0447
39°59'	127 664.0186	478.909 03	0.030 92	-85 406.073 05	-0.765 50
Δ	30.8398 25	0.000 8278	-0.000 0003	0.345 9247	0.000 0447
40°00'	129 514.4081	478.958 70	0.030 90	-85 385.317 57	-0.762 82
Δ	30.8399 13	0.000 8252	-0.000 0002	0.346 0455	0.000 0447
40°01'	131 364.8029	479.008 21	0.030 89	-85 364.554 84	-0.760 14
Δ	30.8400 03	0.000 8225	-0.000 0003	0.346 1665	0.000 0445
40°02'	133 215.2031	479.057 56	0.030 87	-85 343.784 85	-0.757 47
Δ	30.8400 92	0.000 8198	-0.000 0003	0.346 2873	0.000 0447
40°03'	135 065.6086	479.106 75	0.030 85	-85 323.007 61	-0.754 79
Δ	30.8401 80	0.000 8172	-0.000 0003	0.346 4082	0.000 0447
40°04'	136 916.0194	479.155 78	0.030 83	-85 302.223 12	-0.752 11
Δ	30.8402 70	0.000 8143	-0.000 0003	0.346 5290	0.000 0445
40°05'	138 766.4356	479.204 64	0.030 81	-85 281.431 38	-0.749 44
Δ	30.8403 58	0.000 8118	-0.000 0002	0.346 6498	0.000 0447
40°06'	140 616.8571	479.253 35	0.030 80	-85 260.632 39	-0.746 76
Δ	30.8404 47	0.000 8090	-0.000 0003	0.346 7707	0.000 0445

φ	S'	A2	A4	A1	A3
40°07'	142 467.2839	479.301 89	0.030 78	-85 239.826 15	-0.744 09
Δ	30.8405 37	0.000 8062	-0.000 0003	0.346 8913	0.000 0445
40°08'	144 317.7161	479.350 26	0.030 76	-85 219.012 67	-0.741 42
Δ	30.8406 27	0.000 8037	-0.000 0003	0.347 0120	0.000 0445
40°09'	146 168.1537	479.398 48	0.030 74	-85 198.191 95	-0.738 75
Δ	30.8407 15	0.000 8008	-0.000 0003	0.347 1328	0.000 0445
40°10'	148 018.5966	479.446 53	0.030 72	-85 177.363 98	-0.736 08
Δ	30.8408 03	0.000 7983	-0.000 0002	0.347 2533	0.000 0445
40°11'	149 869.0448	479.494 43	0.030 71	-85 156.528 78	-0.733 41
Δ	30.8408 93	0.000 7955	-0.000 0003	0.347 3740	0.000 0445
40°12'	151 719.4984	479.542 16	0.030 69	-85 135.686 34	-0.730 74
Δ	30.8409 82	0.000 7927	-0.000 0003	0.347 4947	0.000 0445
40°13'	153 569.9573	479.589 72	0.030 67	-85 114.836 66	-0.728 07
Δ	30.8410 70	0.000 7902	-0.000 0003	0.347 6152	0.000 0445
40°14'	155 420.4215	479.637 13	0.030 65	-85 093.979 75	-0.725 40
Δ	30.8411 62	0.000 7873	-0.000 0003	0.347 7357	0.000 0443
40°15'	157 270.8912	479.684 37	0.030 63	-85 073.115 61	-0.722 74
Δ	30.8412 48	0.000 7847	-0.000 0002	0.347 8563	0.000 0445
40°16'	159 121.3661	479.731 45	0.030 62	-85 052.244 23	-0.720 07
Δ	30.8413 38	0.000 7820	-0.000 0003	0.347 9767	0.000 0443
40°17'	160 971.8464	479.778 37	0.030 60	-85 031.365 63	-0.717 41
Δ	30.8414 27	0.000 7793	-0.000 0003	0.348 0972	0.000 0443
40°18'	162 822.3320	479.825 13	0.030 58	-85 010.479 80	-0.714 75
Δ	30.8415 17	0.000 7765	-0.000 0003	0.348 2177	0.000 0443
40°19'	164 672.8230	479.871 72	0.030 56	-84 989.586 74	-0.712 09
Δ	30.8416 07	0.000 7738	-0.000 0003	0.348 3380	0.000 0445
40°20'	166 523.3194	479.918 15	0.030 54	-84 968.686 46	-0.709 42
Δ	30.8416 95	0.000 7712	-0.000 0003	0.348 4583	0.000 0443
40°21'	168 373.8211	479.964 42	0.030 52	-84 947.778 96	-0.706 76
Δ	30.8417 83	0.000 7685	-0.000 0002	0.348 5787	0.000 0443
40°22'	170 224.3281	480.010 53	0.030 51	-84 926.864 24	-0.704 10
Δ	30.8418 73	0.000 7658	-0.000 0003	0.348 6990	0.000 0442
40°23'	172 074.8405	480.056 48	0.030 49	-84 905.942 30	-0.701 45
Δ	30.8419 62	0.000 7630	-0.000 0003	0.348 8193	0.000 0443
40°24'	173 925.3582	480.102 26	0.030 47	-84 885.013 14	-0.698 79
Δ	30.8420 52	0.000 7603	-0.000 0003	0.348 9395	0.000 0443
40°25'	175 775.8813	480.147 88	0.030 45	-84 864.076 77	-0.696 13
Δ	30.8421 42	0.000 7577	-0.000 0003	0.349 0598	0.000 0442
40°26'	177 626.4098	480.193 34	0.030 43	-84 843.133 18	-0.693 48
Δ	30.8422 30	0.000 7548	-0.000 0003	0.349 1798	0.000 0443
40°27'	179 476.9436	480.238 63	0.030 41	-84 822.182 39	-0.690 82
Δ	30.8423 18	0.000 7523	-0.000 0002	0.349 3002	0.000 0442
40°28'	181 327.4827	480.283 77	0.030 40	-84 801.224 38	-0.688 17

Δ	30.8424 08	0.000 7495	-0.000 0003	0.349 4203	0.000 0442
40°29'	183 178.0272	480.328 74	0.030 38	-84 780.259 16	-0.685 52
Δ	30.8424 98	0.000 7467	-0.000 0003	0.349 5403	0.000 0443
φ	S’	A_2	A_4	A_1	A_3
40°30'	185 028.5771	480.373 54	0.030 36	-84 759.286 74	-0.682 86
Δ	30.8425 87	0.000 7442	-0.000 0003	0.349 6605	0.000 0442
40°31'	186 879.1323	480.418 19	0.030 34	-84 738.307 11	-0.680 21
Δ	30.8426 75	0.000 7413	-0.000 0003	0.349 7805	0.000 0442
40°32'	188 729.6928	480.462 67	0.030 32	-84 717.320 28	-0.677 56
Δ	30.8427 67	0.000 7387	-0.000 0003	0.349 9005	0.000 0442
40°33'	190 580.2588	480.506 99	0.030 30	-84 696.326 25	-0.674 91
Δ	30.8428 53	0.000 7360	-0.000 0003	0.350 0205	0.000 0440
40°34'	192 430.8300	480.551 15	0.030 28	-84 675.325 02	-0.672 27
Δ	30.8429 45	0.000 7333	-0.000 0002	0.350 1405	0.000 0442
40°35'	194 281.4067	480.595 15	0.030 27	-84 654.316 59	-0.669 62
Δ	30.8430 32	0.000 7305	-0.000 0003	0.350 2605	0.000 0442
40°36'	196 131.9886	480.638 98	0.030 25	-84 633.300 96	-0.666 97
Δ	30.8431 23	0.000 7278	-0.000 0003	0.350 3803	0.000 0440
40°37'	197 982.5760	480.682 65	0.030 23	-84 612.278 14	-0.664 33
Δ	30.8432 12	0.000 7252	-0.000 0003	0.350 5002	0.000 0442
40°38'	199 833.1687	480.726 16	0.030 21	-84 591.248 13	-0.661 68
Δ	30.8433 00	0.000 7225	-0.000 0003	0.350 6202	0.000 0440
40°39'	201 683.7667	480.769 51	0.030 19	-84 570.210 92	-0.659 04
Δ	30.8433 90	0.000 7197	-0.000 0003	0.350 7398	0.000 0440
40°40'	203 534.3701	480.812 69	0.030 17	-84 549.166 53	-0.656 40
Δ	30.8434 80	0.000 7170	-0.000 0002	0.350 8597	0.000 0440
40°41'	205 384.9789	480.855 71	0.030 16	-84 528.114 95	-0.653 76
Δ	30.8435 70	0.000 7143	-0.000 0003	0.350 9795	0.000 0440
40°42'	207 235.5931	480.898 57	0.030 14	-84 507.056 18	-0.651 12
Δ	30.8436 58	0.000 7115	-0.000 0003	0.351 0992	0.000 0440
40°43'	209 086.2126	480.941 26	0.030 12	-84 485.990 23	-0.648 48
Δ	30.8437 47	0.000 7088	-0.000 0003	0.351 2188	0.000 0440
40°44'	210 936.8374	480.983 79	0.030 10	-84 464.917 10	-0.645 84
Δ	30.8438 37	0.000 7062	-0.000 0003	0.351 3385	0.000 0440
40°45'	212 787.4676	481.026 16	0.030 08	-84 443.836 79	-0.643 20
Δ	30.8439 27	0.000 7035	-0.000 0003	0.351 4583	0.000 0438
40°46'	214 638.1032	481.068 37	0.030 06	-84 422.749 29	-0.640 57
Δ	30.8440 15	0.000 7007	-0.000 0003	0.351 5778	0.000 0440
40°47'	216 488.7441	481.110 41	0.030 04	-84 401.654 62	-0.637 93
Δ	30.8441 05	0.000 6982	-0.000 0003	0.351 6973	0.000 0438
40°48'	218 339.3904	481.152 30	0.030 02	-84 380.552 78	-0.635 30
Δ	30.8441 95	0.000 6953	-0.000 0002	0.351 8170	0.000 0440
40°49'	220 190.0421	481.194 02	0.030 01	-84 359.443 76	-0.632 66
Δ	30.8442 83	0.000 6925	-0.000 0003	0.351 9365	0.000 0438
40°50'	222 040.6991	481.235 57	0.029 99	-84 338.327 57	-0.630 03
Δ	30.8443 73	0.000 6898	-0.000 0003	0.352 0560	0.000 0438

