Performance of Cellular CDMA with Cell Site Antenna Arrays, Rayleigh Fading, and Power Control Error

Weichen Ye, *Member, IEEE*, and Alexander M. Haimovich, *Senior Member, IEEE*

***Abstract*—The performance of code-division multiple-access (CDMA) systems is affected by multiple factors such as large-scale fading, small-scale fading, and cochannel interference (CCI). Most of the published research on the performance analysis of CDMA systems usually accounts for subsets of these factors. In this work, it is attempted to provide a comprehensive analysis which joins several of the most important factors affecting the performance of CDMA systems. In particular, new analytical expressions are developed for the outage and bit-error probability of CDMA systems. These expressions account for adverse effects such as path loss, large-scale fading (shadowing), small-scale fading (Rayleigh fading), and CCI, as well as for correcting mechanisms such as power control (compensates for path loss and shadowing), spatial diversity (mitigates against Rayleigh fading), and voice activity gating (reduces CCI). The new expressions may be used as convenient analysis tools that complement computer simulations. Of particular interest are tradeoffs revealed among system parameters, such as maximum allowed power control error versus the number of antennas used for spatial diversity.**

***Index Terms*—Antenna arrays, cellular CDMA, fading power control.**

I. Introduction

SYSTEMS utilizing code-division multiple access (CDMA) are currently being deployed around the country and around the world in response to the ever increasing demand for cellular/personal communications services. Extensive research has been published on the performance analysis of CDMA systems. Fading is among the major factors affecting the performance of such systems. Fading is generally characterized according to its effect over a geographical area. Large-scale fading consists of path loss and shadowing, the latter term referring to fluctuations in the received signal mean power. Large-scale fading is affected by prominent terrain contours between the transmitter and receiver. Small-scale fading is the common reference to the rapid changes in signal amplitude and phase over a small spatial separation. In this work, the combined effect of large- and small-scale fading are considered. The small-scale fading is assumed to be governed by the Rayleigh distribution (Rayleigh fading).

Paper approved by Z. Kostic, the Editor for Wireless Communication of the IEEE Communications Society. Manuscript received June 5, 1998; revised December 29, 1998 and October 4, 1999. This work was supported in part by the Air Force Office of Scientific Research under Grant F49620-97-1-0241.

W. Ye is with Lucent Technologies, Whippany, NJ 07981 USA.

A. M. Haimovich is with the Center for Communications and Signal Processing Research, Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102 USA (e-mail: haimovic@njit.edu).

Publisher Item Identifier S 0090-6778(00)06156-0.

Besides fading, CDMA systems are susceptible to the *near–far* problem. It is well known that owing to the near–far problem, power control is an important system requirement in current design CDMA. The CDMA system capacity (defined as the number of users that can access the system simultaneously) is maximized if each mobile transmitter power level is controlled so that its signal arrives at the base station with the minimum required signal-to-interference ratio (SIR) [1]. Power control loops are designed to compensate for large-scale fading effects, but amplitude variations due to small-scale fading are too rapid to be tracked. Power control circuits, however, have finite accuracy, which implies that CDMA systems are still faced with residual shadowing effects. The signal received at the base station from a power-controlled user can be modeled as governed by the log-normal distribution [2], [3]. The standard deviation of the received signal power is defined as the *power control error* (PCE) and is typically of the order of 1.5–2.5 dB. The same fading that affects the desired user's signal also affects signals from other users which serve as cochannel interference (CCI) to the desired user. Thus, the resultant CCI power can be modeled as the power sum of multiple log-normal contributions. This observation serves as the starting point for the performance analysis of CDMA systems in fading channels. The computation of the distribution of a sum of log-normal variates has been examined in several publications [4]–[6]. A closed-form solution for the distribution is not known; the common method used in these publications is to approximate the sum of log-normal variates as another log-normal variate. This approach is justified by a theorem proved by Marlow that states that under very general conditions, the power sum of independent variates is asymptotically normally distributed [7]. When the number of variates is finite, the power sum of these variates does not strictly follow normal distribution. But the approximation to normal is often used, and its validity was evaluated in [4] and [6]. Schwartz and Yeh developed a technique for the evaluation of the mean and variance of power sums of independent log-normal components [4]. In [5], Schwartz and Yeh's approach was extended to the case of correlated log-normal variates. In Wilkinson's approach, introduced in [4], the power sum of log-normal variates is taken to be normal with a distribution determined by the first and second moments of the sum. Several approaches that can be used to compute the distribution of a sum of log-normal variates were compared in [6]. Therein, it is shown that among the methods considered, Wilkinson's approach provides the most accurate results.

While large-scale fading effects are compensated by power control, small-scale fading can be mitigated by space–time diversity provided by an adaptive antenna array along with a Rake receiver. In addition to fading, CDMA systems are adversely affected by CCI. The effect of CCI needs to be taken into account in any performance analysis of CDMA systems. On two-way telephone connections, measurements have established that a typical voice signal is active less than 50% of the time. Interference in CDMA systems is reduced by providing voice activity gating. This technique involves the monitoring of voice activity such that each transmitter is switched off during periods of no voice activity, thus reducing CCI. A performance analysis needs to take voice gating into account.

Due to the complexity of the mobile communications scenario, most published results account for only some of the adverse effects that were mentioned above. For example, the performance analysis related to the log-normal interference can be found in [6] and [8]–[12], however, the cited work does not consider small-scale fading effects. The performance analysis with both fading and shadowing is considered in several recent publications [13]–[18]. However, results are published either as simulations or as approximations, which generally lack accuracy at low PCE. For accurate predictions of CDMA systems performance, it is of great interest to be able to develop closed-form expressions that simultaneously incorporate the effects of shadowing, PCE, Rayleigh fading, voice activity and space–time processing. This work attempts to provide more complete answers to the reverse-link performance of wireless CDMA, and to develop expressions for the outage probability and probability of bit error, while taking into account multiple effects. The present work also extends [19] in several ways: it analyzes the outage based on the *average* SIR; it considers approximations for the bit-error rate; it considers various extensions such as correlations among users; time diversity; and pilot-aided coherent detection.

The paper is organized as follows. The system model is given in Section II. Section III contains the main theoretical results including outage probability, probability of bit error, and some extensions for more general cases. Numerical results are presented in Section IV, and our conclusions are given in Section V.