φ	S’	A_2	A_4	A_1	A_3
40°51'	223 891.3615	481.276 96	0.029 97	-84 317.204 21	-0.627 40
Δ	30.8444 63	0.000 6873	-0.000 0003	0.352 1755	0.000 0438
40°52'	225 742.0293	481.318 20	0.029 95	-84 296.073 68	-0.624 77
Δ	30.8445 52	0.000 6843	-0.000 0003	0.352 2948	0.000 0438
40°53'	227 592.7024	481.359 26	0.029 93	-84 274.935 99	-0.622 14
Δ	30.8446 42	0.000 6818	-0.000 0003	0.352 4143	0.000 0438
40°54'	229 443.3809	481.400 17	0.029 91	-84 253.791 13	-0.619 51
Δ	30.8447 32	0.000 6790	-0.000 0003	0.352 5338	0.000 0438
40°55'	231 294.0648	481.440 91	0.029 89	-84 232.639 10	-0.616 88
Δ	30.8448 20	0.000 6763	-0.000 0003	0.352 6530	0.000 0438
40°56'	233 144.7540	481.481 49	0.029 87	-84 211.479 92	-0.614 25
Δ	30.8449 10	0.000 6737	-0.000 0002	0.352 7725	0.000 0437
40°57'	234 995.4486	481.521 91	0.029 86	-84 190.313 57	-0.611 63
Δ	30.8449 98	0.000 6708	-0.000 0003	0.352 8917	0.000 0438
40°58'	236 846.1485	481.562 16	0.029 84	-84 169.140 07	-0.609 00
Δ	30.8450 88	0.000 6682	-0.000 0003	0.353 0110	0.000 0437
40°59'	238 696.8538	481.602 25	0.029 82	-84 147.959 41	-0.606 38
Δ	30.8451 78	0.000 6655	-0.000 0003	0.353 1302	0.000 0437
41°00'	240 547.5645	481.642 18	0.029 80	-84 126.771 60	-0.603 76
Δ	30.8452 68	0.000 6627	-0.000 0003	0.353 2495	0.000 0437
41°01'	242 398.2806	481.681 94	0.029 78	-84 105.576 63	-0.601 14
Δ	30.8453 57	0.000 6602	-0.000 0003	0.353 3687	0.000 0437
41°02'	244 249.0020	481.721 55	0.029 76	-84 084.374 51	-0.598 52
Δ	30.8454 47	0.000 6572	-0.000 0003	0.353 4877	0.000 0437
41°03'	246 099.7288	481.760 98	0.029 74	-84 063.165 25	-0.595 90
Δ	30.8455 37	0.000 6547	-0.000 0003	0.353 6070	0.000 0437
41°04'	247 950.4610	481.800 26	0.029 72	-84 041.948 83	-0.593 28
Δ	30.8456 25	0.000 6518	-0.000 0003	0.353 7260	0.000 0437
41°05'	249 801.1985	481.839 37	0.029 70	-84 020.725 27	-0.590 66
Δ	30.8457 15	0.000 6492	-0.000 0003	0.353 8450	0.000 0437
41°06'	251 651.9414	481.878 32	0.029 68	-83 999.494 57	-0.588 04
Δ	30.8458 05	0.000 6465	-0.000 0002	0.353 9642	0.000 0435
41°07'	253 502.6897	481.917 11	0.029 67	-83 978.256 72	-0.585 43
Δ	30.8458 95	0.000 6438	-0.000 0003	0.354 0830	0.000 0437
41°08'	255 353.4434	481.955 74	0.029 65	-83 957.011 74	-0.582 81
Δ	30.8459 83	0.000 6410	-0.000 0003	0.354 2022	0.000 0435
41°09'	257 204.2024	481.994 20	0.029 63	-83 935.759 61	-0.580 20
Δ	30.8460 73	0.000 6382	-0.000 0003	0.354 3210	0.000 0435
41°10'	259 054.9668	482.032 49	0.029 61	-83 914.500 35	-0.577 59
Δ	30.8461 63	0.000 6357	-0.000 0003	0.354 4400	0.000 0437
41°11'	260 905.7366	482.070 63	0.029 59	-83 893.233 95	-0.574 97
Δ	30.8462 52	0.000 6328	-0.000 0003	0.354 5588	0.000 0435
41°12'	262 756.5117	482.108 60	0.029 57	-83 871.960 42	-0.572 36
Δ	30.8463 42	0.000 6302	-0.000 0003	0.354 6777	0.000 0435

φ	S'	A_2	A_4	A_1	A_3
41°13'	264 607.2922	482.146 41	0.029 55	-83 850.679 76	-0.569 75
Δ	30.8464 32	0.000 6275	-0.000 0003	0.354 7965	0.000 0433
41°14'	266 458.0781	482.184 06	0.029 53	-83 829.391 97	-0.567 15
Δ	30.8465 22	0.000 6247	-0.000 0003	0.354 9153	0.000 0435