II. System Model

The system model represents the reverse link of a single-cell CDMA system which serves K_u users, and has a base station with an M-element antenna array. The received signals are assumed to undergo independent, flat Rayleigh fading. It is further assumed that the fading is slowly varying, such that the low-pass equivalent channel seen by each antenna can be characterized by a complex-valued scalar. The system is assumed interference-limited with negligible thermal noise. The CDMA reverse-link receiver model is shown in Fig. 1. The complex envelope of the signal received at the base station is then expressed by the M-dimensional vector

$$\mathbf{x}(t) = \sqrt{\lambda_1}\, m_1(t-\tau_1)u_1(t-\tau_1)\mathbf{c}_1 + \sum_{k=2}^{K_u} \epsilon_k \sqrt{\lambda_k}\, m_k(t-\tau_k)u_k(t-\tau_k)\mathbf{c}_k \quad (1)$$

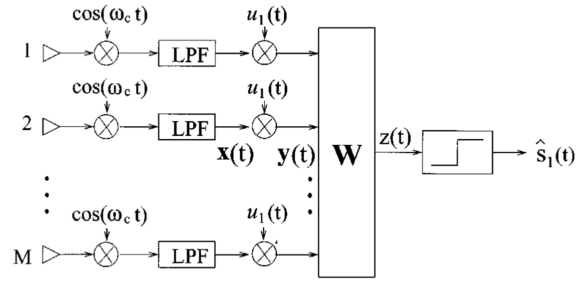

Fig. 1. CDMA reverse-link receiver.

where the first and second terms, respectively, represent the desired signal and the CCI, λ_k, $k = 1, \cdots, K_u$, are the powers of the received signals, $\mathbf{c}_k$ are normalized complex Gaussian channel vectors with $E[\mathbf{c}_k\mathbf{c}_k^H] = \mathbf{I}$, $\mathbf{I}$ is the $M \times M$ identity matrix, the superscript denotes transpose and complex conjugate, $m_k(t)$ are nonreturn to zero waveforms of the users' data, $u_k(t)$ are the spreading sequences, ϵ_k are binary random variables indicating the users' voice activity, τ_k are the users' delays. Let $m_k(t) = \sum_i s_k(i)h(t-iT_s)$, where $h(t)$ is the basic pulse shape, T_s is the symbol interval, i is the symbol interval index, and $s_k(i) \in \{-1, 1\}$ are the users' binary data. It is assumed that $E[s_k(i)] = 0$ and $E[s_k(i)s_l(j)] = \delta_{kl}\delta_{ij}$, where $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ otherwise. The signature waveforms are normalized to unit energy over the symbol interval. For convenience, $\tau_1 = 0$. In a system that provides a single service (such as voice) with the same bit-error rate and with perfect power control, all λ_k's are equal. The received powers λ_k, $k = 1, \cdots, K_u$, are the result of path loss, shadowing, and imperfect power control, and are modeled as independent, identically distributed (i.i.d.) random variables with log-normal distribution. If λ_k has a log-normal distribution, then the received power expressed in decibels, $\alpha_k = 10\log_{10}\lambda_k$, has a normal distribution with mean m_α and variance σ_α^2. The standard deviation of α_k is the PCE measured in decibels. Since $\alpha_k < m_\alpha$ with probability $1/2$, $10^{m_\alpha/10}$ is the median value of λ_k. The voice activity ϵ_k is modeled as a Bernoulli (p) random variable with $\Pr(\epsilon_k = 1) = p$, where p is the *voice activity factor*.

Following spread spectrum demodulation and sampling at the symbol interval, the received signal can be written

$$\mathbf{y}(i) = \int_{iT_s}^{(i+1)T_s} \mathbf{x}(t)u_1(t)\,dt = \sqrt{\lambda_1}s_1(i)\mathbf{c}_1 + \sum_{k=2}^{K_u}\epsilon_k\sqrt{\lambda_k}\left(s_k(i-1)\rho_k^- + s_k(i)\rho_k^+\right)\mathbf{c}_k \quad (2)$$

where

$$\rho_k^- = \int_{iT_s}^{iT_s+\tau_k} u_k(t-\tau_k)u_1(t)\,dt$$
$$\rho_k^+ = \int_{iT_s+\tau_k}^{(i+1)T_s} u_k(t-\tau_k)u_1(t)\,dt$$

are the correlations between user signatures. It is assumed that the correlations above are independent of the symbol interval index i. If the CCI is spatially white, the optimum output SIR is provided by maximal-ratio combining (MRC). The array weight vector $\mathbf{w}$ acts then as a channel matched filter, $\mathbf{w} = \mathbf{c}_1$. The array output is expressed

$$z(i) = \mathbf{w}^H \mathbf{y}(i) = \mathbf{c}_1^H \mathbf{y}(i) = \phi_s(i) + \phi_j(i) \tag{3}$$

where

$$\phi_s(i) = \sqrt{\lambda_1} s_1(i) \mathbf{c}_1^H \mathbf{c}_1 \tag{4}$$

and

$$\phi_j(i) = \mathbf{c}_1^H \sum_{k=2}^{K_u} \epsilon_k \sqrt{\lambda_k} (s_k(i-1)\rho_k^- + s_k(i)\rho_k^+) \mathbf{c}_k \tag{5}$$

are the desired signal and CCI, respectively, at the array output. Over the duration of a bit, it is assumed that the interference can be approximated by an equivalent source using the following expression:

$$\phi_j(i) = s(i) \mathbf{c}_1^H \sum_{k=2}^{K_u} \epsilon_k \sqrt{\eta_k \lambda_k} \mathbf{c}_k \tag{6}$$

where $s(i)$ combines the total interference bit effects during the bit duration and η_k is a gain factor incorporating the effect of cross correlation with the desired user's signal. This model is a worst case of sorts, in which interference sources are not independent, however, it has the advantage of being analytically tractable (see also [20]). The instantaneous output SIR is then written

$$\gamma = \frac{|\phi_s(i)|^2}{|\phi_j(i)|^2}. \tag{7}$$

III. Analysis of PCE and Fading Effects

In this section, relations are established for performance measures such as outage and bit-error rate, as a function of PCE, Rayleigh fading, and voice activity.

A. Computation of Outage Probability

The outage probability provides an indicator of how often the communication link's quality is under a specified acceptable level. The system capacity is generally computed for a prescribed outage level. The outage with respect to the *instantaneous* SIR was studied in [19]. In some cases (for example, when the mobiles transmit voice rather than data), it is more suitable to consider the outage based on the *average* SIR (the averaging is over the Rayleigh fading).