φ	S'	A_2	A_4	A_1	A_3
41°15'	268 308.8694	482.221 54	0.029 51	-83 808.097 05	-0.564 54
Δ	30.8466 10	0.000 6220	-0.000 0003	0.355 0342	0.000 0435
41°16'	270 159.6660	482.258 86	0.029 49	-83 786.795 00	-0.561 93
Δ	30.8467 02	0.000 6193	-0.000 0003	0.355 1528	0.000 0433
41°17'	272 010.4681	482.296 02	0.029 47	-83 765.485 83	-0.559 33
Δ	30.8467 88	0.000 6165	-0.000 0002	0.355 2717	0.000 0435
41°18'	273 861.2754	482.333 01	0.029 46	-83 744.169 53	-0.556 72
Δ	30.8468 80	0.000 6138	-0.000 0003	0.355 3902	0.000 0433
41°19'	275 712.0882	482.369 84	0.029 44	-83 722.846 12	-0.554 12
Δ	30.8469 70	0.000 6112	-0.000 0003	0.355 5090	0.000 0433
41°20'	277 562.9064	482.406 51	0.029 42	-83 701.515 58	-0.551 52
Δ	30.8470 58	0.000 6083	-0.000 0003	0.355 6275	0.000 0435
41°21'	279 413.7299	482.443 01	0.029 40	-83 680.177 93	-0.548 91
Δ	30.8471 48	0.000 6057	-0.000 0003	0.355 7462	0.000 0433
41°22'	281 264.5588	482.479 35	0.029 38	-83 658.833 16	-0.546 31
Δ	30.8472 38	0.000 6030	-0.000 0003	0.355 8647	0.000 0432
41°23'	283 115.3931	482.515 53	0.029 36	-83 637.481 28	-0.543 72
Δ	30.8473 28	0.000 6002	-0.000 0003	0.355 9833	0.000 0433
41°24'	284 966.2328	482.551 54	0.029 34	-83 616.122 28	-0.541 12
Δ	30.8474 17	0.000 5977	-0.000 0003	0.356 1017	0.000 0433
41°25'	286 817.0778	482.587 40	0.029 32	-83 594.756 18	-0.538 52
Δ	30.8475 07	0.000 5947	-0.000 0003	0.356 2203	0.000 0433
41°26'	288 667.9282	482.623 08	0.029 30	-83 573.382 96	-0.535 92
Δ	30.8475 97	0.000 5922	-0.000 0003	0.356 3387	0.000 0432
41°27'	290 518.7840	482.658 61	0.029 28	-83 552.002 64	-0.533 33
Δ	30.8476 87	0.000 5893	-0.000 0003	0.356 4572	0.000 0433
41°28'	292 369.6452	482.693 97	0.029 26	-83 530.615 21	-0.530 73
Δ	30.8477 77	0.000 5867	-0.000 0003	0.356 5755	0.000 0432
41°29'	294 220.5118	482.729 17	0.029 24	-83 509.220 68	-0.528 14
Δ	30.8478 65	0.000 5838	-0.000 0003	0.356 6938	0.000 0432

Form 2. Inverse Problem.

GEODETIC COORDINATES FROM STATE PLANE (N , E) ⟶ (φ , λ)

$$Q = \frac{E - 150{,}000.}{1{,}000{,}000.}$$

$$\varphi = \varphi' + B_2 \times Q^2 + B_4 \times Q^4 - {}^*\text{2nd order correction}$$

$$\lambda = \lambda_0 + B_1 \times Q + B_3 \times Q^3 + B_5 \times Q^5 - {}^*\text{2nd order correction}$$

Where:

λ	-	Geodetic Longitude in D°.ddddd (Decimal Degrees)
φ	-	Geodetic Latitude in D°.ddddd (Decimal Degrees)
λ_0	-	74°.5 (NJ's Geodetic Longitude of Central Meridian)
φ'	-	Footpoint Latitude. Tabulated as a function of N.
E_0	-	150,000m (NJ's Grid Easting Origin Shift)
E	-	Grid Easting (m)
B_2,B_4,B_1,B_3,B_5	-	Tabulated coefficients as a function of N (Northing)

*2nd order corrections (table 5.5) and are necessary only for accuracies better than ±0.001mm or ±0.0001".

~~~~~~

COMPUTATION FORM

Station: ____________________

Given:

N= m	E= m	$Q = \frac{E-150{,}000}{1{,}000{,}000} =$

Compute:

	φ'	B2	B4
$^*N_{2000}$			
$\Delta \times dN^{**}$			
Sum			
+		$B_2 \times Q^2$	
+		$B_4 \times Q^4$	
φ^d =			
φ =	° ′ ″		

B1	B3	B3

	74.5	NJ const.
+		$B_1 \times Q$
+		$B_3 \times Q^3$
+		$B_5 \times Q^5$
λ^d =		
λ =	° ′	

$^*N_{2000}$ = The largest value of N in the table (i.e. 2000m interval), not exceeding N

Example: If N=135,952.481, then N_{2000} = 134,000

$dN^{**} = N - N_{2000}$ Example: If N=135,952.481, dN = 135,952.481-134,000=1,952.481

For better than ±1mm accuracy subtract the respective 2nd order correction from φ and λ.
~~~~~~

COMPUTING GEODETIC COORDINATES FROM STATE PLANE

Station: Example

Given:

N= 4844.018 m	E= 114614.732	$Q = \frac{E-150{,}000}{1{,}000{,}000}$ = -0.35385268

Compute:

	φ'	B₂	B₄
$^{*}N_{2000}$	38.869368868	-0.5685382	0.00805
$\Delta_{\times} dN^{**}$	0.007603624	-0.0001535	
Sum	38.876972492	-0.5686917	0.00805
+	-0.000712069	$B_2 \times Q^2$	
+	0.000000013	$B_4 \times Q^4$	
φ^d =	38.87626043		
φ =	38°52′34.5376″		

B_1	B_3	B_3
-11.52381930	0.108485	-0.00193
-0.00122818	0.000045	
-11.52504748	0.108530	-0.00193

	74.5	NJ const.
+	0.407816893	$B_1 \times Q$
+	-0.000004809	$B_3 \times Q^3$
+	0	$B_5 \times Q^5$
λ^d =	74.90781208	
λ =	74°54′28.1234″	

$^{*}N_{2000}$ = The largest value of N in the table (i.e. 2000m interval), not exceeding N
Example: If N=135,952.481, then N_{2000} = 134,000

$dN^{**} = N - N_{2000}$ Example: If N=135,952.481, dN = 135,952.481-134,000=1,952.481

For better than ±1mm accuracy subtract the respective 2nd order correction from φ and λ.

TABLE 5.3 COMPUTING GEODETIC COORDINATES FROM STATE PLANE

N	φ '	B_2	B_4	B_1	B_3	B_5
0	38.8333 33333	-0.567 8114	0.00803	-11.5180 0647	0.108273	-0.00192
Δ/m	0.0000 09008 8975	-0.000 0001 8165		-0.0000 0145 2560	0.000000 0530	
2 000	38.8513 51128	-0.568 1747	0.00804	-11.5209 1159	0.108379	-0.00192
Δ/m	0.0000 09008 8700	-0.000 0001 8175		-0.0000 0145 3855	0.000000 0530	
4 000	38.8693 68868	-0.568 5382	0.00805	-11.5238 1930	0.108485	-0.00193
Δ/m	0.0000 09008 8415	-0.000 0001 8185		-0.0000 0145 5160	0.000000 0530	
6 000	38.8873 86551	-0.568 9019	0.00806	-11.5267 2962	0.108591	-0.00193
Δ/m	0.0000 09008 8140	-0.000 0001 8190		-0.0000 0145 6465	0.000000 0530	
8 000	38.9054 04179	-0.569 2657	0.00807	-11.5296 4255	0.108697	-0.00193
Δ/m	0.0000 09008 7860	-0.000 0001 8200		-0.0000 0145 7765	0.000000 0530	
10 000	38.9234 21751	-0.569 6297	0.00807	-11.5325 5808	0.108803	-0.00194
Δ/m	0.0000 09008 7585	-0.000 0001 8210		-0.0000 0145 9075	0.000000 0530	
12 000	38.9414 39268	-0.569 9939	0.00808	-11.5354 7623	0.108909	-0.00194
Δ/m	0.0000 09008 7300	-0.000 0001 8220		-0.0000 0146 0380	0.000000 0535	
14 000	38.9594 56728	-0.570 3583	0.00809	-11.5383 9699	0.109016	-0.00194
Δ/m	0.0000 09008 7025	-0.000 0001 8230		-0.0000 0146 1690	0.000000 0535	