Let the outage be defined as the probability that the average output SIR, γ_E, falls below a prescribed threshold γ_{th}, $P_{oE} = \Pr(\gamma_E < \gamma_{\text{th}})$, where the *average* SIR γ_E is given by

$$\gamma_E = E\left[\frac{|\phi_s(i)|^2}{|\phi_j(i)|^2}\right]. \tag{8}$$

For maximal-ratio combining, the average output SIR

$$\gamma_E = \sum_{m=1}^{M} \mu_m \tag{9}$$

where μ_m is the single-element input SIR. In our system model, the input SIR is assumed to be equal at all elements, $\mu_m = \mu$, then $\gamma_E = M\ \mu$, where μ is defined by

$$\mu = \frac{\lambda}{\mathcal{J}} = \frac{\lambda}{\sum_{k=2}^{K_u} \epsilon_k \eta_k \lambda_k}. \tag{10}$$

It is noted that the *average* SIR is conditioned on the realization of μ. For full characterization of γ_E, it is necessary to determine the density of μ. Since $\epsilon_k \in \{0, 1\}$, $\mathcal{J}$ is the sum of log-normal random variables. The number of elements in the sum is $\sum_{k=2}^{K_u} \epsilon_k$. Following Wilkinson's method [4], [6], $\mathcal{J}$ is approximated by a log-normal random variable; it is then proceeded to match $E[\mathcal{J}]$ and $E[\mathcal{J}^2]$ with the corresponding cumulants of the log-normal distribution. In [19], expressions are found for the mean m_g and variance σ_g of the normal variate $g = \ln \mu$. It follows that the outage probability can be written:

$$P_{oE} = \Pr\left(g < \ln\frac{\gamma_{\text{th}}}{M}\right). \tag{11}$$

Since g is normally distributed, (11) can be expressed

$$P_{oE} = 1 - Q\left(\frac{\ln\dfrac{\gamma_{\text{th}}}{M} - m_g}{\sigma_g}\right) \tag{12}$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

is the Gaussian tail function.

Relation (12) provides the closed-form computation of the *average* SIR outage probability as a function of Rayleigh fading, PCE, and voice activity. When the communication link's quality is sensitive to the *instantaneous* SIR, the result in [19] should be used to evaluate the outage probability.

B. Computation of the Probability of Bit Error

Computation of the bit error requires knowledge of the distribution of the interference at the array output. Due to dissimilar shadowing and fading affecting the various users, interferers are not identically distributed, hence, the central limit theorem cannot be strictly invoked to claim the Gaussian property. Nevertheless, the Gaussian property is often assumed in such analyses [13], [16], [21], [22]. In this paper, the Gaussian assumption is validated by a chi-square test presented in the next section.

For binary phase-shift keying (BPSK) and Gaussian interference, the conditional bit-error probability (BEP) is given by

$$b = P(e \mid \gamma) = Q(\sqrt{2\gamma}) \tag{13}$$

where γ is the *instantaneous* SIR and is given by

$$\gamma = \frac{\lambda_1 |\mathbf{c}_1^H \mathbf{c}_1|^2}{\left|\mathbf{c}_1^H \sum_{k=2}^{K_u} \epsilon_k \sqrt{\eta_k \lambda_k} \mathbf{c}_k\right|^2}. \tag{14}$$

The variate γ is a function of Rayleigh fading, shadowing (PCE), and voice activity. The distribution of γ is required to determine the average probability of bit error. The conditional density of γ was found in [23], [20]

$$f_\gamma(\gamma \mid \mu) = \frac{M(\gamma/\mu)^{M-1}}{\mu(1+\gamma/\mu)^{M+1}}. \tag{15}$$

where μ was defined in (10). The BEP b is a function of the instantaneous SIR γ, hence a random variable. The density of b can be found from [24, p. 125]

$$f_b(b \mid \mu) = \left. \frac{f_\gamma(\gamma \mid \mu)}{|db/d\gamma|} \right|_{\gamma=(1/2)[Q^{-1}(b)]^2} \tag{16}$$

where Q^{-1} is the inverse function of Q. After a few manipulations, it can be shown that

$$\frac{db}{d\gamma} = -\frac{1}{2\sqrt{\gamma\pi}}e^{-\gamma}. \tag{17}$$

Substituting (17) into (16), one has the conditional density of b

$$f_b(b \mid \mu) = \left. \frac{M(\gamma/\mu)^{M-1}}{\mu(1+\gamma/\mu)^{M+1}} 2\sqrt{\gamma\pi}e^{\gamma} \right|_{\gamma=(1/2)[Q^{-1}(b)]^2}. \tag{18}$$

The function Q^{-1} can be evaluated by numerical integration. The unconditional density of b can be obtained by averaging over the distribution of μ

$$f_b(b) = \int_0^\infty f_b(b \mid \mu) f_\mu(\mu)\, d\mu. \tag{19}$$

Replacing μ by $\mu = e^g$, one gets

$$f_b(b) = \int_0^\infty f_b(b \mid e^g) f_g(g) e^g\, dg = E_g[G(b,g)] \tag{20}$$

where $f_g(g) = f_\mu(g)/|dg/d\mu|_{\mu=e^g}$ and $G(b,g) = f_b(b|e^g)e^g$. Since μ has a log-normal distribution, $g = \ln \mu$ is normally distributed. An exact closed-form result for the integral in (20) is not available, however, an approximation exists for $E_g[G(b,g)]$ when g has a normal distribution. The approximation is valid for an arbitrary probability function $G(b,g)$ and is expressed in terms of the mean m_g and the standard deviation σ_g [25]

$$f_b(b) \cong \frac{2}{3} G(b, m_g) + \frac{1}{6} G(b, m_g + \sqrt{3}\sigma_g) + \frac{1}{6} G(b, m_g - \sqrt{3}\sigma_g). \tag{21}$$

The computation of the mean and variance of g, m_g, and σ_g, respectively, is discussed in [19], thus, the previous relation expresses $f_b(b)$ in terms of known quantities. To reiterate, $f_b(b)$ accounts for the effects of Rayleigh fading, shadowing (PCE), and voice activity. The BEP density function in (21) is a more complete characterization of system performance than the more common average BEP. The latter can be obtained by using the density of b, or from the following argument. The conditional BEP can be expressed as

$$\begin{aligned} P(e \mid \mu) &= \int_0^\infty P(e \mid \gamma) f_\gamma(\gamma \mid \mu)\, d\gamma \\ &= M\mu \int_0^\infty Q(\sqrt{2\gamma})\, \frac{\gamma^{M-1}}{(\mu+\gamma)^{M+1}} d\gamma. \end{aligned} \tag{22}$$

Following [26], (22) can be expressed utilizing hypergeometric functions

$$\begin{aligned} P(e \mid \mu) = \frac{M}{2\Gamma(M+1)} & [2\mu\Gamma(M+1) \\ & \cdot {}_2F_2(M+1, 1; 3/2, 2; \mu) - 2\sqrt{\mu}\Gamma(M+1/2) \\ & \cdot {}_2F_2(M+1/2, 1/2; 1/2, 3/2; \mu) + \Gamma(M)] \end{aligned} \tag{23}$$

where ${}_2F_2(\cdot)$ is the standard generalized hypergeometric function [27]. Expression (23) can be evaluated numerically by using software packages such as Maple, Mathematica, etc. Alternatively, (22) can be evaluated numerically. The unconditional BEP is found by averaging $P(e \mid \mu)$ over the distribution of μ

$$P_e = \int_0^\infty P(e \mid \mu) f_\mu(\mu)\, d\mu. \tag{24}$$

In terms of the normal random variable g, $g = \ln \mu$, one gets

$$P_e = \int_{-\infty}^\infty P(e \mid g) f_g(g)\, dg = E_g[P(e \mid g)]. \tag{25}$$

Finally, using the same approach as in (21), one can obtain the average probability of bit error

$$\begin{aligned} P_e &= E_g[P(e \mid g)] \\ &= E_g[H(g)] \\ &\cong \frac{2}{3}H(e^{m_g}) + \frac{1}{6}H(e^{m_g+\sqrt{3}\sigma_g}) + \frac{1}{6}H(e^{m_g-\sqrt{3}\sigma_g}) \end{aligned} \tag{26}$$

where we introduced $H(g) = P(e \mid g)$ as a notational convenience.

C. Extensions of Previous Results

Some of the results obtained previously can be extended to more general cases. These include: other cell interference, correlation of shadowing among users, channel with time dispersion, pilot tone effect, and performance analysis in terms of Erlang capacity. These effects are discussed below.

1) Other Cell Interference: CCI is caused by intercell interference as well as intracell interference. This is particularly true for CDMA, for which the reuse factor is one. The other cell interference was studied in [28] and [3]. If the same traffic load is assumed in all cells, the effect of CCI introduced by users of all other cells is equivalent to the effect of CCI from qK_u users of the home cell, where q is a factor that is determined empirically. With $K'_u \triangleq K_u(1+q)$, all the results developed so far apply with K_u replaced by K'_u.

2) Correlation of Shadowing Among Users: The analysis of the outage probability assumed independent shadowing among users. In practice, when signals received from different users are shadowed by the same obstacles in the vicinity of the base station, the shadowing affecting the users may be correlated. This requires some modifications in the computation of the quantities m_g and σ_g used in determining the outage probability P_{oE}. In [19], it is shown that the normal variate g (ln of the input SIR at each antenna element), is expressed in terms of the first and second moments of the interference power $\mathcal{J}$, $E[\mathcal{J}]$, and $E[\mathcal{J}^2]$, respectively. The computation of $E[\mathcal{J}]$, as shown in

[19], is not affected by the correlation assumption. However, in the presence of correlation, the computation of $E[\mathcal{J}^2]$ is different than it is presented in [19]. We proceed now with this computation. By assumption, the power of the kth user λ_k, has a log-normal distribution, hence $\alpha_k = \ln \lambda_k$ has a normal distribution with some mean m_α and some variance σ_α^2. The shadowing correlation coefficient is defined as

$$r_{kj} = \frac{E[(\alpha_k - m_\alpha)(\alpha_j - m_\alpha)]}{\sigma_\alpha^2} \tag{27}$$

where α_k and α_j are assumed identically distributed. For simplicity, we assume $r_{kj} = r$ for $k \neq j$ and $k, j = 1, \cdots, K_u$. We have

$$\begin{aligned} E[\mathcal{J}^2] &= E\left[\left(\sum_{k=2}^{K_u} \epsilon_k \eta_k e^{\alpha_k}\right)^2\right] \\ &= \sum_{k=2}^{K_u} \eta_k^2 E[\epsilon_k^2 e^{2\alpha_k}] + 2\sum_{k=2}^{K_u} \sum_{j=k+1}^{K_u} \eta_k \eta_j E[\epsilon_k e^{\alpha_k} \epsilon_j e^{\alpha_j}] \\ &= pe^{2m_\alpha + 2\sigma_\alpha^2} \sum_{k=2}^{K_u} \eta_k^2 + p^2 e^{2m_\alpha} e^{\sigma_\alpha^2 (1+r)} \sum_{k=2}^{K_u} \sum_{j\neq k} \eta_k \eta_j \\ &\equiv e^{\xi_2} \end{aligned} \tag{28}$$

where p is the voice activity factor defined earlier. Using terminology from [19], the interference mean is expressed $E[\mathcal{J}] = e^{\xi_1}$. Letting $\mathcal{J} = e^\beta$, and since $\mathcal{J}$ is log-normal (hence β is normal), we also have

$$E[\mathcal{J}] = E[e^\beta] = e^{m_\beta + \sigma_\beta^2/2} = e^{\xi_1} \tag{29}$$

$$E[\mathcal{J}^2] = E[e^{2\beta}] = e^{2m_\beta + 2\sigma_\beta^2} = e^{\xi_2}. \tag{30}$$

Solving for m_β and σ_β^2, we obtain

$$m_\beta = 2\xi_1 - \frac{1}{2}\xi_2 \tag{31}$$

$$\sigma_\beta^2 = \xi_2 - 2\xi_1. \tag{32}$$

Now, from (10) and the log-normality of λ_1 and $\mathcal{J}$, it follows that $\mu = \lambda_1/\mathcal{J}$ is also log-normal. We have

$$g = \ln \mu = \ln \frac{\lambda_1}{\mathcal{J}} = \alpha_1 - \beta. \tag{33}$$

The density function of μ is determined from the mean and variance of g, which can be expressed in terms of known quantities

$$m_g = m_\alpha - m_\beta \tag{34}$$

$$\sigma_g^2 = \sigma_\alpha^2 + \sigma_\beta^2 - 2r_{\alpha\beta}\sigma_\alpha\sigma_\beta \tag{35}$$

where $r_{\alpha\beta}$ is the correlation efficient between α_1 and β. Once m_g and σ_g^2 were obtained, the computation of P_{oE} can be completed as in the uncorrelated case using (12).

The results above can be extended to the general case when r_{kj} are not equal, and α_k have different mean values and variances (see [5] and [6] for more details about the expression of e^{ξ_2} and calculation of $r_{\alpha\beta}$).