N	φ ‘	B_2	B_4	B_1	B_3	B_5
16 000	38.9774 74133	-0.570 7229	0.00810	-11.5413 2037	0.109123	-0.00195
Δ/m	0.0000 09008 6745	-0.000 0001 8240		-0.0000 0146 2995	0.000000 0530	
18 000	38.9954 91482	-0.571 0877	0.00811	-11.5442 4636	0.109229	-0.00195
Δ/m	0.0000 09008 6465	-0.000 0001 8245		-0.0000 0146 4310	0.000000 0535	
20 000	39.0135 08775	-0.571 4526	0.00811	-11.5471 7498	0.109336	-0.00195
Δ/m	0.0000 09008 6190	-0.000 0001 8255		-0.0000 0146 5620	0.000000 0540	
22 000	39.0315 26013	-0.571 8177	0.00812	-11.5501 0622	0.109444	-0.00196
Δ/m	0.0000 09008 5905	-0.000 0001 8265		-0.0000 0146 6935	0.000000 0535	
24 000	39.0495 43194	-0.572 1830	0.00813	-11.5530 4009	0.109551	-0.00196
Δ/m	0.0000 09008 5630	-0.000 0001 8275		-0.0000 0146 8250	0.000000 0535	
26 000	39.0675 60320	-0.572 5485	0.00814	-11.5559 7659	0.109658	-0.00196
Δ/m	0.0000 09008 5350	-0.000 0001 8285		-0.0000 0146 9565	0.000000 0540	
28 000	39.0855 77390	-0.572 9142	0.00815	-11.5589 1572	0.109766	-0.00197
Δ/m	0.0000 09008 5070	-0.000 0001 8290		-0.0000 0147 0880	0.000000 0540	
30 000	39.1035 94404	-0.573 2800	0.00816	-11.5618 5748	0.109874	-0.00197
Δ/m	0.0000 09008 4790	-0.000 0001 8305		-0.0000 0147 2200	0.000000 0540	
32 000	39.1216 11362	-0.573 6461	0.00816	-11.5648 0188	0.109982	-0.00197
Δ/m	0.0000 09008 4515	-0.000 0001 8310		-0.0000 0147 3520	0.000000 0540	
34 000	39.1396 28265	-0.574 0123	0.00817	-11.5677 4892	0.110090	-0.00198
Δ/m	0.0000 09008 4230	-0.000 0001 8320		-0.0000 0147 4840	0.000000 0540	
36 000	39.1576 45111	-0.574 3787	0.00818	-11.5706 9860	0.110198	-0.00198
Δ/m	0.0000 09008 3955	-0.000 0001 8330		-0.0000 0147 6165	0.000000 0545	
38 000	39.1756 61902	-0.574 7453	0.00819	-11.5736 5093	0.110307	-0.00198
Δ/m	0.0000 09008 3675	-0.000 0001 8340		-0.0000 0147 7485	0.000000 0540	
40 000	39.1936 78637	-0.575 1121	0.00820	-11.5766 0590	0.110415	-0.00199
Δ/m	0.0000 09008 3395	-0.000 0001 8350		-0.0000 0147 8815	0.000000 0545	
42 000	39.2116 95316	-0.575 4791	0.00820	-11.5795 6353	0.110524	-0.00199
Δ/m	0.0000 09008 3115	-0.000 0001 8355		-0.0000 0148 0135	0.000000 0545	
44 000	39.2297 11939	-0.575 8462	0.00821	-11.5825 2380	0.110633	-0.00199
Δ/m	0.0000 09008 2835	-0.000 0001 8365		-0.0000 0148 1470	0.000000 0545	
46 000	39.2477 28506	-0.576 2135	0.00822	-11.5854 8674	0.110742	-0.00200
Δ/m	0.0000 09008 2560	-0.000 0001 8375		-0.0000 0148 2795	0.000000 0545	
48 000	39.2657 45018	-0.576 5810	0.00823	-11.5884 5233	0.110851	-0.00200
Δ/m	0.0000 09008 2275	-0.000 0001 8385		-0.0000 0148 4125	0.000000 0545	
50 000	39.2837 61473	-0.576 9487	0.00824	-11.5914 2058	0.110960	-0.00201
Δ/m	0.0000 09008 1995	-0.000 0001 8395		-0.0000 0148 5455	0.000000 0550	
52 000	39.3017 77872	-0.577 3166	0.00825	-11.5943 9149	0.111070	-0.00201
Δ/m	0.0000 09008 1720	-0.000 0001 8405		-0.0000 0148 6790	0.000000 0550	
54 000	39.3197 94216	-0.577 6847	0.00825	-11.5973 6507	0.111180	-0.00201
Δ/m	0.0000 09008 1440	-0.000 0001 8415		-0.0000 0148 8125	0.000000 0545	
56 000	39.3378 10504	-0.578 0530	0.00826	-11.6003 4132	0.111289	-0.00202
Δ/m	0.0000 09008 1155	-0.000 0001 8420		-0.0000 0148 9455	0.000000 0550	
58 000	39.3558 26735	-0.578 4214	0.00827	-11.6033 2023	0.111399	-0.00202
Δ/m	0.0000 09008 0880	-0.000 0001 8435		-0.0000 0149 0800	0.000000 0555	
60 000	39.3738 42911	-0.578 7901	0.00828	-11.6063 0183	0.111510	-0.00202
Δ/m	0.0000 09008 0600	-0.000 0001 8440		-0.0000 0149 2130	0.000000 0550	
62 000	39.3918 59031	-0.579 1589	0.00829	-11.6092 8609	0.111620	-0.00203
Δ/m	0.0000 09008 0320	-0.000 0001 8450		-0.0000 0149 3475	0.000000 0550	

N	φ '	B_2	B_4	B_1	B_3	B_5
64 000	39.4098 75095	-0.579 5279	0.00830	-11.6122 7304	0.111730	-0.00203
Δ/m	0.0000 09008 0040	-0.000 0001 8460		-0.0000 0149 4815	0.000000 0555	
66 000	39.4278 91103	-0.579 8971	0.00830	-11.6152 6267	0.111841	-0.00203
Δ/m	0.0000 09007 9760	-0.000 0001 8470		-0.0000 0149 6155	0.000000 0555	
68 000	39.4459 07055	-0.580 2665	0.00831	-11.6182 5498	0.111952	-0.00204
Δ/m	0.0000 09007 9480	-0.000 0001 8475		-0.0000 0149 7500	0.000000 0555	
70 000	39.4639 22951	-0.580 6360	0.00832	-11.6212 4998	0.112063	-0.00204
Δ/m	0.0000 09007 9200	-0.000 0001 8490		-0.0000 0149 8845	0.000000 0555	
72 000	39.4819 38791	-0.581 0058	0.00833	-11.6242 4767	0.112174	-0.00204
Δ/m	0.0000 09007 8920	-0.000 0001 8500		-0.0000 0150 0190	0.000000 0555	
74 000	39.4999 54575	-0.581 3758	0.00834	-11.6272 4805	0.112285	-0.00205
Δ/m	0.0000 09007 8640	-0.000 0001 8505		-0.0000 0150 1540	0.000000 0560	
76 000	39.5179 70303	-0.581 7459	0.00835	-11.6302 5113	0.112397	-0.00205
Δ/m	0.0000 09007 8360	-0.000 0001 8515		-0.0000 0150 2885	0.000000 0560	
78 000	39.5359 85975	-0.582 1162	0.00835	-11.6332 5690	0.112509	-0.00206
Δ/m	0.0000 09007 8080	-0.000 0001 8525		-0.0000 0150 4235	0.000000 0555	
80 000	39.5540 01591	-0.582 4867	0.00836	-11.6362 6537	0.112620	-0.00206
Δ/m	0.0000 09007 7800	-0.000 0001 8535		-0.0000 0150 5590	0.000000 0560	
82 000	39.5720 17151	-0.582 8574	0.00837	-11.6392 7655	0.112732	-0.00206
Δ/m	0.0000 09007 7520	-0.000 0001 8545		-0.0000 0150 6940	0.000000 0560	
84 000	39.5900 32655	-0.583 2283	0.00838	-11.6422 9043	0.112844	-0.00207
Δ/m	0.0000 09007 7240	-0.000 0001 8555		-0.0000 0150 8295	0.000000 0565	
86 000	39.6080 48103	-0.583 5994	0.00839	-11.6453 0702	0.112957	-0.00207
Δ/m	0.0000 09007 6960	-0.000 0001 8565		-0.0000 0150 9650	0.000000 0560	
88 000	39.6260 63495	-0.583 9707	0.00840	-11.6483 2632	0.113069	-0.00207
Δ/m	0.0000 09007 6680	-0.000 0001 8570		-0.0000 0151 1010	0.000000 0565	
90 000	39.6440 78831	-0.584 3421	0.00840	-11.6513 4834	0.113182	-0.00208
Δ/m	0.0000 09007 6400	-0.000 0001 8585		-0.0000 0151 2365	0.000000 0565	
92 000	39.6620 94111	-0.584 7138	0.00841	-11.6543 7307	0.113295	-0.00208
Δ/m	0.0000 09007 6120	-0.000 0001 8590		-0.0000 0151 3725	0.000000 0565	
94 000	39.6801 09335	-0.585 0856	0.00842	-11.6574 0052	0.113408	-0.00208
Δ/m	0.0000 09007 5840	-0.000 0001 8605		-0.0000 0151 5085	0.000000 0565	
96 000	39.6981 24503	-0.585 4577	0.00843	-11.6604 3069	0.113521	-0.00209
Δ/m	0.0000 09007 5560	-0.000 0001 8610		-0.0000 0151 6450	0.000000 0565	
98 000	39.7161 39615	-0.585 8299	0.00844	-11.6634 6359	0.113634	-0.00209
Δ/m	0.0000 09007 5275	-0.000 0001 8620		-0.0000 0151 7810	0.000000 0570	
100 000	39.7341 54670	-0.586 2023	0.00845	-11.6664 9921	0.113748	-0.00210
Δ/m	0.0000 09007 5000	-0.000 0001 8630		-0.0000 0151 9180	0.000000 0565	
102 000	39.7521 69670	-0.586 5749	0.00846	-11.6695 3757	0.113861	-0.00210
Δ/m	0.0000 09007 4715	-0.000 0001 8640		-0.0000 0152 0540	0.000000 0570	
104 000	39.7701 84613	-0.586 9477	0.00846	-11.6725 7865	0.113975	-0.00210
Δ/m	0.0000 09007 4440	-0.000 0001 8650		-0.0000 0152 1915	0.000000 0570	
106 000	39.7881 99501	-0.587 3207	0.00847	-11.6756 2248	0.114089	-0.00211
Δ/m	0.0000 09007 4155	-0.000 0001 8660		-0.0000 0152 3280	0.000000 0570	
108 000	39.8062 14332	-0.587 6939	0.00848	-11.6786 6904	0.114203	-0.00211
Δ/m	0.0000 09007 3880	-0.000 0001 8670		-0.0000 0152 4650	0.000000 0570	
110 000	39.8242 29108	-0.588 0673	0.00849	-11.6817 1834	0.114317	-0.00211
Δ/m	0.0000 09007 3595	-0.000 0001 8680		-0.0000 0152 6020	0.000000 0575	

N	φ '	B_2	B_4	B_1	B_3	B_5
112 000	39.8422 43827	-0.588 4409	0.00850	-11.6847 7038	0.114432	-0.00212
Δ/m	0.0000 09007 3315	-0.000 0001 8685		-0.0000 0152 7395	0.000000 0575	
114 000	39.8602 58490	-0.588 8146	0.00851	-11.6878 2517	0.114547	-0.00212
Δ/m	0.0000 09007 3035	-0.000 0001 8700		-0.0000 0152 8770	0.000000 0570	
116 000	39.8782 73097	-0.589 1886	0.00851	-11.6908 8271	0.114661	-0.00213
Δ/m	0.0000 09007 2750	-0.000 0001 8705		-0.0000 0153 0150	0.000000 0575	
118 000	39.8962 87647	-0.589 5627	0.00852	-11.6939 4301	0.114776	-0.00213
Δ/m	0.0000 09007 2475	-0.000 0001 8720		-0.0000 0153 1520	0.000000 0580	
120 000	39.9143 02142	-0.589 9371	0.00853	-11.6970 0605	0.114892	-0.00213
Δ/m	0.0000 09007 2195	-0.000 0001 8725		-0.0000 0153 2905	0.000000 0575	
122 000	39.9323 16581	-0.590 3116	0.00854	-11.7000 7186	0.115007	-0.00214
Δ/m	0.0000 09007 1910	-0.000 0001 8735		-0.0000 0153 4280	0.000000 0575	
124 000	39.9503 30963	-0.590 6863	0.00855	-11.7031 4042	0.115122	-0.00214
Δ/m	0.0000 09007 1630	-0.000 0001 8745		-0.0000 0153 5665	0.000000 0580	
126 000	39.9683 45289	-0.591 0612	0.00856	-11.7062 1175	0.115238	-0.00214
Δ/m	0.0000 09007 1350	-0.000 0001 8760		-0.0000 0153 7045	0.000000 0580	
128 000	39.9863 59559	-0.591 4364	0.00857	-11.7092 8584	0.115354	-0.00215
Δ/m	0.0000 09007 1070	-0.000 0001 8765		-0.0000 0153 8430	0.000000 0580	
130 000	40.0043 73773	-0.591 8117	0.00858	-11.7123 6270	0.115470	-0.00215
Δ/m	0.0000 09007 0790	-0.000 0001 8775		-0.0000 0153 9815	0.000000 0580	
132 000	40.0223 87931	-0.592 1872	0.00858	-11.7154 4233	0.115586	-0.00216
Δ/m	0.0000 09007 0510	-0.000 0001 8785		-0.0000 0154 1205	0.000000 0580	
134 000	40.0404 02033	-0.592 5629	0.00859	-11.7185 2474	0.115702	-0.00216
Δ/m	0.0000 09007 0225	-0.000 0001 8795		-0.0000 0154 2590	0.000000 0585	
136 000	40.0584 16078	-0.592 9388	0.00860	-11.7216 0992	0.115819	-0.00216
Δ/m	0.0000 09006 9945	-0.000 0001 8805		-0.0000 0154 3985	0.000000 0585	
138 000	40.0764 30067	-0.593 3149	0.00861	-11.7246 9789	0.115936	-0.00217
Δ/m	0.0000 09006 9665	-0.000 0001 8815		-0.0000 0154 5370	0.000000 0585	
140 000	40.0944 44000	-0.593 6912	0.00862	-11.7277 8863	0.116053	-0.00217
Δ/m	0.0000 09006 9385	-0.000 0001 8825		-0.0000 0154 6765	0.000000 0585	
142 000	40.1124 57877	-0.594 0677	0.00863	-11.7308 8216	0.116170	-0.00217
Δ/m	0.0000 09006 9105	-0.000 0001 8830		-0.0000 0154 8160	0.000000 0585	
144 000	40.1304 71698	-0.594 4443	0.00864	-11.7339 7848	0.116287	-0.00218
Δ/m	0.0000 09006 8820	-0.000 0001 8845		-0.0000 0154 9555	0.000000 0585	
146 000	40.1484 85462	-0.594 8212	0.00864	-11.7370 7759	0.116404	-0.00218
Δ/m	0.0000 09006 8540	-0.000 0001 8855		-0.0000 0155 0955	0.000000 0590	
148 000	40.1664 99170	-0.595 1983	0.00865	-11.7401 7950	0.116522	-0.00219
Δ/m	0.0000 09006 8260	-0.000 0001 8865		-0.0000 0155 2350	0.000000 0590	
150 000	40.1845 12822	-0.595 5756	0.00866	-11.7432 8420	0.116640	-0.00219
Δ/m	0.0000 09006 7980	-0.000 0001 8870		-0.0000 0155 3750	0.000000 0585	
152 000	40.2025 26418	-0.595 9530	0.00867	-11.7463 9170	0.116757	-0.00219
Δ/m	0.0000 09006 7700	-0.000 0001 8885		-0.0000 0155 5150	0.000000 0595	
154 000	40.2205 39958	-0.596 3307	0.00868	-11.7495 0200	0.116876	-0.00220
Δ/m	0.0000 09006 7415	-0.000 0001 8895		-0.0000 0155 6550	0.000000 0590	
156 000	40.2385 53441	-0.596 7086	0.00869	-11.7526 1510	0.116994	-0.00220
Δ/m	0.0000 09006 7135	-0.000 0001 8900		-0.0000 0155 7960	0.000000 0590	
158 000	40.2565 66868	-0.597 0866	0.00870	-11.7557 3102	0.117112	-0.00221
Δ/m	0.0000 09006 6855	-0.000 0001 8915		-0.0000 0155 9360	0.000000 0595	