3) Frequency-Selective Channel: The reverse-link channel is assumed frequency-selective with L resolvable paths. A Rake receiver is used to track and combine the paths. Following spread-spectrum demodulation, the signal received at the antenna array from the lth path can be written as an M-dimensional vector

$$\begin{aligned} \mathbf{y}_l(i) &= \sqrt{\lambda_{1l}} s_1(i)\mathbf{c}_{1l} + \sum_{\substack{n=1 \\ n\neq l}}^{L} \sqrt{\lambda_{1n}} \\ &\quad \cdot (s_1(i-1)\rho_{1ln}^- + s_1(i)\rho_{1ln}^+)\mathbf{c}_{1n} \\ &\quad + \sum_{k=2}^{K_u} \sum_{n=1}^{L} \epsilon_k \sqrt{\lambda_{kn}} (s_k(i-1)\rho_{kln}^- + s_k(i)\rho_{kln}^+)\mathbf{c}_{kn}, \\ &\qquad l = 1, \cdots, L \end{aligned} \tag{36}$$

where $k = 1, \cdots, K_u$ is the user index, $n = 1, \cdots, L$ is the path index, λ_{kn} are the received signal powers, $\mathbf{c}_{kn}$ are the channel vectors

$$\rho_{kln}^- = \int_{iT_s}^{iT_s + \tau_{kn}} u_k(t - \tau_{kn}) u_1(t - \tau_{1l})\, dt$$

$$\rho_{kln}^+ = \int_{iT_s + \tau_{kn}}^{(i+1)T_s} u_k(t - \tau_{kn}) u_1(t - \tau_{1l})\, dt$$

τ_{kl} and τ_{kn} are the delays. Assume that the cross correlations are independent of the path l, i.e., $\rho_{kln}^{-/+} = \rho_{kn}^{-/+}$. The various terms in (36) represent the desired signal, self-interference, and CCI, respectively. In the following, it is assumed that the self-interference contribution is negligible in comparison with the CCI. Signal vectors associated with the different paths, $\mathbf{y}_l(i)$, $l = 1, \cdots, L$, are stacked to form an ML-dimensional vector, $\mathbf{y}(i)$, and grouped according to components related to the desired signal and CCI, yielding the expression (see [29] for details)

$$\mathbf{y}(i) = \sqrt{\lambda_1} s_1(i)\mathbf{c}_1 + \mathbf{j}(i) \tag{37}$$

where $\mathbf{y}(i) = [\mathbf{y}_1^T(i), \cdots, \mathbf{y}_L^T(i)]^T$, $\mathbf{c}_1 = [\mathbf{c}_{11}^T, \cdots, \mathbf{c}_{1L}^T]^T$. The first term in the relation above represents the desired signal, $\mathbf{j}(i)$ is the interference, the superscript "T" denotes transpose. With MRC in both space (antenna array) and time (Rake) domains, the output is equivalent to applying MRC to the stacked vector in (37). The MRC weight vector is then given by $\mathbf{w} = \mathbf{c}_1$. Similar to the approach taken earlier, it is assumed that the interference can be expressed as an equivalent source

$$\mathbf{j}(i) = s(i) \sum_{k=2}^{K_u} \sum_{n=1}^{L} \epsilon_k \sqrt{\eta_{kn}\lambda_{kn}} \mathbf{c}_{kn} \tag{38}$$

where $s(i)$ is the CCI source bit, η_{kn} is a gain factor representing the cross correlation between codes. The double sum over the scaled ML-dimensional Gaussian distributed vectors $\mathbf{c}_{kn}$ is equivalent to another Gaussian vector

$$\sum_{k=2}^{K_u} \sum_{n=1}^{L} \epsilon_k \sqrt{\eta_{kn}\lambda_{kn}} \mathbf{c}_{kn} = \sqrt{\mathcal{J}} \mathbf{c}_p \tag{39}$$

where

$$\mathcal{J} = \sum_{k=2}^{K_u} \sum_{n=1}^{L} \epsilon_k \eta_{kn} \lambda_{kn} = \sum_{k=2}^{K_u} \epsilon_k \nu_k \lambda_k \tag{40}$$

and $\nu_k = \sum_{n=1}^{L} \eta_{kn}$. The expression in (40) is similar in form to (38), thus, the evaluation of the outage probability can continue as in the flat channel case. The distribution of the output SIR is obtained from (15), by substituting M with ML, to account for the additional diversity paths provided by the frequency-selective channel

$$f_\gamma(\gamma \mid \mu) = \frac{ML(\gamma/\mu)^{ML-1}}{\mu(1+\gamma/\mu)^{ML+1}}. \tag{41}$$

All other results in the section hold by substituting M with ML.

4) Pilot-Aided Coherent Detection: In this case, the reverse link is assumed with pilot-aided coherent detection and perfect channel estimation. The power-split ratio for the pilot is r_p. Subsequently, the fraction of the total transmitted power that is used for the information traffic is $1/(1+r_p)$. With this model, the instantaneous output SIR is given by $\gamma' = \kappa\gamma$, where $\kappa = 1/(1+r_p)$, and γ was given in (14). Therefore, distribution of the output SIR γ' is given by

$$f_{\gamma'}(\gamma' \mid \mu) = \frac{M(\gamma'/\kappa\mu)^{M-1}}{\kappa\mu(1+\gamma'/\kappa\mu)^{M+1}} \tag{42}$$

and P_{oE} and P_e can be modified accordingly. In particular, the outage probability is given by

$$P_{oE} = 1 - Q\left(\frac{\ln \dfrac{\gamma_{\rm th}}{\kappa M} - m_g}{\sigma_g}\right). \tag{43}$$

5) Performance in Terms of Erlang Capacity: For a communication system, capacity may be measured in terms of the number of users per cell, or in terms of offered traffic intensity in Erlangs. Wireless system can be modeled as having Poisson traffic arrival, exponentially distributed service time, finite number of servers, and no waiting room (expressed as an $M/M/N/N$ queue). The system capacity of frequency-division multiple access (FDMA) or time-division multiple access (TDMA) is obtained by analyzing the blocking probability of an $M/M/N/N$ queue. Since users in CDMA systems all share a common spectral frequency band, the blocking condition of a CDMA system could be defined in a different way from that of FDMA or TDMA [8]. The blocking condition in CDMA systems will be defined as the case when the average SIR at the base station falls below a prescribed level. This is the same definition as the outage referred to earlier in this section. For a finite number of communication channels, Poisson distribution cannot strictly represent the number of active users per cell. Nevertheless, the approximation of Poisson distribution is often used in the analysis [8], [3], [30]. Then, the distribution of the number of active users in the system is given by

$$P[K_u = k] = \frac{\zeta^k}{k!}e^{-\zeta}, \qquad k = 0, 1, 2, \cdots \tag{44}$$

where ζ is determined by both call arrival rate and service rate. The mean value and variance of K_u are given by $\zeta = E[K_u] = \mathrm{var}[K_u]$. For simplicity, we assume that the factors η_k, defined following (6), are equal, $\eta_k = \eta$. The first and second moment of the interference $\mathcal{J}$ are to be averaged over K_u. From (38), and with $\lambda_k = e^{\alpha_k}$, we have

$$\begin{aligned} E[\mathcal{J}] &= E_{K_u}\left\{E_{\epsilon_k,\alpha_k}\left[\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{\alpha_k}\right]\right\} \\ &= (\zeta-1)p\eta e^{m_\alpha+\sigma_\alpha^2/2} \\ &\equiv e^{\xi_1} \end{aligned} \tag{45}$$