N	φ ‘	B_2	B_4	B_1	B_3	B_5
160 000	40.2745 80239	-0.597 4649	0.00871	-11.7588 4974	0.117231	-0.00221
Δ/m	0.0000 09006 6570	-0.000 0001 8920		-0.0000 0156 0770	0.000000 0595	
162 000	40.2925 93553	-0.597 8433	0.00872	-11.7619 7128	0.117350	-0.00221
Δ/m	0.0000 09006 6295	-0.000 0001 8935		-0.0000 0156 2180	0.000000 0595	
164 000	40.3106 06812	-0.598 2220	0.00872	-11.7650 9564	0.117469	-0.00222
Δ/m	0.0000 09006 6010	-0.000 0001 8940		-0.0000 0156 3585	0.000000 0595	
166 000	40.3286 20014	-0.598 6008	0.00873	-11.7682 2281	0.117588	-0.00222
Δ/m	0.0000 09006 5725	-0.000 0001 8955		-0.0000 0156 5000	0.000000 0595	
168 000	40.3466 33159	-0.598 9799	0.00874	-11.7713 5281	0.117707	-0.00223
Δ/m	0.0000 09006 5450	-0.000 0001 8960		-0.0000 0156 6410	0.000000 0600	
170 000	40.3646 46249	-0.599 3591	0.00875	-11.7744 8563	0.117827	-0.00223
Δ/m	0.0000 09006 5165	-0.000 0001 8975		-0.0000 0156 7825	0.000000 0595	
172 000	40.3826 59282	-0.599 7386	0.00876	-11.7776 2128	0.117946	-0.00223
Δ/m	0.0000 09006 4885	-0.000 0001 8985		-0.0000 0156 9240	0.000000 0600	
174 000	40.4006 72259	-0.600 1183	0.00877	-11.7807 5976	0.118066	-0.00224
Δ/m	0.0000 09006 4605	-0.000 0001 8990		-0.0000 0157 0655	0.000000 0600	
176 000	40.4186 85180	-0.600 4981	0.00878	-11.7839 0107	0.118186	-0.00224
Δ/m	0.0000 09006 4320	-0.000 0001 9005		-0.0000 0157 2075	0.000000 0605	
178 000	40.4366 98044	-0.600 8782	0.00879	-11.7870 4522	0.118307	-0.00225
Δ/m	0.0000 09006 4040	-0.000 0001 9010		-0.0000 0157 3495	0.000000 0600	
180 000	40.4547 10852	-0.601 2584	0.00880	-11.7901 9221	0.118427	-0.00225
Δ/m	0.0000 09006 3760	-0.000 0001 9025		-0.0000 0157 4915	0.000000 0605	
182 000	40.4727 23604	-0.601 6389	0.00880	-11.7933 4204	0.118548	-0.00225
Δ/m	0.0000 09006 3475	-0.000 0001 9030		-0.0000 0157 6340	0.000000 0600	
184 000	40.4907 36299	-0.602 0195	0.00881	-11.7964 9472	0.118668	-0.00226
Δ/m	0.0000 09006 3195	-0.000 0001 9045		-0.0000 0157 7765	0.000000 0605	
186 000	40.5087 48938	-0.602 4004	0.00882	-11.7996 5025	0.118789	-0.00226
Δ/m	0.0000 09006 2915	-0.000 0001 9050		-0.0000 0157 9185	0.000000 0605	
188 000	40.5267 61521	-0.602 7814	0.00883	-11.8028 0862	0.118910	-0.00227
Δ/m	0.0000 09006 2630	-0.000 0001 9065		-0.0000 0158 0615	0.000000 0610	
190 000	40.5447 74047	-0.603 1627	0.00884	-11.8059 6985	0.119032	-0.00227
Δ/m	0.0000 09006 2355	-0.000 0001 9070		-0.0000 0158 2045	0.000000 0605	
192 000	40.5627 86518	-0.603 5441	0.00885	-11.8091 3394	0.119153	-0.00227
Δ/m	0.0000 09006 2065	-0.000 0001 9085		-0.0000 0158 3470	0.000000 0610	
194 000	40.5807 98931	-0.603 9258	0.00886	-11.8123 0088	0.119275	-0.00228
Δ/m	0.0000 09006 1790	-0.000 0001 9095		-0.0000 0158 4905	0.000000 0610	
196 000	40.5988 11289	-0.604 3077	0.00887	-11.8154 7069	0.119397	-0.00228
Δ/m	0.0000 09006 1505	-0.000 0001 9100		-0.0000 0158 6340	0.000000 0610	
198 000	40.6168 23590	-0.604 6897	0.00888	-11.8186 4337	0.119519	-0.00229
Δ/m	0.0000 09006 1220	-0.000 0001 9115		-0.0000 0158 7770	0.000000 0610	
200 000	40.6348 35834	-0.605 0720	0.00889	-11.8218 1891	0.119641	-0.00229
Δ/m	0.0000 09006 0945	-0.000 0001 9125		-0.0000 0158 9205	0.000000 0615	
202 000	40.6528 48023	-0.605 4545	0.00889	-11.8249 9732	0.119764	-0.00229
Δ/m	0.0000 09006 0660	-0.000 0001 9135		-0.0000 0159 0645	0.000000 0610	
204 000	40.6708 60155	-0.605 8372	0.00890	-11.8281 7861	0.119886	-0.00230
Δ/m	0.0000 09006 0375	-0.000 0001 9140		-0.0000 0159 2085	0.000000 0615	
206 000	40.6888 72230	-0.606 2200	0.00891	-11.8313 6278	0.120009	-0.00230

Δ/m	0.0000 09006 0100	-0.000 0001 9155		-0.0000 0159 3520	0.000000 0615	

N	φ ‘	B_2	B_4	B_1	B_3	B_5
208 000	40.7068 84250	-0.606 6031	0.00892	-11.8345 4982	0.120132	-0.00231
Δ/m	0.0000 09005 9810	-0.000 0001 9165		-0.0000 0159 4965	0.000000 0615	
210 000	40.7248 96212	-0.606 9864	0.00893	-11.8377 3975	0.120255	-0.00231
Δ/m	0.0000 09005 9535	-0.000 0001 9175		-0.0000 0159 6410	0.000000 0615	
212 000	40.7429 08119	-0.607 3699	0.00894	-11.8409 3257	0.120378	-0.00232
Δ/m	0.0000 09005 9250	-0.000 0001 9185		-0.0000 0159 7850	0.000000 0620	
214 000	40.7609 19969	-0.607 7536	0.00895	-11.8441 2827	0.120502	-0.00232
Δ/m	0.0000 09005 8970	-0.000 0001 9195		-0.0000 0159 9300	0.000000 0620	
216 000	40.7789 31763	-0.608 1375	0.00896	-11.8473 2687	0.120626	-0.00232
Δ/m	0.0000 09005 8685	-0.000 0001 9205		-0.0000 0160 0745	0.000000 0615	
218 000	40.7969 43500	-0.608 5216	0.00897	-11.8505 2836	0.120749	-0.00233
Δ/m	0.0000 09005 8405	-0.000 0001 9215		-0.0000 0160 2195	0.000000 0625	
220 000	40.8149 55181	-0.608 9059	0.00898	-11.8537 3275	0.120874	-0.00233
Δ/m	0.0000 09005 8120	-0.000 0001 9225		-0.0000 0160 3645	0.000000 0620	
222 000	40.8329 66805	-0.609 2904	0.00899	-11.8569 4004	0.120998	-0.00234
Δ/m	0.0000 09005 7840	-0.000 0001 9235		-0.0000 0160 5095	0.000000 0620	
224 000	40.8509 78373	-0.609 6751	0.00899	-11.8601 5023	0.121122	-0.00234
Δ/m	0.0000 09005 7560	-0.000 0001 9250		-0.0000 0160 6550	0.000000 0625	
226 000	40.8689 89885	-0.610 0601	0.00900	-11.8633 6333	0.121247	-0.00234
Δ/m	0.0000 09005 7275	-0.000 0001 9255		-0.0000 0160 8005	0.000000 0625	
228 000	40.8870 01340	-0.610 4452	0.00901	-11.8665 7934	0.121372	-0.00235
Δ/m	0.0000 09005 6995	-0.000 0001 9265		-0.0000 0160 9465	0.000000 0625	
230 000	40.9050 12739	-0.610 8305	0.00902	-11.8697 9827	0.121497	-0.00235
Δ/m	0.0000 09005 6710	-0.000 0001 9280		-0.0000 0161 0920	0.000000 0625	
232 000	40.9230 24081	-0.611 2161	0.00903	-11.8730 2011	0.121622	-0.00236
Δ/m	0.0000 09005 6430	-0.000 0001 9285		-0.0000 0161 2380	0.000000 0625	
234 000	40.9410 35367	-0.611 6018	0.00904	-11.8762 4487	0.121747	-0.00236
Δ/m	0.0000 09005 6145	-0.000 0001 9300		-0.0000 0161 3840	0.000000 0630	
236 000	40.9590 46596	-0.611 9878	0.00905	-11.8794 7255	0.121873	-0.00237
Δ/m	0.0000 09005 5865	-0.000 0001 9310		-0.0000 0161 5300	0.000000 0630	
238 000	40.9770 57769	-0.612 3740	0.00906	-11.8827 0315	0.121999	-0.00237
Δ/m	0.0000 09005 5585	-0.000 0001 9320		-0.0000 0161 6770	0.000000 0630	
240 000	40.9950 68886	-0.612 7604	0.00907	-11.8859 3669	0.122125	-0.00237
Δ/m	0.0000 09005 5300	-0.000 0001 9325		-0.0000 0161 8230	0.000000 0630	
242 000	41.0130 79946	-0.613 1469	0.00908	-11.8891 7315	0.122251	-0.00238
Δ/m	0.0000 09005 5015	-0.000 0001 9340		-0.0000 0161 9700	0.000000 0630	
244 000	41.0310 90949	-0.613 5337	0.00909	-11.8924 1255	0.122377	-0.00238
Δ/m	0.0000 09005 4735	-0.000 0001 9350		-0.0000 0162 1170	0.000000 0635	
246 000	41.0491 01896	-0.613 9207	0.00910	-11.8956 5489	0.122504	-0.00239
Δ/m	0.0000 09005 4455	-0.000 0001 9360		-0.0000 0162 2635	0.000000 0630	
248 000	41.0671 12787	-0.614 3079	0.00911	-11.8989 0016	0.122630	-0.00239
Δ/m	0.0000 09005 4170	-0.000 0001 9375		-0.0000 0162 4110	0.000000 0635	
250 000	41.0851 23621	-0.614 6954	0.00911	-11.9021 4838	0.122757	-0.00240
Δ/m	0.0000 09005 3890	-0.000 0001 9380		-0.0000 0162 5585	0.000000 0635	
252 000	41.1031 34399	-0.615 0830	0.00912	-11.9053 9955	0.122884	-0.00240
Δ/m	0.0000 09005 3605	-0.000 0001 9390		-0.0000 0162 7055	0.000000 0640	