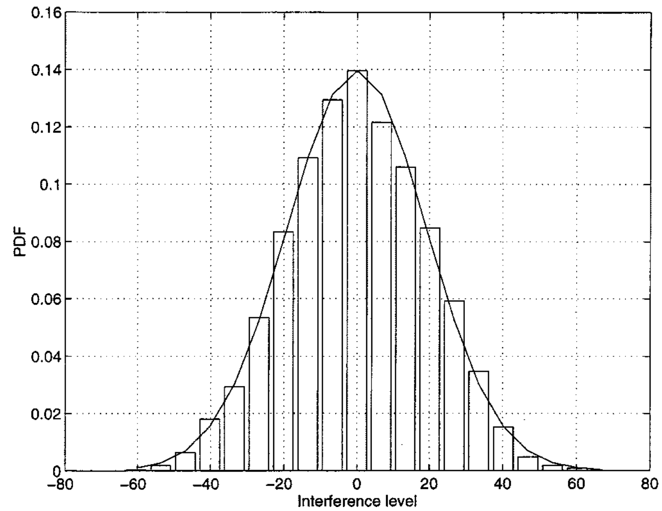

Fig. 2. Gaussian fit to the interference histogram.

and

$$\begin{aligned} E[\mathcal{J}^2] &= E_{K_u}\left\{E_{\epsilon_k,\alpha_k}\left[\left(\sum_{k=2}^{K_u}\epsilon_k\eta_k e^{\alpha_k}\right)^2\right]\right\} \\ &= (\zeta-1)p\eta^2 e^{2m_\alpha+2\sigma_\alpha^2} \\ &\quad + (\zeta^2-2\zeta+2)p^2\eta^2 e^{2m_\alpha+\sigma_\alpha^2} \\ &\equiv e^{\xi_2}. \end{aligned} \tag{46}$$

Using these expressions, we can proceed to compute the relation between outage P_{oE} and Erlang capacity (ζ). The parameter ζ is amended to $\zeta(1+q)$ for $K_u' = K_u(1+q)$, if taking into account the other cell interference.

IV. Numerical Results

Results in this section are derived from computer simulations of a CDMA system employing BPSK modulation and BPSK spreading, with a voice activity factor of $p = 3/8$ and a spreading ratio of 85. The spreading ratio corresponds to an information data rate of 14.4 kb/s and a signal bandwidth of 1.23 MHz. Unless specified otherwise, the number of antenna elements assumed in the simulations was $M = 4$. The channel was assumed flat and subject to Rayleigh fading and shadowing.

First, the validity of the Gaussian approximation for the CCI was evaluated for $K_u = 30$ users, and PCE = 1.5 dB. To that end, the histogram of the interference level was generated and compared to the theoretical Gaussian curve. This is shown in Fig. 2. Additionally, a chi-square test following the method presented in [24] was applied to evaluate the goodness of the fit. The sample space was partitioned into 21 disjoint intervals corresponding to a test with 20 degrees of freedom. Standard chi-square test tables show that for 20 degrees of freedom, the threshold for a 1% significance level is 37.57. Calculated from the simulation and averaged over 200 Monte–Carlo runs, the chi-square statistic D^2 was 22.14, which does not exceed the threshold. It is concluded that the Gaussian approximation is valid for the interference.

The outage probability with respect to the *average* SIR is plotted in Fig. 3 as a function of the capacity (number of

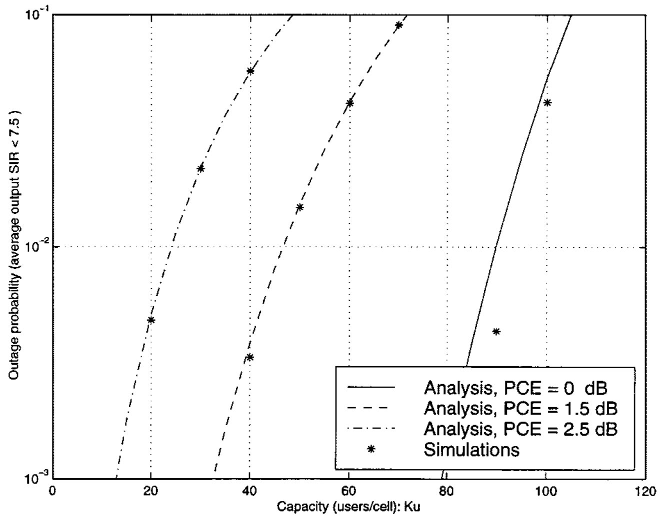

Fig. 3. Outage probability versus capacity (users/cell), based on *average* SIR, for four antenna elements $M = 4$, various PCE's, and $p = 3/8$.

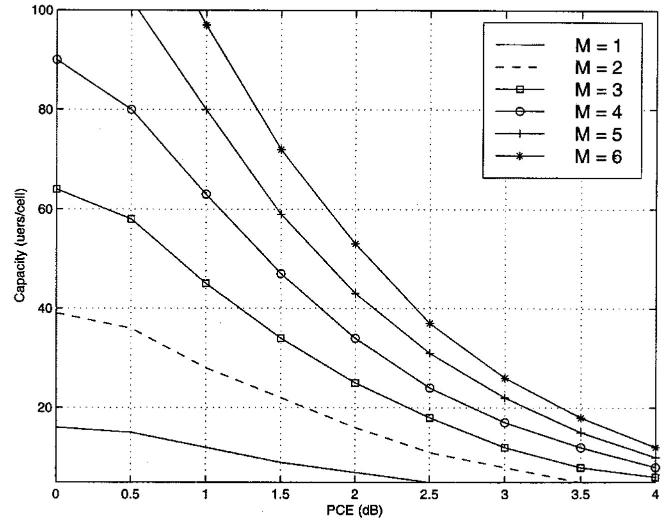

Fig. 5. Capacity (users/cell) versus PCE for $P_{oE}\ (\gamma_E < 7.5) = 0.01$. Analytical results for $M = 1$–6, and $p = 3/8$.