N	φ ‘	B_2	B_4	B_1	B_3	B_5
254 000	41.1211 45120	-0.615 4708	0.00913	-11.9086 5366	0.123012	-0.00241
Δ/m	0.0000 09005 3320	-0.000 0001 9405		-0.0000 0162 8535	0.000000 0635	
256 000	41.1391 55784	-0.615 8589	0.00914	-11.9119 1073	0.123139	-0.00241
Δ/m	0.0000 09005 3045	-0.000 0001 9410		-0.0000 0163 0010	0.000000 0640	
258 000	41.1571 66393	-0.616 2471	0.00915	-11.9151 7075	0.123267	-0.00241
Δ/m	0.0000 09005 2755	-0.000 0001 9425		-0.0000 0163 1490	0.000000 0640	
260 000	41.1751 76944	-0.616 6356	0.00916	-11.9184 3373	0.123395	-0.00242
Δ/m	0.0000 09005 2475	-0.000 0001 9435		-0.0000 0163 2970	0.000000 0640	
262 000	41.1931 87439	-0.617 0243	0.00917	-11.9216 9967	0.123523	-0.00242
Δ/m	0.0000 09005 2195	-0.000 0001 9445		-0.0000 0163 4455	0.000000 0640	
264 000	41.2111 97878	-0.617 4132	0.00918	-11.9249 6858	0.123651	-0.00243
Δ/m	0.0000 09005 1910	-0.000 0001 9455		-0.0000 0163 5935	0.000000 0640	
266 000	41.2292 08260	-0.617 8023	0.00919	-11.9282 4045	0.123779	-0.00243
Δ/m	0.0000 09005 1630	-0.000 0001 9465		-0.0000 0163 7425	0.000000 0645	
268 000	41.2472 18586	-0.618 1916	0.00920	-11.9315 1530	0.123908	-0.00244
Δ/m	0.0000 09005 1345	-0.000 0001 9475		-0.0000 0163 8905	0.000000 0645	
270 000	41.2652 28855	-0.618 5811	0.00921	-11.9347 9311	0.124037	-0.00244
Δ/m	0.0000 09005 1060	-0.000 0001 9485		-0.0000 0164 0400	0.000000 0645	
272 000	41.2832 39067	-0.618 9708	0.00922	-11.9380 7391	0.124166	-0.00244
Δ/m	0.0000 09005 0785	-0.000 0001 9500		-0.0000 0164 1890	0.000000 0645	
274 000	41.3012 49224	-0.619 3608	0.00923	-11.9413 5769	0.124295	-0.00245
Δ/m	0.0000 09005 0495	-0.000 0001 9505		-0.0000 0164 3375	0.000000 0650	
276 000	41.3192 59323	-0.619 7509	0.00924	-11.9446 4444	0.124425	-0.00245
Δ/m	0.0000 09005 0215	-0.000 0001 9520		-0.0000 0164 4875	0.000000 0645	
278 000	41.3372 69366	-0.620 1413	0.00925	-11.9479 3419	0.124554	-0.00246
Δ/m	0.0000 09004 9930	-0.000 0001 9530		-0.0000 0164 6365	0.000000 0650	
280 000	41.3552 79352	-0.620 5319	0.00926	-11.9512 2692	0.124684	-0.00246
Δ/m	0.0000 09004 9650	-0.000 0001 9540		-0.0000 0164 7865	0.000000 0650	
282 000	41.3732 89282	-0.620 9227	0.00927	-11.9545 2265	0.124814	-0.00247
Δ/m	0.0000 09004 9365	-0.000 0001 9550		-0.0000 0164 9360	0.000000 0650	
284 000	41.3912 99155	-0.621 3137	0.00928	-11.9578 2137	0.124944	-0.00247
Δ/m	0.0000 09004 9085	-0.000 0001 9560		-0.0000 0165 0860	0.000000 0655	
286 000	41.4093 08972	-0.621 7049	0.00928	-11.9611 2309	0.125075	-0.00248
Δ/m	0.0000 09004 8800	-0.000 0001 9570		-0.0000 0165 2360	0.000000 0650	
288 000	41.4273 18732	-0.622 0963	0.00929	-11.9644 2781	0.125205	-0.00248
Δ/m	0.0000 09004 8520	-0.000 0001 9585		-0.0000 0165 3865	0.000000 0655	
290 000	41.4453 28436	-0.622 4880	0.00930	-11.9677 3554	0.125336	-0.00249
Δ/m	0.0000 09004 8235	-0.000 0001 9590		-0.0000 0165 5370	0.000000 0655	
292 000	41.4633 38083	-0.622 8798	0.00931	-11.9710 4628	0.125467	-0.00249
Δ/m	0.0000 09004 7950	-0.000 0001 9605		-0.0000 0165 6870	0.000000 0655	
294 000	41.4813 47673	-0.623 2719	0.00932	-11.9743 6002	0.125598	-0.00249
Δ/m	0.0000 09004 7670	-0.000 0001 9615		-0.0000 0165 8380	0.000000 0660	
296 000	41.4993 57207	-0.623 6642	0.00933	-11.9776 7678	0.125730	-0.00250
Δ/m	0.0000 09004 7385	-0.000 0001 9625		-0.0000 0165 9890	0.000000 0655	
298 000	41.5173 66684	-0.624 0567	0.00934	-11.9809 9656	0.125861	-0.00250
Δ/m	0.0000 09004 7105	-0.000 0001 9635		-0.0000 0166 1400	0.000000 0660	

Form 3. Meridian Convergence γ

$$Q = \frac{E - 150,000.}{1,000,000.}$$

$$\gamma = D_1 \times Q + D_3 \times Q^3$$

Where:

γ	-	Meridian Convergence in D°.ddddd (Decimal Degrees)
E_0	-	150,000m (NJ's Grid Easting Origin Shift)
E	-	Grid Easting (m)
D_1, D_3	-	Tabulated coefficients as a function of N (Northing)

Station: ____

N= m	E= m	$Q = \frac{E-150,000}{1,000,000} =$

	D_1	D_3
$^{*}N_{10000}$		
$\Delta \times dN^{**}$		
Sum		

		$D_1 \times Q$
+		$D_3 \times Q^3$
$\gamma^d =$		
γ ° ' " =	° ′ ″	

$^{*}N_{10000}$ = The largest value of N in the table (i.e. 10,000m interval), not exceeding N

Example: If N=135,952.481, then N_{10000} = 130,000

$dN^{**} = N - N_{10000}$ Example: If N=135,952.481, dN = 135,952.481-130,000=5,952.481