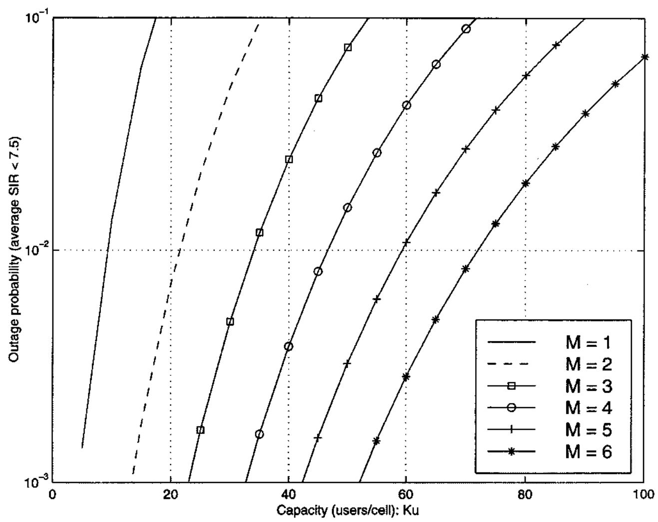

Fig. 4. Outage probability versus capacity (users/cell), based on *average* SIR. Analytical results for PCE = 1.5 dB, $M = 1$–6, and $p = 3/8$.

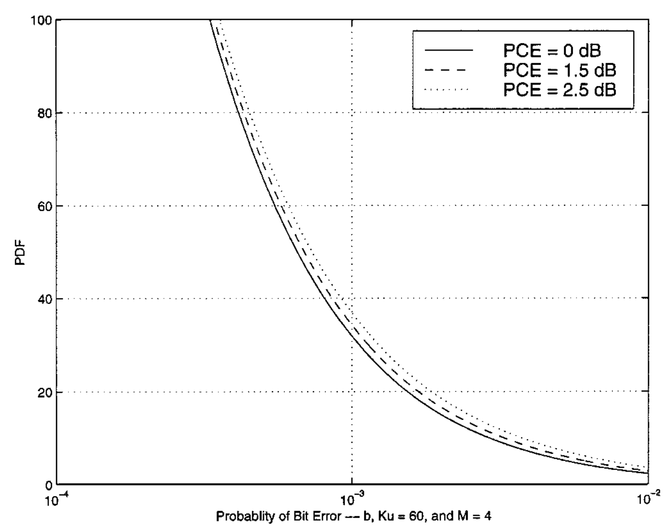

Fig. 6. PDF of probability of bit error with $K_u = 60$ and $M = 4$.

users/cell) with the PCE as a parameter. For two-antenna selective diversity at cell site, adequate reverse-link performance ($P_e < 10^{-3}$) is achievable with an array input of $\mathrm{SIR} < 5$ [9], which is equivalent to array output $\mathrm{SIR} < 7.5$. If we use the same array output SIR requirement for the receiver in Fig. 1, then the outage threshold is set at $\gamma_{\mathrm{th}} = 7.5$. The analytical curves in Fig. 3 are computed from (12), and the simulation curves are based on 1 000 000 samples. For an outage of 10^{-2}, the system capacity is approximately 90, 47, and 24 users/cell for PCE = 0, 1.5, and 2.5 dB, respectively. Consequently, for PCE = 1.5–2.5 dB, the system capacity degrades 48%–73% compared to the case of perfect power control. The effect of space diversity on the outage probability for *average* SIR is shown in Fig. 4. For an outage of $P_{oE} = 10^{-2}$, the system capacity is about 9–72 users/cell for $M = 1$–6, i.e., the average capacity increase for each additional degree of space diversity is about 13 users/cell. A clear illustration of the tradeoffs between the effects of antenna arrays and PCE can be found in Fig. 5. The figure shows the capacity (computed analytically for $P_{oE}(\gamma_E < 7.5) = 0.01$) as a function of PCE and the number of antenna elements. The figure shows that for capacity of 30 users/cell, a two-element receiver at the base station requires the PCE to be less than 1 dB, while a six-element receiver can relax this requirement to 2.8 dB. The figure can be used to find the system capacity for a given PCE and for different number of antenna elements. For example, when PCE = 2.5 dB, the system capacity increases from 10 users/cell to 38 users/cell for an increase in the number of antennas from $M = 2$–6. Clearly, these results however, do not take into account effects such as coding and interleaving.

Figs. 6 and 7 depict the distribution of the probability of bit error (b) with various values of PCE and number of antennas. These probability density function (pdf) curves shift toward to lower value of b as PCE decreases and the number of antennas increases. Fig. 8 displays the analytical results for the average probability of bit error as a function of the number of users per cell. The system parameters are the same as in Fig. 3. If the desired performance is $P_e = 10^{-3}$, capacity is, respectively, 55,

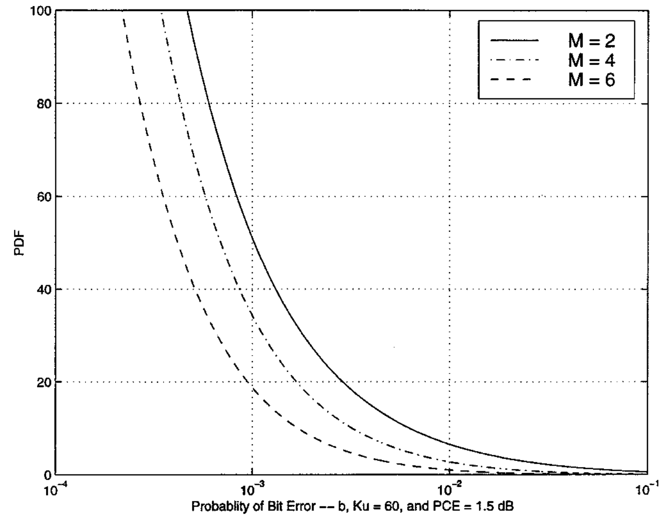

Fig. 7. PDF of probability of bit error with $K_u = 60$ and PCE $\text{PCE} = 1.5$ dB.

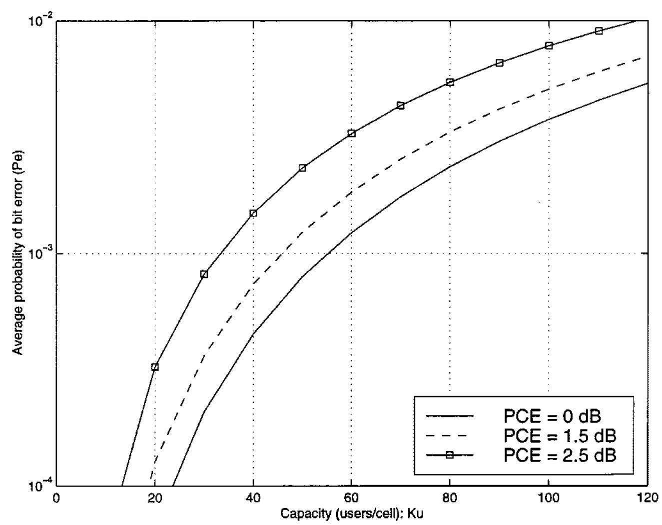

Fig. 8. Average probability of bit error versus capacity. Four antenna elements, $M = 4$, various PCE's, and $p = 3/8$.

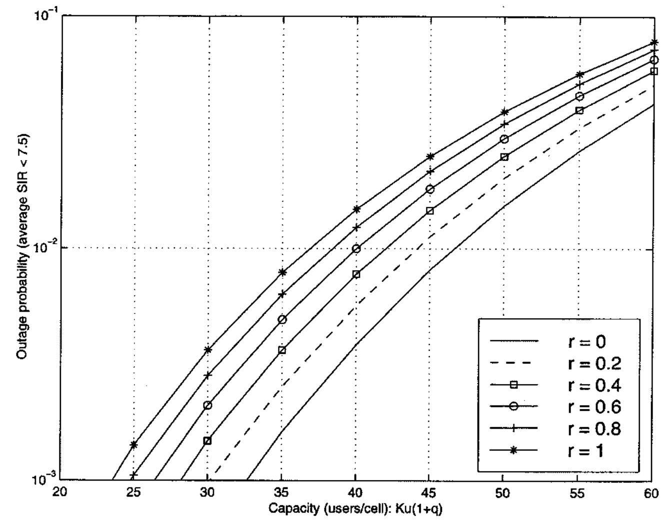

Fig. 9. Outage probability versus capacity (users/cell), based on *average* SIR. Four antenna elements $M = 4$, four resolvable paths $L = 4$, PCE = 1.5 dB, different values of correlation coefficient r, and $p = 3/8$.

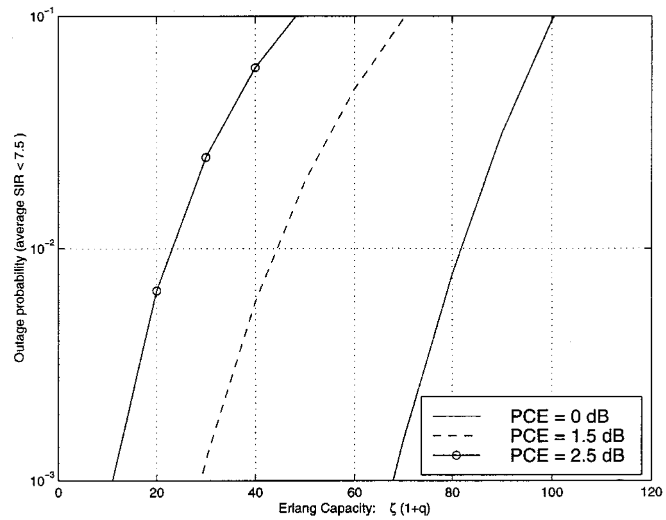

Fig. 10. Outage probability versus Erlang capacity, based on *average* SIR. Four antenna elements $M = 4$, one resolvable path $L = 1$, $r = 0$, different PCE's, and $p = 3/8$.

45, and 32 users/cell for PCE = 0, 1.5, and 2.5 dB. In a CDMA system with a PCE from 1.5 to 2.5 dB, the system capacity degrades from 18% to 42% compared to the case of perfect power control.

Finally, we examined some of the extensions discussed in the previous sections. Fig. 9 shows curves of outage probability versus capacity for PCE = 1.5 dB, $M = 4$ antennas, $L = 4$ time diversity paths, and different values of the correlation coefficient (r). For r from 0.2 to 1, the system capacity degrades from 6% to 21% compared to the case of uncorrelated shadowing ($r = 0$). Erlang capacity is shown in Fig. 10. The figure depicts curves of outage probability versus Erlang capacity, $\zeta(1+q)$, for $M = 4$ antennas, $L = 1$ resolvable path, $r = 0$, voice activity $p = 3/8$, and PCE = 0, 1.5, and 2.5 dB, respectively. This figure parallels the results of Fig. 3.

V. Conclusions

In this paper, we studied the reverse-link performance of cellular CDMA systems, with space–time processing, Rayleigh fading, shadowing, PCE, and voice activity gating. The performance was analyzed in terms of outage probability for average output SIR, as well as average probability of bit error. Analytical results were obtained that provide simple, but accurate approximations that can be used to evaluate system performance. All parameters needed for the computations can be obtained from measured data. The analysis shows that space–time processing provided by cell site antenna arrays along with a Rake receiver compensates for performance degradations due to PCE in cellular CDMA systems. Computer simulations provided a good match to the analytical expressions developed in the context. It is noticed that the exact system performance improvement due to adaptive antenna arrays varies with the fading environment and cell layout. Since the implementation of adaptive antenna arrays introduces more digital signal processing and larger time delays, the interaction among the adaptive antenna, coding, interleaver, call processing, etc., requires further evaluation via simulations and field tests.

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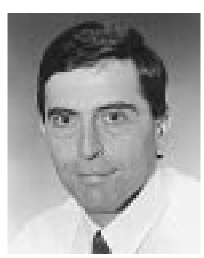
Weichen Ye (S'95–M'98) received the B.S. and M.S. degrees in electrical engineering from Shanghai Jiao Tong University, Shanghai, China, in 1988 and 1991, respectively, and the Ph.D. degree in electrical engineering from the New Jersey Institute of Technology, Newark, NJ, in 1998.

From 1991 to 1995, he worked for the Test and Measurement Organization of Hewlett-Packard Company in China. In 1998, he joined Network Wireless Systems of Lucent Technologies in Whippany, NJ. Currently, he is a Member of Technical Staff at Lucent CDMA Radio Technology Performance Group. His research interests are in the areas of communication theory, array signal processing, CDMA radio performance analysis and optimization, and power control and call processing algorithms in 2G and 3G CDMA systems.



Alexander M. Haimovich (M'82–SM'98) received the B.Sc. degree in electrical engineering from the Technion, Haifa, Israel, in 1977, the M.Sc. degree in electrical engineering from Drexel University, Philadelphia, PA, in 1983, and the Ph.D. degree in systems from the University of Pennsylvania, Philadelphia, in 1989.

He is an Associate Professor at the New Jersey Institute of Technology, Newark, NJ. His research interests include smart antennas, space–time coding, turbo-codng, ultra-wideband communications, and adaptive array processing for radar.