COMPUTING MERIDIAN CONVERGENCE - γ

Station: Example

N= 4844.018 m

E= 114614.732 m

$Q = \frac{E-150,000}{1,000,000} = -0.035385268$

	D_1	D_3
*N_{10000}	7.22244780	-0.097048
$\Delta_{\times}dN^{**}$	0.01125796	-0.000271
Sum	7.23370576	-0.097319

	-0.255966617	$D_1 \times Q$
+	0.000004312	$D_3 \times Q^3$
$\gamma^d =$	-0.255962305	
γ ° ‘ “=	-0°15′21.46″	

*N_{10000} = The largest value of N in the table (i.e. 10,000m interval), not exceeding N

Example: If N=135,952.481, then N_{10000} = 130,000

$dN^{**} = N - N_{10000}$ Example: If N=135,952.481, dN = 135,952.481-130,000=5,952.481

TABLE 5.4. COMPUTING MERIDIAN CONVERGENCE

N	D_1	D_3
0	7.2224 4780	-0.097 048
Δ/m	0.0000 0232 4095	-0.000 000 0559
10 000	7.2456 8875	-0.097 607
Δ/m	0.0000 0232 9951	-0.000 000 0564
20 000	7.2689 8826	-0.098 171
Δ/m	0.0000 0233 5841	-0.000 000 0567
30 000	7.2923 4667	-0.098 738
Δ/m	0.0000 0234 1766	-0.000 000 0571
40 000	7.3157 6433	-0.099 309
Δ/m	0.0000 0234 7724	-0.000 000 0575
50 000	7.3392 4157	-0.099 884
Δ/m	0.0000 0235 3718	-0.000 000 0579
60 000	7.3627 7875	-0.100 463
Δ/m	0.0000 0235 9745	-0.000 000 0583
70 000	7.3863 7620	-0.101 046
Δ/m	0.0000 0236 5808	-0.000 000 0588
80 000	7.4100 3428	-0.101 634
Δ/m	0.0000 0237 1906	-0.000 000 0591
90 000	7.4337 5334	-0.102 225
Δ/m	0.0000 0237 8039	-0.000 000 0595
100 000	7.4575 3373	-0.102 820
Δ/m	0.0000 0238 4209	-0.000 000 0599
110 000	7.4813 7582	-0.103 419
Δ/m	0.0000 0239 0414	-0.000 000 0604
120 000	7.5052 7996	-0.104 023
Δ/m	0.0000 0239 6655	-0.000 000 0607
130 000	7.5292 4651	-0.104 630
Δ/m	0.0000 0240 2933	-0.000 000 0612
140 000	7.5532 7584	-0.105 242
Δ/m	0.0000 0240 9248	-0.000 000 0616

N	D_1	D_3
150 000	7.5773 6832	-0.105 858
Δ/m	0.0000 0241 5599	-0.000 000 0621
160 000	7.6015 2431	-0.106 479
Δ/m	0.0000 0242 1988	-0.000 000 0625
170 000	7.6257 4419	-0.107 104
Δ/m	0.0000 0242 8414	-0.000 000 0629
180 000	7.6500 2833	-0.107 733
Δ/m	0.0000 0243 4878	-0.000 000 0634
190 000	7.6743 7711	-0.108 367
Δ/m	0.0000 0244 1380	-0.000 000 0638
200 000	7.6987 9091	-0.109 005
Δ/m	0.0000 0244 7921	-0.000 000 0642
210 000	7.7232 7012	-0.109 647
Δ/m	0.0000 0245 4499	-0.000 000 0648
220 000	7.7478 1511	-0.110 295
Δ/m	0.0000 0246 1117	-0.000 000 0651
230 000	7.7724 2628	-0.110 946
Δ/m	0.0000 0246 7774	-0.000 000 0657
240 000	7.7971 0402	-0.111 603
Δ/m	0.0000 0247 4470	-0.000 000 0661
250 000	7.8218 4872	-0.112 264
Δ/m	0.0000 0248 1206	-0.000 000 0666
260 000	7.8466 6078	-0.112 930
Δ/m	0.0000 0248 7982	-0.000 000 0671
270 000	7.8715 4060	-0.113 601
Δ/m	0.0000 0249 4797	-0.000 000 0675
280 000	7.8964 8857	-0.114 276
Δ/m	0.0000 0250 1655	-0.000 000 0680
290 000	7.9215 0512	-0.114 956
Δ/m	0.0000 0250 8552	-0.000 000 0686

TABLE 5.5 SECOND ORDER CORRECTION

Second Order Correction for Northing (N) in Meters

	Longitude λ					
$\Delta\varphi''$	73°30'	74°00'	74°30'	75°00'	75°30'	75°45'
10	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
20	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
30	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
40	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
50	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003

Second Order Correction for Easting (E) in Meters

	Longitude λ					
$\Delta\varphi''$	73°30'	74°00'	74°30'	75°00'	75°30'	75°45'
10	0.0005	0.0003	0.0000	0.0003	0.0005	0.0006
20	0.0008	0.0004	0.0000	0.0003	0.0007	0.0009
30	0.0009	0.0004	0.0000	0.0004	0.0008	0.0010
40	0.0008	0.0004	0.0000	0.0003	0.0007	0.0009
50	0.0005	0.0003	0.0000	0.0003	0.0005	0.0006

Second Order Correction for Latitude (j) in Seconds of Arc

	Easting					
ΔN_m	0	50 000	100 000	150 000	200 000	250 000
500	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002
1000	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003	-0.00003
1500	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002	-0.00002

Second Order Correction for Longitude (l) in Seconds of arc

	Easting					
ΔN_m	0	50 000	100 000	150 000	200 000	250 000
200	0.00007	0.00005	0.00002	0.00000	-0.00002	-0.00005
400	0.00012	0.00008	0.00004	0.00000	-0.00004	-0.00008
600	0.00016	0.00011	0.00005	0.00000	-0.00005	-0.00011
800	0.00018	0.00012	0.00006	0.00000	-0.00005	-0.00012
1000	0.00019	0.00013	0.00007	0.00000	-0.00007	-0.00013
1200	0.00018	0.00012	0.00006	0.00000	-0.00005	-0.00012
1400	0.00016	0.00011	0.00005	0.00000	-0.00005	-0.00011
1600	0.00012	0.00008	0.00004	0.00000	-0.00004	-0.00008
1800	0.00007	0.00005	0.00002	0.00000	-0.00002	-0.00005

APPENDIX A

The New Jersey Law Concerning State Plane Coordinate System

51:3-7. Official survey base established; plane co-ordinates

51:3-7. The official survey base for New Jersey shall be a system of plane co-ordinates to be known as the New Jersey system of plane co-ordinates, said system being defined as a transverse Mercator projection of the Geodetic Reference System of 1980, having a central meridian 74` 30' west from Greenwich on which meridian the scale is set at one part in 10,000 too small. All co-ordinates of the system are expressed in meters, the x co-ordinate being measured easterly along the grid and the y co-ordinate being measured northerly along the grid, the origin of the co-ordinates being on the meridian 74` 30' west from Greenwich at the intersection of the parallel 38` 50' north latitude, such origin being given the co-ordinates x=150,000 meters; y=0 meters. The precise position of said system shall be as marked on the ground by triangulation or traverse stations established in conformity with the standards adopted by the National Geodetic Survey, formerly the United States Coast and Geodetic Survey for first and second-order work, whose geodetic positions have been rigidly adjusted on the North American Datum of 1983 or the most recently published adjustment by the National Geodetic Survey, and whose plane co-ordinates have been computed on the system defined. The New Jersey co-ordinate system defined by the North American Datum of 1927 may be used concurrently with or in lieu of the system defined by the North American Datum of 1983 for a period of 36 months after the effective date of this amendatory act, P.L.1989, c.218.

Standard conversions from meters to feet shall be the adopted standards of the National Oceanic and Atmospheric Administration.

Amended 1989,c.218,s.1.

51:3-8. Connecting property surveys with system of co-ordinates

51:3-8. Any triangulation or traverse station established as described in section 51:3-7 of this title shall be used in establishing a connection between a property survey and the above-mentioned system of rectangular co-ordinates.

Amended 1989, c.218, s.2.

51:3-9. Indorsement of surveys

No survey of lands hereinafter made shall have indorsed thereon any legend or other statement indicating that it is based upon the New Jersey system of plane co-ordinates unless the co-ordinates have been established on that system as herein defined.